## C H A P T E R

## 33

## Leaming objectives TRANSFORMER: <br> > Three-phase Transformers <br>  <br> > Star/Star orY/Y

 Connection> Delta-Delta orConnection
> Wye/Delta orY/ Connection
> Delta/Wye or/Y Connection
> Open-Delta orV-V Connection
> PowerSupplied by V-V Bank
> Scott Connection or T-T Connection
> Three-phase to Two-Phase Conversion a nd vic e-versa
$>$ Parallel Operation of 3phase Transformers
> Instrument Transformers
$>$ Current Transformers
> Potential Transformers


Three phase transformers are used throughout industry to change values of three phase voltage and current. Three phase power is the most common way in which power is produced.

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### 33.1. Three-Phase Transformer

Large scale generation of electric power is usually 3-phase at generated voltages of 13.2 kV or somewhat higher. Transmission is generally accomplished at higher voltages of $110,132,275,400$ and 750 kV for which purpose 3-phase transformers are necessary to step up the generated voltage to that of the transmission line. Next, at load centres, the transmission voltages are reduced to distribution voltages of 6,600, 4,600 and 2,300 volts. Further, at most of the consumers, the distribution voltages are still reduced to utilization voltages of 440,220 or 110 volts. Years ago, it was a common


Three phase transformer inner circuits


Fig. 33.1
practice to use suitably interconnected three single-phase transformers instead of a single 3-phase transformer. But these days, the latter is gaining popularity because of improvement in design and manufacture but principally because of better acquaintance of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a 3 phase transformer are that it occupies less floor space for equal rating, weighs less, costs about $15 \%$ less and further, that only one unit is to be handled and connected.

Like single-phase transformers, the three-phase transformers are also of the core type or shell type. The basic principle of a 3-phase transformer is illustrated in Fig. 33.1 in which only primary windings have been shown interconnected in star and put across 3phase supply. The three cores are $120^{\circ}$ apart and their empty legs are shown in
contact with each other. The centre leg, formed by these three, carries the flux produced by the threephase currents $I_{R}, I_{Y}$ and $I_{B}$. As at any instant $I_{R}+I_{Y}$ $+I_{B}=0$, hence the sum of three fluxes is also zero. Therefore, it will make no difference if the common leg is removed. In that case any two legs will act as the return for the third just as in a 3-phase system any two conductors act as the return for the current


Fig. 33.2 (a)
in the third conductor. This improved design is shown in Fig. 33.2 (a) where dotted rectangles indicate the three windings and numbers in the cores and yokes represent the directions and magnitudes of fluxes at a particular instant. It will be seen that at any instant, the amount of 'up' flux in any leg is equal to the sum of 'down' fluxes in the other two legs. The core type transformers are usually wound with circular cylindrical coils.

In a similar way, three single-phase shell type transformers can be combined together to form a 3phase shell type unit as shown in Fig. 33.2(b). But some saving in iron can be achieved in


Fig. 33.2 (b)
Fig. 33.3


Single-Phase Transformer Cores
constructing a single 3-phase transformer as shown in Fig. 33.3. It does not differ from three singlephase transformers put side by side. Saving in iron is due to the joint use of the magnetic paths between the coils. The three phases, in this case, are more independent than they are in the core type transformers, because each phase has a magnetic circuit independent of the other.

One main drawback in a 3-phase transformer is that if any one phase becomes disabled, then the whole transformer has to be ordinarily removed from service for repairs (the shell type may be operated open
$\Delta$ or Vee but this is not always feasible). However, in the case of a 3-phase bank of single-phase transformers, if one transformer goes out of order, the system can still be run open- $\Delta$ at reduced capacity or the faulty transformer can be readily replaced by a single spare.

### 33.2. Three-phase Transformer Connections

There are various methods available for transforming 3-phase voltages to higher or lower 3-phase voltages i.e. for handling a considerable amount of power. The most common connections are (i) $Y-Y(i i) \Delta-\Delta($ iii $) Y-\Delta(i v) \Delta-Y(v)$ open-delta or $V-V(v i)$ Scott connection or $T-T$ connection.

### 33.3. Star/Star or $Y / Y$ Connection

This connection is most economical for small, high-voltage transformers because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only $1 /$ $\sqrt{3}$ of line voltage). In Fig. 33.4 a bank of 3 transformers connected in $Y$ on both the primary and the secondary sides is shown. The ratio of line voltages on the primary and secondary sides is the same as the transformation ratio of each transformer. However, there is a phase shift of $30^{\circ}$ between the phase voltages and line voltages both on the primary and secondary sides. Of course, line voltages on both sides as well as primary voltages are respectively in phase with each other. This connection works


Fig. 33.4
 satisfactorily only if the load is balanced. With the unbalanced load to the neutral, the neutral point shifts thereby making the three line-to-neutral (i.e. phase) voltages unequal. The effect of unbalanced loads can be illustrated by placing a single load between phase (or coil) $a$ and the neutral on the secondary side. The power to the load has to be supplied by primary phase (or coil) $A$. This primary coil $A$ cannot supply the required power because it is in series with primaries $B$ and $C$ whose secondaries are open. Under these conditions, the primary coils $B$ and $C$ act as very high impedances so that primary coil $A$ can obtain but very little current through them from the line. Hence, secondary coil $a$ cannot supply any appreciable power. In fact, a very low resistance approaching a short-circuit may be connected between point $a$ and the neutral and only a very small amount of current will flow. This, as said above, is due to the reduction of voltage $E_{\text {an }}$ because of neutral shift. In other words, under short-circuit conditions, the neutral is pulled too much towards coil $a$. This reduces $E_{\text {an }}$ but increases $E_{b n}$ and $E_{c n}$ (however line voltage $E_{A B}, E_{B C}$ and $E_{C A}$ are unaffected). On the primary side, $E_{A N}$ will be
practically reduced to zero whereas $E_{B N}$ and $E_{C N}$ will rise to nearly full primary line voltage. This difficulty of shifting (or floating) neutral can be obviated by connecting the primary neutral (shown dotted in the figure) back to the generator so that primary coil $A$ can take its required power from between its line and the neutral. It should be noted that if a single phase load is connected between the lines $a$ and $b$, there will be a similar but less pronounced neutral shift which results in an overvoltage on one or more transformers.

Another advantage of stabilizing the primary neutral by connecting it to neutral of the generator is that it eliminates distortion in the secondary phase voltages. This is explained as follows. For delivering a sine wave of voltage, it is necessary to have a sine wave of flux in the core, but on account of the characteristics of iron, a sine wave of flux requires a third harmonic component in the exciting current. As the frequency of this component is thrice the frequency of the circuit, at any given instant, it tends to flow either towards or away from the neutral point in all the three transformers. If the primary neutral is isolated, the triple frequency current cannot flow. Hence, the flux in the core cannot be a sine wave and so the voltages are distorted. But if the primary neutral is earthed i.e. joined to the generator neutral, then this provides a path for the triple-frequency currents and e.m.fs. and the difficulty is overcome. Another way of avoiding this trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in $\Delta$ and provides a circuit in which the triple-frequency component of the magnetising current can flow (with an isolated neutral, it could not). In that case, a sine wave of voltage applied to the primary will result in a sine wave of phase voltage in the secondary. As said above, the advantage of this connection is that insulation is stressed only to the extent of line to neutral voltage i.e. $58 \%$ of the line voltage.

### 33.4. Delta-Delta or $\Delta-\Delta$ Connection

This connection is economical for large, low-voltage transformers in which insulation problem is not so urgent, because it increases the number of turns/phase. The transformer connections and voltage triangles are shown in Fig. 33.5. The ratio of transformation between primary and secondary line voltage is exactly the same as that of each transformer. Further, the secondary voltage triangle $a b c$ occupies the same relative position as the primary voltage triangle $A B C$ i.e. there is no angular displacement between the two. Moreover, there is no internal phase shift between phase and line voltages on either side as was the case in $Y-Y$ connection. This connection has the following advantages :

1. As explained above, in order that the output voltage be sinusoidal, it is necessary that the magnetising current of the transformer must contain a third harmonic component. In this case, the third harmonic component of the magnetising current can flow in the $\Delta$-connected transformer primaries without flowing in the line wires. The three phases are $120^{\circ}$ apart which is $3 \times 120=360^{\circ}$ with respect to the third harmonic, hence it merely circulates in the $\Delta$. Therefore, the flux is sinusoidal which results in sinusoidal voltages.
2. No difficulty is experienced from unbalanced loading as was the case in $Y-Y$ connection. The three-phase voltages remain practically constant regardless of load imbalance.
3. An added advantage of this connection is that if one transformer becomes disabled, the system can continue to operate in open-delta or in $V-V$ although with reduced available capacity. The reduced capacity is $58 \%$ and not $66.7 \%$ of the normal value, as explained in Art. 33.7.

$0^{\circ}$ Angular Displacement

Fig. 33.5

$30^{\circ}$ Angular Displacement


$30^{\circ}$ Angular Displacement

Fig. 33.7

### 33.5. Wye/Delta or $Y / \Delta$ Connection

The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is $Y$-connected with grounded neutral as shown in Fig. 33.6. The ratio between the secondary and primary line voltage is $1 / \sqrt{3}$ times the transformation ratio of each transformer. There is a $30^{\circ}$ shift between the primary and secondary line voltages which means that a $Y-\Delta$ transformer bank cannot be paralleled with either a $Y-Y$ or a $\Delta-\Delta$ bank. Also, third harmonic currents flows in the $\Delta$ to provide a sinusoidal flux.

### 33.6. Delta/Wye or $\Delta / Y$ Connection

This connection is generally employed where it is necessary to step up the voltage as for example, at the beginning of high tension transmission system. The connection is shown in Fig. 33.7. The neutral of the secondary is grounded for providing 3-phase 4-wire service. In recent years, this connection has gained considerable popularity because it can be used to serve both the 3-phase power equipment and single-phase lighting circuits.

This connection is not open to the objection of a floating neutral and voltage distortion because the existence of a $\Delta$-connection allows a path for the third-harmonic currents. It would be observed that the primary and secondary line voltages and line currents are out of phase with each other by $30^{\circ}$. Because of this $30^{\circ}$ shift, it is impossible to parallel such a bank with a $\Delta-\Delta$ or $Y-Y$ bank of transformers even though the voltage ratios are correctly adjusted. The ratio of secondary to primary voltage is $\sqrt{3}$ times the transformation ratio of each transformer.

Example 33.1. A 3-phase, $50-\mathrm{Hz}$ transformer has a delta-connected primary and star-connected secondary, the line voltages being 22,000 V and 400 V respectively. The secondary has a starconnected balanced load at 0.8 power factor lagging. The line current on the primary side is 5 A . Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in $k W$ ?

Solution. It should be noted that in three-phase transformers, the phase transformation ratio is equal to the turn ratio but the terminal or line voltages depend upon the method of connection employed. The $\Delta / Y$ connection is shown in Fig. 33.8.


Fig. 33.8
Phase voltage on primary side
$=22,000 \mathrm{~V}$
Phase voltage on secondary side $=400 / \sqrt{3}$
$\therefore \quad K=400 / 22,000 \times \sqrt{3}=1 / 55 \sqrt{3}$
Primary phase current
$=5 / \sqrt{3} \mathrm{~A}$
Secondary phase current
$=\frac{5}{\sqrt{3}} \div \frac{1}{55 \sqrt{3}}=275 \mathrm{~A}$
Secondary line current
$=275 \mathrm{~A}$
$\therefore$ Output
$=\sqrt{3} V_{L} I_{L} \cos \phi=\sqrt{3} \times 400 \times 275 \times 0.8=\mathbf{1 5 . 2 4} \mathbf{k} \mathbf{W}$.
Example 33.2. A 500-kVA, 3-phase, 50-Hz transformer has a voltage ratio (line voltages) of $33 / 11-k V$ and is delta/star connected. The resistances per phase are : high voltage $35 \Omega$, low voltage $0.876 \Omega$ and the iron loss is 3050 W . Calculate the value of efficiency at full-load and one-half of fullload respectively (a) at unity p.f. and (b) 0.8 p.f.
(Electrical Machinery, Madras Univ. 1985)
Solution. Transformation ratio $K=\frac{11,000}{\sqrt{3} \times 33,000}=\frac{1}{3 \sqrt{3}}$
Per phase

$$
\begin{aligned}
R_{02} & =0.876+(1 / 3 \sqrt{3})^{2} \times 35=2.172 \Omega \\
& =\frac{500,000}{\sqrt{3} \times 11,000}=\frac{500}{11 \sqrt{3}} \mathrm{~A}
\end{aligned}
$$

Full-load condition
Full load total Cu loss $=3 \times(500 / 11 \sqrt{3})^{2} \times 2.172=4,490 \mathrm{~W}$; Iron loss $=3,050 \mathrm{~W}$
Total full-load losses

$$
=4,490+3,050=7,540 \mathrm{~W} ; \text { Output at }
$$ unity p.f. $=500 \mathrm{~kW}$

$\therefore$ F.L. efficiency $=500,000 / 507,540$ $=0.9854$ or $\mathbf{9 8 . 5 4 \%}$; Output at 0.8 p.f. $=400 \mathrm{~kW}$
$\therefore$ Efficiency $=400,000 / 407.540$

$$
=0.982 \text { or } 98.2 \%
$$

## Half-load condition

Output at unity p.f. $=250 \mathrm{~kW}$

$$
\begin{gathered}
\text { Cu losses }=(1 / 2)^{2} \times 4,490 \\
=1,222 \mathbf{W}
\end{gathered}
$$

Total losses

$$
=3,050+1,122=4,172 \mathrm{~W}
$$



```
\(\therefore \quad \eta=250,000 / 254,172=0.9835=98.35 \%\)
Output at 0.8 p.f. \(\quad=200 \mathrm{~kW} \quad \therefore \quad \eta=\mathbf{2 0 0 , 0 0 0} / \mathbf{2 0 4}, \mathbf{1 7 2}=\mathbf{0} .98 \quad\) or \(\mathbf{9 8 \%}\)
```

Example 33.3. A 3-phase, 6,600/415-V, 2,000-kVA transformer has a per unit resistance of 0.02 and a per unit leakage reactance of 0.1. Calculate the Cu loss and regulation at full-load 0.8 p.f. lag.
(Electrical Machines-I, Bombay Univ. 1987)
Solution. As seen from Art. $27-16, \% R=\% \mathrm{Cu}$ loss $=\frac{\mathrm{Cu} \text { loss }}{V A} \times 100$
Now, $\% R=0.02 \times 100=2 \% \quad \therefore \quad 2=\frac{C u \text { loss }}{2,000} \times 100 \quad \therefore \mathrm{Cu}$ loss $=40 \mathrm{~kW}$
Now, percentage leakage reactance $=0.1 \times 100=10 \%$

$$
\text { Regn. }=v_{r} \cos \phi+v_{x} \sin \phi=2 \times 0.8+10 \times 0.6=7.6 \%
$$

Example 33.4. A $120-\mathrm{kVA}, 6,000 / 400-\mathrm{V}$, Y/Y 3-ph, $50-\mathrm{Hz}$ transformer has an iron loss of 1,600 W. The maximum efficiency occurs at 3/4 full load.

Find the efficiencies of the transformer at
(i) full-load and 0.8 power factor
(ii) half-load and unity power factor
(iii) the maximum efficiency.
(Elect. Technology Utkal Univ. 1987)

Solution. Since maximum efficiency occurs at $3 / 4$ full-load, Cu loss at $3 / 4$ full-load equals iron loss of $1,600 \mathrm{~W}$.

Cu loss at 3/4 F.L.
(i) F.L. output at 0.8 p.f.

$$
=120 \times 0.8=96 \mathrm{~kW}=96,000 \mathrm{~W}
$$

Total loss

$$
=1,600+2,845=4,445 \mathrm{~W}
$$

$\therefore$

$$
\eta=\frac{96,000}{100,445} \times 100=95.57 \%
$$

(ii) Cu loss at $1 / 2$ full-load

Total loss
$=710+1,600=2310 \mathrm{~W}$
Output at $1 / 2$ F.L. and u.p.f. is $=60 \mathrm{~kW}=60,000 \mathrm{~W} ; \eta=\frac{60,000}{62,310} \times 100=\mathbf{9 6 . 5 7 \%}$
(iii) Maximum efficiency occurs at $3 / 4$ full-load when iron loss equals Cu loss.
Total loss
$=2 \times 1,600=3,200 \mathrm{~W}$
Output at u.p.f.
$=(3 / 4) \times 120=90 \mathrm{~kW}=90,000 \mathrm{~W}$
Input

$$
=90,000+3,200=93,200 \mathrm{~W} \therefore \eta=\frac{90,000}{93,200} \times 100=96.57 \%
$$

$$
=1,600 \mathrm{~W} ; \mathrm{Cu} \text { loss at F.L. }=1,600 \times(4 / 3)^{2}=2,845 \mathrm{~W}
$$

$$
=(1 / 2)^{2} \times 2,845=710 \mathrm{~W}
$$

$$
=710+1,600=2310 \mathrm{~W}
$$

Example 33.5. A 3-phase transformer, ratio 33/6.6-kV, $\Delta / Y, 2-M V A$ has a primary resistance of 8 $\Omega$ per phase and a secondary resistance of 0.08 ohm per phase. The percentage impedance is $7 \%$. Calculate the secondary voltage with rated primary voltage and hence the regulation for full-load 0.75 p.f. lagging conditions.
(Elect. Machine-I, Nagpur, Univ. 1993)
Solution. F.L. secondary current $=\frac{2 \times 10^{6}}{\sqrt{3} \times 6.6 \times 10^{3}}=175 \mathrm{~A}$
$K=6.6 / \sqrt{3} \times 33=1 / 8.65 ; R_{02}=0.08+8 / 8.65^{2}=0.1867 \Omega$ per phase
Now, secondary impedance drop per phase $=\frac{7}{100} \times \frac{6,600}{\sqrt{3}}=266.7 \mathrm{~V}$

$$
\therefore \quad Z_{02}=266.7 / 175=1.523 \Omega \text { per phase }
$$

$$
X_{02}=\sqrt{Z_{02}^{2}-R_{02}^{2}}=\sqrt{1.523^{2}-0.1867^{2}}=1.51 \Omega / \text { phase }
$$

Drop per phase $=I_{2}\left(R_{02} \cos \phi+X_{02} \sin \phi\right)=175(0.1867 \times 0.75+1.51 \times 0.66)=200 \mathrm{~V}$
Secondary voltage/phase

$$
=6,600 / \sqrt{3}=3,810 \mathrm{~V} \quad \therefore \quad V_{2}=3,810-200=3,610 \mathrm{~V}
$$

$\therefore$ Secondary line voltage $\quad=3,610 \times \sqrt{3}=6,250 \mathrm{~V}$
\%regn. $\quad=200 \times 100 / 3,810=5.23 \%$
Example 33.6. A 100-kVA, 3-phase, $50-\mathrm{Hz} 3,300 / 400 \mathrm{~V}$ transformer is $\Delta$-connected on the h.v. side and $Y$-connected on the l.v. side. The resistance of the h.v. winding is $3.5 \Omega$ per phase and that of the l.v. winding $0.02 \Omega$ per phase. Calculate the iron losses of the transformer at normal voltage and frequency if its full-load efficiency be $95.8 \%$ at 0.8 p.f. (lag).
(A.C. Machines-I, Jadavpur Univ. 1989)

Solution. F.L. output

$$
\begin{aligned}
& =100 \times 0.8=80 \mathrm{~kW} ; \text { Input }=80 / 0.958=83.5 \mathrm{~kW} \\
& =\text { Input }- \text { Output }=83.5-80=3.5 \mathrm{~kW}
\end{aligned}
$$

Total loss
Let us find full-load Cu losses for which purpose, we would first calculate $R_{02}$.

$$
\begin{aligned}
K & =\frac{\text { secondary voltage } / \text { phase }}{\text { primary voltage/phase }}=\frac{400 / \sqrt{3}}{3,300}=\frac{4}{33 \sqrt{3}} \\
R_{02} & =R_{2}+K^{2} R_{1}=0.02+(4 / \sqrt{3} \times 33)^{2} \times 3.5=0.037 \Omega
\end{aligned}
$$

Full-load secondary phase current is $I_{2}=100,000 / \sqrt{3} \times 400=144.1 \mathrm{~A}$
Total Cu loss

$$
\begin{array}{ll}
\text { Total Cu loss } & =3 I_{2}^{2} R_{02}=3 \times 144.1^{2} \times 0.037=2,305 \mathrm{~W} \\
\text { Iron loss } & =\text { Total loss }- \text { F.L. } \text { Cu loss }=3,500-2,305=\mathbf{1 , 1 9 5} \mathbf{~ W}
\end{array}
$$

Example 33.7. A 5,000-kVA, 3-phase transformer, $6.6 / 33-\mathrm{kV}$, $\Delta / Y$, has a no-load loss of 15 kW and a full-load loss of 50 kW . The impedance drop at full-load is $7 \%$. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV .

```
Solution. Full-load \(\quad I_{2}=5 \times 10^{6} / \sqrt{3} \times 33,000=87.5 \mathrm{~A}\)
Impedance drop/phase \(\quad=7 \%\) of \((33 / \sqrt{3})=7 \%\) of \(19 \mathrm{kV}=1,330 \mathrm{~V}\)
\(\therefore \quad Z_{02}=1,330 / 87.5=15.3 \Omega /\) phase; F.L. Cu loss \(=50-15=35 \mathrm{~kW}\)
\(\therefore \quad 3 I_{2} R_{02}=35,000 ; \quad R_{02}=35,000 / 3 \times 8.75^{2}=1.53 \Omega /\) phase
\(\therefore \quad X_{02}=\sqrt{15.3^{2}-1.53^{2}}=\mathbf{1 5 . 2 3} \Omega\)
```

When load is $3,200 \mathrm{~kW}$ at 0.8 p.f.

$$
I_{2}=3,200 / \sqrt{3} \times 33 \times 0.8=70 \mathrm{~A} ; \text { drop }=70(1.53 \times 0.8+15.23 \times 0.6)=725 \mathrm{~V} / \text { phase }
$$

$\therefore \quad \%$ regn.

$$
=\frac{725 \times 100}{19,000}=3.8 \%
$$

Primary voltage will have to be increased by $3.8 \%$.

$$
\therefore \text { Primary voltage } \quad=6.6+3.8 \% \text { of } 6.6=6.85 \mathrm{kV}=6,850 \mathrm{~V}
$$

Example 33.8. A 3-phase transformer has its primary connected in $\Delta$ and its secondary in Y. It has an equivalent resistance of $1 \%$ and an equivalent reactance of $6 \%$. The primary applied voltage is $6,600 \mathrm{~V}$. What must be the ratio of transformation in order that it will deliver 4,800 V at full-load current and 0.8 power factor (lag)?
(Elect. Technology-II, Magadh Univ. 1991)
Solution. Percentage regulation

$$
=v_{r} \cos \phi+v_{x} \sin \phi
$$

$$
=1 \times 0.8+6 \times 0.6=4.4 \%
$$

Induced secondary e.m.f. (line value)
$=4,800+4.4 \%$ of $4,800=5,010 \mathrm{~V}$, as in Fig. 33.9.
Secondary phase voltage
$=5,010 / \sqrt{3}=2,890 \mathrm{~V}$
Transformation ratio


Fig. 33.9
$K=2,890 / 6,600=\mathbf{0 . 4 3 7}$.
Example 33.9. A 2000-kVA, 6,600/400-V, 3-phase transformer is delta-connected on the high voltage side and star-connected on the low-voltage side. Determine its \% resistance and \% reactance drops, \% efficiency and \% regulation on full load 0.8 p.f. leading given the following data :
S.C. test ; H.V. data : $400 \mathrm{~V}, 175 \mathrm{~A}$ and 17 kW
O.C. test; L.V. data: $400 \mathrm{~V}, 150 \mathrm{~A}$ and 15 kW
(Basic Elect., Machines Nagpur Univ. 1993)
Solution. From S.C. test data, we have
Primary voltage/phase $\quad=400 \mathrm{~V}$; Primary current $/$ phase $=175 / \sqrt{3}=100 \mathrm{~A}$

$$
\begin{aligned}
\therefore \quad Z_{01} & =\frac{400}{101}=3.96 \Omega \\
I_{1}^{2} R_{01} & =\frac{17000}{3} \text { or } R_{01}=0.555 \Omega ; X_{01}=\sqrt{3.96^{2}-0.555^{2}}=3.92 \Omega \\
\% R & =\frac{I_{1} R_{01}}{V_{1}} \times 100=\frac{101 \times 0.555}{6,600} \times 100=0.849 \\
\% X & =\frac{I_{1} X_{01}}{V_{1}} \times 100=\frac{101 \times 3.92}{6,600} \times 100=\mathbf{6} \\
\% \text { regn } & =v_{r} \cos \phi-v_{x} \sin \phi=0.49 \times 0.8-6 \times 0.6=-2.92 \%
\end{aligned}
$$

Full-load primary line current can be found from

$$
\sqrt{3} \times 6,600 \times I_{1}=2000 \times 1,000 ; I_{1}=175 \mathrm{~A}
$$

It shows that S.C. test has been carried out under full-load conditions.

$$
\text { Total losses } \quad=17+15=32 \mathrm{~kW} \text {; F.L. output }=2,000 \times 0.8=1600 \mathrm{~kW}
$$

$$
\eta=1,600 / 1,632=0.98 \text { or } \mathbf{9 8 \%}
$$

Example 33.10. A 3-ph, delta/star connected $11,000 / 440 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer takes a line current of 5 amp , when secondary Load of 0.8 Lagging p.f. is connected. Determine each coilcurrent and output of transformer.
(Amravati Univ. 1999)
Solution. Due to delta/star connections the voltage ratings of the two sides on per phase basis are:

Primary coil rating $=11,000 \mathrm{~V}$, Secondary coil rating $=\frac{440}{\sqrt{3}}=254$ volts
Primary coil-current $\quad=5 / \sqrt{3}=2.887 \mathrm{amp}$
Each coil is delivering equal volt. amps.
Since three phase volt $-\mathrm{amps}=3 \times 11,000 \times 5 / \sqrt{3}$

$$
=95266
$$

Volts amps/phase $=31755$
This corresponds to the secondary coil-current of $I_{2}$, given by

$$
I_{2}=\frac{31755}{254}=125 \mathrm{amp} . \text { This is shown in Fig. 33.10. }
$$

Total Output of transformer, in $\mathrm{kVA}=31,755$
Since, the p.f. given is 0.8 lagging.
The total output power in $\mathrm{kW}=31,755 \times 0.80=\mathbf{2 5 . 4} \mathbf{k W}$


Fig. 33.10. Transformer coil currents
Example 33.11. A load of 1000 kVA at 0.866 p.f. lagging is supplied by two 3 phase transformers of 800 kVA capacity operating in parallel. Ratio of transformation is same : 6600/400 V, deltal star. If the equivalent impedances referred to secondary are $(0.005+j .015)$ ohm and $(0.012+j$ 0.030) ohm per phase respectively. Calculate load and power factor of each transformer.
(Amravati Univ. 1999)

```
Solution. Total load
\[
\begin{aligned}
& =1000 \mathrm{kVA} \\
\cos \phi & =0.866 \mathrm{Lag}, \phi=30^{\circ} \mathrm{lag} \\
& =866 \mathrm{~kW}
\end{aligned}
\]
Total power output
```

Secondary current with star connection,

$$
I_{2}=\frac{1000}{\sqrt{3} \times 440} \times 1000=1312.2 \mathrm{amp}
$$

If the two transformers are identified as $A$ and $B$, with their parameters with subscripts of $a$ and $b$, we have:

$$
\begin{aligned}
\dot{\mathbf{Z}}_{a} & =0.005+j 0.015=0.0158 \angle 71.56^{\circ} \mathrm{ohm} \\
\dot{\mathbf{Z}}_{b} & =0.012+j 0.030=0.0323 \angle 68.2^{\circ} \mathrm{ohm} \\
\dot{\mathbf{Z}}_{a}+\dot{\mathbf{Z}}_{b} & =0.017+j 0.045=0.0481 \angle 69.3^{\circ} \mathrm{ohm} \\
\dot{\mathbf{I}}_{2 a} & =\text { secondary current of transformer } A \\
\dot{\mathbf{I}}_{2 b} & =\text { secondary current of transformer } B \\
\dot{\mathbf{I}}_{2 a} & =\frac{\dot{\mathbf{Z}}_{b}}{\dot{\mathbf{Z}}_{b}+\dot{\mathbf{Z}}_{a}} \times \dot{\mathbf{I}}_{2}=\frac{0.0323 \angle 68.2^{\circ}}{0.0481 \angle 69.3^{\circ}} \times 1312 \angle-30^{\circ} \\
& =88.1 \angle-31.1^{\circ} \\
\dot{\mathbf{I}}_{2 b} & =\frac{\dot{\mathbf{Z}}_{a}}{\dot{\mathbf{Z}}_{a}+\dot{\mathbf{Z}}_{b}} \times \dot{\mathbf{I}}_{2}=\frac{0.0158 \angle 71.56^{\circ}}{0.0481 \angle 69.3^{\circ}} \times 1312 \angle-30^{\circ} \\
& =43.1 \angle-27.74^{\circ}
\end{aligned}
$$

For Transformer $A$

$$
\text { Load } \quad=3 \times 254 \times 881 \times 10^{-3}=671.3 \mathrm{kVA}
$$

| Power factor <br> For Transformer $\boldsymbol{B}$ | $=\cos 31.1^{\circ} \mathrm{lag}=0.856$ lag |
| :--- | :--- |
| Load | $=3 \times 254 \times 431 \times 10^{-3}=328.4 \mathrm{kVA}$ |
| Power factor | $=\cos 23.74^{\circ} \mathrm{lag}=0.885 \mathrm{lag}$ |

Check: Total kW gives a check.
1000 kVA at 0.866 lag means 866 kW .
Output, in kW, of transformer $A=671.3 \times 0.856=574.6 \mathrm{~kW}$
Output in kW of transformer $B=328.4 \times 0.885=\mathbf{2 9 0 . 6} \mathbf{~ k W}$
Sum of these two outputs $\quad=574.6+290.6=865.2 \mathrm{~kW}$
Note. Total kVAR also gives a check.
Depending on leading or lagging p.f., appropriate sign (+ve or $-v e$ ) must be assigned to the kVAR-term.

## Tutorial Problem No. 33.1

1. A 3-phase, star-connected alternator generates $6,360 \mathrm{~V}$ per phase and supplies 500 kW at a p.f. 0.9 lagging to a load through a step-down transformer of turns $40: 1$. The transformer is delta connected on the primary side and star-connected on the secondary side. Calculate the value of the line volts at the load. Calculate also the currents in $(a)$ alternator windings $(b)$ transformer primary windings $(c)$ transformen secondary windings.
[476 V (a) 29.1 A (b) $16.8 \mathrm{~A}(c) 672 \mathrm{~A}]$
2. A $11,000 / 6,600 \mathrm{~V}, 3-\phi$, transformer has a star-connected primary and a delta-connected secondary. It supplies a 6.6 kV motor having a star-connected stator, developing 969.8 kW at a power factor of 0.9 lagging and an efficiency of 92 per cent. Calculate (i) motor line and phase currents (ii) transformer secondary current and (iii) transformer primary current.

$$
\left[(a) \text { Motor } ; I_{L}=I_{p h}=126.3 \mathrm{~A}(b) \text { phase current } 73 \mathrm{~A}(c) 75.8 \mathrm{~A}\right]
$$

### 33.7. Open-Delta or $V-V$ connection

If one of the transformers of a $\Delta-\Delta$ is removed and 3-phase supply is connected to the primaries as shown in Fig. 33.11, then three equal 3-phase voltages will be available at the secondary terminals on noload. This method of transforming 3-phase power by means of only two transformers is called the open $-\Delta$ or $V-V$ connection.

## It is employed:

1. when the three-phase load is too small to warrant the installation of full three-phase transformer bank.
2. when one of the transformers in a $\Delta-\Delta$ bank is disabled, so that service is continued although at reduced capacity, till the faulty transformer is repaired or a new one is substituted.
3. when it is anticipated that in future the load will increase necessitating the closing of open delta.
One important point to note is that the total load that can be carried by a $V-V$ bank is not two-third of the capacity of a $\Delta-\Delta$ bank but it is only $57.7 \%$ of it. That is a reduction of $15 \%$ (strictly, $15.5 \%$ ) from its normal rating.


Fig. 33.11 Suppose there is $\Delta-\Delta$ bank of three $10-\mathrm{kVA}$ transformers. When one transformer is removed, then it runs in $V-V$. The total rating of the two transformers is 20 kVA . But the capacity of the $V-V$ bank is not the sum of the transformer kVA ratings but only 0.866 of it i.e. $20 \times 0.866=17.32$ (or $30 \times 0.57=17.3 \mathrm{kVA}$ ). The fact that the ratio of V-capacity to $\Delta$-capacity is $1 / \sqrt{3}=57.7 \%$ (or nearly $58 \%$ ) instead of $66 \frac{2}{3}$ per cent can be proved as follows :

As seen from Fig. 33.12 (a)
$\Delta-\Delta$ capacity $=\sqrt{3} \cdot V_{L} \cdot I_{L}=\sqrt{3} \cdot V_{L}\left(\sqrt{3} \cdot I_{S}\right)=3 V_{L} I_{S}$
In Fig. $33.12(b)$, it is obvious that when $\Delta-\Delta$ bank becomes $V-V$ bank, the secondary line current $I_{L}$ becomes equal to the secondary phase current $I_{S}$.

$$
\begin{array}{llrl}
\therefore & V-V \text { capacity } & =\sqrt{3} \cdot V_{L} I_{L}=\sqrt{3} V_{L} \cdot I_{S} \\
\therefore & \frac{V-V \text { capacity }}{\Delta-\Delta \text { capacity }}=\frac{\sqrt{3} \cdot V_{L} I_{S}}{3 V_{L} I_{S}}=\frac{1}{\sqrt{3}}=0.577 \text { or } 58 \text { per cent }
\end{array}
$$

It means that the 3-phase load which can be carried without exceeding the ratings of the transformers is 57.7 per cent of the original load rather than the expected $66.7 \%$.


Fig. 33.12
It is obvious from above that when one transformer is removed from a $\Delta-\Delta$ bank.

1. the bank capacity is reduced from 30 kVA to $30 \times 0.577=17.3 \mathrm{kVA}$ and not to 20 kVA as might be thought off-hand.
2. only $86.6 \%$ of the rated capacity of the two remaining transformers is available (i.e. $20 \times 0.866$ $=17.3 \mathrm{kVA}$ ). In other words, ratio of operating capacity to available capacity of an open- $\Delta$ is 0.866 . This factor of 0.866 is sometimes called the utility factor.
3. each transformer will supply $57.7 \%$ of load and not $50 \%$ when operating in $V-V$ (Ex. 33.13).

However, it is worth noting that if three transformers in a $\Delta-\Delta$ bank are delivering their rated load* and one transformer is removed, the overload on each of the two remaining transformers is $73.2 \%$ because

$$
\frac{\text { total load in } V-V}{V A / \text { transformer }}=\frac{\sqrt{3} \cdot V_{L} I_{S}}{V_{L} I_{S}}=\sqrt{3}=1.732
$$

This over-load may be carried temporarily but some provision must be made to reduce the load if overheating and consequent breakdown of the remaining two transformers is to be avoided.

The disadvantages of this connection are :

1. The average power factor at which the $V$-bank operates is less than that of the load. This power factor is actually $86.6 \%$ of the balanced load power factor. Another significant point to note is that, except for a balanced unity power factor load, the two transformers in the $V-V$ bank operate at different power factors (Art. 33.8).
2. Secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

It may, however, be noted that if two transformers are operating in $V-V$ and loaded to rated capacity (in the above example, to 17.3 kVA ), the addition of a third transformer increases the total capacity by $\sqrt{3}$ or $173.2 \%$ (i.e. to 30 kVA ). It means that for an increase in cost of $50 \%$ for the third transformer, the increase in capacity is $73.2 \%$ when converting from a $V-V$ system to a $\Delta-\Delta$ system.

[^0]
### 33.8. Power Supplied by $V-V$ Bank

When a $V-V$ bank of two transformers supplies a balanced 3-phase load of power factor $\cos \phi$, then one transformer operates at a p.f. of $\cos \left(30^{\circ}-\phi\right)$ and the other at $\cos \left(30^{\circ}+\phi\right)$. Consequently, the two transformers will not have the same voltage regulation.
$\therefore \quad P_{1}=\mathrm{kVA} \cos \left(30^{\circ}-\phi\right)$ and $P_{2}=\mathrm{kVA} \cos \left(30^{\circ}+\phi\right)$
(i) When $\phi=0$ i.e. load p.f. $=1$

Each transformer will have a p.f. $=\cos 30^{\circ}=0.866$
(ii) When $\phi=30^{\circ}$ i.e. load p.f. $=\mathbf{0 . 8 6 6}$.

In this case, one transformer has a p.f. of $\cos \left(30^{\circ}-30^{\circ}\right)=1$ and the other of $\cos \left(30^{\circ}+30^{\circ}\right)$ $=0.866$.
(iii) When $\phi=60^{\circ}$ i.e. load p.f. $=\mathbf{0 . 5}$

In this case, one transformer will have a p.f. $=\cos \left(30-60^{\circ}\right)=\cos \left(-30^{\circ}\right)=0.866$ and the other of $\cos \left(30^{\circ}+60^{\circ}\right)=0$. It means that one of the transformers will not supply any load whereas the other having a p.f. $=0.866$ will supply the entire load.

Example 33.12. What should be the $k V A$ rating of each transformer in a $V-V$ bank when the 3-phase balanced load is 40 kVA? If a third similar transformer is connected for operation, what is the rated capacity? What percentage increase in rating is affected in this way?

Solution. As pointed out earlier, the kVA rating of each transformer has to be $15 \%$ greater.
$\therefore \quad \mathrm{kVA} /$ trasformer
$=(40 / 2) \times 1.15=23$
$\Delta-\Delta$ bank rating
$=23 \times 3=69 ;$ Increase $=[(69-40) / 40] \times 100=72.5 \%$

Example 33.13. A $\Delta-\Delta$ bank consisting of three 20-kVA, 2300/230-V transformers supplies a load of 40 kVA . If one transformer is removed, find for the resulting $V-V$ connection
(i) kVA load carried by each transformer
(ii) per cent of rated load carried by each transformer
(iii) total $k V A$ rating of the $V-V$ bank
(iv) ratio of the $V-V$ bank to $\Delta-\Delta$ bank transformer ratings.
(v) per cent increase in load on each transformer when bank is converted into $V-V$ bank.

Solution. (i) As explained earlier in Art. 33.7, $\frac{\text { total kVA load in } V-V \text { bank }}{V A / \operatorname{transformer}}=\sqrt{3}$
$\therefore \quad$ kVA load supplied by each of the two transformers $=40 / \sqrt{3}=23.1 \mathrm{kVA}$
Obviously, each transformer in $V-V$ bank does not carry $50 \%$ of the original load but $57.7 \%$.
(ii) per cent of rated load $=\frac{\mathrm{kVA} \text { load/transformer }}{\mathrm{kVA} \text { rating/transformer }}=\frac{23.1}{20}=\mathbf{1 1 5 . 5} \%$
carried by each transformer.
Obviously, in this case, each transformer is overloaded to the extent of 15.5 per cent.*
(iii) kVA rating of the $V-V$ bank $=(2 \times 20) \times 0.866=34.64 \mathrm{kVA}$
(iv) $\frac{V-V \text { rating }}{\Delta-\Delta \text { rating }}=\frac{34.64}{60}=0.577$ or $\mathbf{5 7 . 7 \%}$

[^1]As seen, the rating is reduced to $57.7 \%$ of the original rating.
(v) Load supplied by each transformer in $\Delta-\Delta$ bank $=40 / 3=13.33 \mathrm{kVA}$
$\therefore$ Percentage increase in load supplied by each transformer

$$
=\frac{\mathrm{kVA} \text { load/transformer in } V-V \text { bank }}{\mathrm{kVA} \text { load/transformer in } \Delta-\Delta \text { bank }}=\frac{23.1}{13.3}=1.732=\mathbf{1 7 3 . 2} \%
$$

It is obvious that each transformer in the $\Delta-\Delta$ bank supplying 40 kVA was running underloaded ( 13.33 vs 20 kVA ) but runs overloaded ( 23.1 vs 20 kVA ) in $V-V$ connection.

Example 33.14. A balanced 3-phase load of 150 kW at $1000 \mathrm{~V}, 0.866$ lagging power factor is supplied from 2000 V, 3-phase mains through single-phase transformers (assumed to be ideal) connected in (i) delta-delta (ii) Vee-Vee. Find the current in the windings of each transformer and the power factor at which they operate in each case. Explain your calculations with circuit and vector diagrams.

## Solution. (i) Delta-Delta Connection

$$
\begin{aligned}
\sqrt{3} V_{L} I_{L} \cos \phi & =150,000 \\
\sqrt{3} \times 1000 \times I_{L} \times 0.866 & =150,000 \therefore I_{L}=100 \mathrm{~A}
\end{aligned}
$$

$\therefore$ Secondary line current $=100 \mathrm{~A}$; secondary phase current $=100 / \sqrt{3}=57.7 \mathrm{~A}$
Transformation ratio $=1000 / 2000=1 / 2$
$\therefore$ Primary phase current $=57.7 / 2=\mathbf{2 8 . 8 5} \mathrm{A}$
(ii) Vee-Vee Connection

Let $I$ be the secondary line current which is also the phase current in $V-V$ connection. Then

$$
\sqrt{3} \times 1000 \times I \times 0.866=150,000 \quad \therefore \quad I=100 \mathrm{~A}
$$

$\therefore$ Secondary phase current $=\mathbf{1 0 0} \mathbf{A}$; primary phase current $=100 \times 1 / 2=\mathbf{5 0} \mathbf{A}$
Transformer power factor $\quad=86.6$ per cent of $0.866=\mathbf{0 . 7 5}$ (lag).
Example 33.15. (a) Two identical 1-phase transformers are connected in open-delta across 3-phase mains and deliver a balanced load of 3000 kW at 11 kV and 0.8 p.f. lagging. Calculate the line and phase currents and the power factors at which the two transformers are working.
(b) If one more identical unit is added and the open delta is converted to closed delta, calculate the additional load of the same power factor that can now be supplied for the same temperature rise. Also calculate the phase and line currents.
(Elect. Machinery-I, Madras Univ. 1987)
Solution. (a) If $I$ is the line current, then

$$
\sqrt{3} \times 11,000 \times I \times 0.8=3,000,000 \quad \mathbf{I}=197 \mathrm{~A}
$$

Since, this also represents the phase current,
$\therefore$ Secondary phase current $=\mathbf{1 9 7} \mathbf{A}$; Transformer p.f. $=86.6$ per cent of $0.8=\mathbf{0 . 6 9 3}$

$$
\begin{array}{ll}
\text { (b) Additional load } & =72.5 \text { per cent of } 3000=2175 \mathrm{~kW} \\
\text { Total load } & =3000+2175=5175 \mathrm{~kW}
\end{array}
$$

Now,

$$
\sqrt{3} \times V_{L} I_{L} \cos \phi=5,175,000 \text { or } \sqrt{3} \times 11,000 \times I_{L} \times 0.8=5,175,000
$$

$\therefore \quad I_{L}=340 \mathrm{~A} ;$ phase current $=340 / \sqrt{3}=196 \mathrm{~A}$
Example 33.16. Two transformers connected in open delta supply a 400-kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V . What is the (a) kVA supplied by each transformer ? (b) kW supplied by each transformer? (Elect. Machines-I, Gwalior Univ. 1991)

Solution. As stated in Art 33.7, the ratio of operating capacity to available capacity in an open- $\Delta$ is 0.866 . Hence, kVA of each transformer is one-half of the total kVA load divided by 0.866 .
(a) kVA of each transformer $=\frac{(400 / 2)}{0.866}=231 \mathrm{kVA}$
(b) As stated in Art 33.8, the two transformers have power factors of $\cos \left(30^{\circ}-\phi\right)$ and $\cos \left(30^{\circ}+\phi\right)$.
$\therefore \quad P_{1}=\mathrm{kVA} \cos \left(30^{\circ}-\phi\right)$ and $P_{2}=\mathrm{kVA} \cos (30+\phi)$
Now, load p.f. $\quad=\cos \phi=0.866 ; \quad \phi=\cos ^{-1}(0.866)=30^{\circ}$
$\therefore \quad P_{1}=231 \times \cos 0^{\circ}=\mathbf{2 3 1} \mathrm{kW} ; P_{2}=231 \times \cos 60^{\circ}=\mathbf{1 1 5 . 5} \mathrm{kW}$
Obviously, $P_{1}+P_{2}$ must equal $400 \times 0.86=346.5 \mathrm{~kW}$

## Tutorial Problem No. 33.2

1. Three $1100 / 110-\mathrm{V}$ transformers connected delta-delta supply a lighting load of 100 kW . One of the transformers is damaged and removed for repairs. Find
(a) What currents were flowing in each transformer when the three transformers were in service ?
(b) What current flows in each transformer when the third is removed? and
(c) The output kVA of each transformer if the transformers connected in open $\Delta$ supply the full-load with normal heating?
$[(a)$ primary $=30.3 \mathrm{~A}$; secondary $303 \mathrm{~A}(b)$ primary $=30.3 \sqrt{\mathbf{3}} \mathrm{~A}$; secondary $=303 \sqrt{\mathbf{3}} \mathrm{~A}$
(c) 33.33 kVA$]$
(Elect. Machines-I, Gwalior Univ. Apr. 1977)

### 33.9. Scott Connection or $T$ - TConnection

This is a connection by which 3-phase to 3-phase transformation is accomplished with the help of two transformers as shown in Fig. 33.13. Since it was first proposed by Charles $F$. Scott, it is frequently referred to as Scott connection. This connection can also be used for 3-phase to 2-phase transformation as explained in Art. 33.10.

One of the transformers has centre taps both on the primary and secondary windings (Fig. 33.13) and is known as the main transformer. It forms the horizontal member of the connection (Fig. 33.14).

The other transformer has a 0.866 tap and is known as teaser transformer. One end of both the primary and secondary of the teaser transformer is joined to the centre taps on both primary and secondary of the main transformer respectively as shown in Fig. 33.14 (a). The other end $A$ of the teaser primary and the two ends $B$ and $C$ of


Fig. 33.13 the main transformer primary are connected to the 3-phase supply.

The voltage diagram is shown in Fig. 33.14 (a) where the 3-phase supply line voltage is assumed to be 100 V and a transformation ratio of unity. For understanding as to how 3-phase transformation results from this arrangement, it is desirable to think of the primary and secondary vector voltages as forming geometrical $T_{S}^{\prime}$ (from which this connection gets its name).


Fig. 33.14
In the primary voltage $T$ of Fig. $33.14(a), E_{D C}$ and $E_{D B}$ are each $50 V$ and differ in phase by $180^{\circ}$, because both coils $D B$ and $D C$ are on the same magnetic circuit and are connected in opposition. Each side of the equilateral triangle represents 100 V . The voltage $E_{D A}$ being the altitude of the equilateral triangle is equal to $(\sqrt{3} / 2) \times 100=86.6 \mathrm{~V}$ and lags behind the voltage across the main by $90^{\circ}$. The same relation holds good in the secondary winding so that $a b c$ is a symmetrical 3-phase system.

With reference to the secondary voltage triangle of Fig. 33.14 (b), it should be noted that for a load of unity power factor, current $I_{d b}$ lags behind voltage $E_{d b}$ by $30^{\circ}$ and $I_{d c}$ leads $E_{d c}$ by $30^{\circ}$. In other words, the teaser transformer and each half of the main transformer, all operate at different power factors.

Obviously, the full rating of the transformers is not being utilized. The teaser transformer operates at only 0.866 of its rated voltage and the main transformer coils operate at $\cos 30^{\circ}=0.866$ power factor, which is equivalent to the main transformer's coils working at 86.6 per cent of their kVA rating. Hence the capacity to rating ratio in a $T-T$. connection is $86.6 \%$ the same as in $V-V$ connection if two identical units are used, although heating in the two cases is not the same.

If, however, both the teaser primary and secondary windings are designed for 86.6 volts only, then they will be operating at full rating, hence the combined rating of the arrangement would become $(86.6+86.6) /(100+86.6)$ $=0.928$ of its total rating.* In other words, ratio of kVA utilized to that available would be 0.928 which makes this connection more economical than open- $\Delta$ with its


Fig. 33.15 ratio of 0.866 .

* Alternatively, VA capacity available is $=V_{L} I_{L}+\left(0.866 V_{L}\right) I_{\mathrm{L}}=1.866 V_{L} I_{L}$ where $I_{L}$ is the primary line current. Since 3-phase power is supplied, volt-amperes actually utilized $=1.732 V_{L} I_{L}$. Hence, ratio of kVA actually utilized to those available is $=1.732 V_{L} I_{L} / 1.866 V_{L} I_{L}=0.928$.

Fig. 33.15 shows the secondary of the $T-T$ connection with its different voltages based on a nominal voltage of 100 V . As seen, the neutral point $n$ is one third way up from point $d$. If secondary voltage and current vector diagram is drawn for load power factor of unity, it will be found that

1. current in teaser transformer is in phase with the voltage.
2. in the main transformer, current leads the voltage by $30^{\circ}$ across one half but lags the voltage by $30^{\circ}$ across the other half as shown in Fig. 33.14 (b).
Hence, when a balanced load of p.f. $=\cos \phi$, is applied, the teaser current will lag or lead the voltage by $\Phi$ while in the two halves of the main transformer, the angle between current and voltage will be $\left(30^{\circ}-\Phi\right)$ and $\left(30^{\circ}+\Phi\right)$. The situation is similar to that existing in a $V-V$ connection.

Example 33.17. Two T-connected transformers are used to supply a 440-V, 33-kVA balanced load from a balanced 3-phase supply of 3300 V . Calculate (a) voltage and current rating of each coil (b) kVA rating of the main and teaser transformer.

Solution. (a) Voltage across main primary is 3300 V whereas that across teaser primary is $=0.866 \times 3300=2858 \mathrm{~V}$.

The current is the same in the teaser and the main and equals the line current.
$\therefore \quad I_{L P}=33,000 / \sqrt{3} \times 3300=5.77 \mathrm{~A}$
-Fig. 33.16
The secondary main voltage equals the line voltage of 440 V whereas teaser secondary voltage $=0.866 \times 440=381 \mathrm{~V}$.

The secondary line current, $I_{L S}=I_{L P} / k=5.77 /(440 / 3300)=43.3 \mathrm{~A}$ as shown in Fig. 33.16.


Fig. 33.16
(b) Main kVA
$=3300 \times 5.77 \times 10^{-3}=19 \mathrm{kVA}$
Teaser kVA

$$
=0.866 \times \text { main } \mathrm{kVA}=0.866 \times 19=16.4 \mathrm{kVA}
$$

### 33.10. Three-phase to Two-phase Conversion and vice-versa

This conversion is required to supply two-phase furnaces, to link two-phase circuit with 3-phase system and also to supply a 3-phase apparatus from a 2-phase supply source. For this purpose, Scott connection as shown in Fig. 33.17 is employed. This connection requires two transformers of different ratings although for interchangeability and provision of spares, both transformers may be identical but having suitable tappings.


If, in the secondaries of Fig. 33.14 (b), points $c$ and $d$ are connected as shown in Fig. 33.18 (b), then a 2-phase, 3-wire system is obtained. The voltage $E_{d c}$ is 86.6 V but $\mathrm{E}_{\mathrm{Cb}}=100 \mathrm{~V}$, hence the


Fig. 33.19
resulting 2-phase voltages will be unequal. However, as shown in Fig. 33.19 (a) if the 3-phase line is connected to point $A_{1}$, such that $D A_{1}$ represents $86.6 \%$ of the teaser primary turns (which are the same as that of main primary), then this will increase the volts/turn in the ratio of $100: 86.6$, because now 86.6 volts are applied across 86.6 per cent of turns and not $100 \%$ turns. In other words, this will make volts/ turn the same both in primary of the teaser and that of the main transformer. If the secondaries of both the transformers have the same number of turns, then secondary voltage will be equal in magnitude as shown, thus resulting in a symmetrical 2-phase, 3-wire system.

Consider the same connection drawn slightly differently as in Fig. 33.20. The primary of the main transformer having $N_{1}$ turns is connected between terminals $C B$ of a 3-phase supply. If supply line voltage is $V$, then obviously $V_{A B}=V_{B C}=V_{C A}=V$ but voltage between $A$ and $D$ is $V \times \sqrt{3} / 2$. As said


Fig. 33.20
Fig. 33.21
above, the number of turns between $A$ and $D$ should be also $(\sqrt{3} / 2) N_{1}$ for making volt/turn the same in both primaries. If so, then for secondaries having equal turns, the secondary terminal voltages will be equal in magnitude although in phase quadrature.

It is to be noted that point $D$ is not the neutral point of the primary supply because its voltage with respect to any line is not $V / \sqrt{3}$. Let $N$ be the neutral point. Its position can be determined as follows. Voltage of $N$ with respect to $A$ must be $V / \sqrt{3}$ and since $D$ to $A$ voltage is $V \times \sqrt{3} / 2$, hence $N$ will be $(\sqrt{3} V / 2-V / \sqrt{3})$ $=0.288 \mathrm{~V}$ or 0.29 V from $D$. Hence, $N$ is above $D$ by a number of turns equal to $29 \%$ of $N_{1}$. Since 0.288 is one-third of 0.866 , hence $N$ divides the teaser winding $A D$ in the ratio $2: 1$.

Let the teaser secondary supply a current $I_{2 T}$ at unity power factor. If we neglect the magnetizing current $I_{0}$, then teaser primary current is $I_{1 T}=I_{2 T} \times$ transformation ratio.
$\therefore \quad I_{1 T}=I_{2 T} \times N_{2} /\left(\sqrt{3} N_{1} / 2\right)=(2 / \sqrt{3}) \times\left(N_{2} / N_{1}\right) \times I_{2 T}=1.15\left(N_{2} / N_{1}\right) I_{2 T}=1.15 K I_{2 T}$ where $K=N_{2} / N_{1}=$ transformation ratio of main transformer. The current is in phase with star voltage of the primary supply (Fig. 33.21).

The total current $I_{1 M}$ in each half of the primary of the main transformer consists of two parts :
(i) One part is that which is necessary to balance the main secondary current $I_{2 M}$. Its value is

$$
=I_{2 M} \times \frac{N_{2}}{N_{1}}=K I_{2 M}
$$

(ii) The second part is equal to one-half of


Fig. 33.22 the teaser primary current i.e. $\frac{1}{2} I_{1 T}$. This is so because the main transformer primary forms a return path for the teaser primary current which divides itself into two halves at mid-point $D$ in either direction. The value of each half is $=I_{1 T} / 2=1.15 K I_{2 T} / 2=0.58 K I_{2 T}$.

Hence, the currents in the lines $B$ and $C$ are obtained vectorially as shown in Fig. 33.22. It should be noted that as the two halves of the teaser primary current flow in opposite directions from point $D$, they have no magnetic effect on the core and play no part at all in balancing the secondary ampere-turns of the main transformer.

The line currents thus have rectangular components of $K I_{2 M}$ and $0.58, K I_{2 T}$ and, as shown in Fig. 33.22 , are in phase with the primary star voltages $V_{N B}$ and $V_{N C}$ and are equal to the teaser primary current. Hence, the three-phase side is balanced when the two-phase load of unity power factor is balanced.


Fig. 33.23
Fig. 33.23 (a) illustrates the condition corresponding to a balanced two-phase load at a lagging power factor of 0.866 . The construction is the same as in Fig. 33.22. It will be seen that the 3-phase side is again balanced. But under these conditions, the main transformer rating is $15 \%$ greater than that of the teaser, because its voltage is $15 \%$ greater although its current is the same.

Hence, we conclude that if the load is balanced on one side, it would always be balanced on the other side.
The conditions corresponding to an unbalanced two-phase load having different currents and power factors are shown in Fig. 33.23 (b). The geometrical construction is similar to those explained in Fig. 33.22 and 33.23 (a).

Summarizing the above we have :

1. Teaser transformer primary has $\sqrt{3} / 2$ times the turns of main primary. But volt/turn is the same. Their secondaries have the same turns which results in equal secondary terminal voltages.
2. If main primary has $N_{1}$ turns and main secondary has $N_{2}$ turns, then main transformation ratio is $N_{2} / N_{1}$. However, the transformation ratio of teaser is

$$
N_{2} /\left(\sqrt{3} N_{1} / 2\right)=1.15 N_{2} / N_{1}=1.15 \mathrm{~K}
$$

3. If the load is balanced on one side, it is balanced on the other side as well.
4. Under balanced load conditions, main transformer rating is $15 \%$ greater than that of the teaser.
5. The currents in either of the two halves of main primary are the vector sum of $K I_{2 M}$ and $0.58 ~ K I_{2 T}$ (or $\frac{1}{2} I_{1 T}$ ).
Example 33.18. Two transformers are required for a Scott connection operating from a 440-V, 3-phase supply for supplying two single-phase furnaces at 200 V on the two-phase side. If the total output is 150 kVA , calculate the secondary to primary turn ratio and the winding currents of each transformer.

Solution. Main Transformer
Primary volts
$=440 \mathrm{~V}$; secondary volts $=200 \mathrm{~V} \therefore \frac{N_{2}}{N_{1}}=\frac{200}{440}=\frac{1}{2.2}$
Secondary current
$=150,000 / 2 \times 200=375 \mathrm{~A}$
$\therefore$ Primary current
$=375 \times 1 / 2.2=197 \mathrm{~A}$
Teaser Transformer
Primary volts
$=(\sqrt{3} / 2 \times 440)=381 \mathrm{~V}:$ Secondary volts $=200 \mathrm{~V}$

$$
\left.\frac{\text { secondary turns }}{\text { primary turns }}=\frac{200}{381}=\frac{1}{1.905} \text { (also teaser ratio }=1.15 \times 1 / 2.2=1 / 1.905\right)
$$

Example 33.19. Two single-phase furnaces working at 100 V are connected to 3300-V, 3-phase mains through Scott-connected transformers. Calculate the current in each line of the 3-phase mains when the power taken by each furnace is $400-\mathrm{kW}$ at a power factor of 0.8 lagging. Neglect losses in the transformers.
(Elect. Machines-III, South Gujarat Univ. 1988)
Solution. Here

$$
\begin{aligned}
& K=100 / 3,300=1 / 33 \text { (main transformer) } \\
& I_{2}=\frac{400,000}{0.8 \times 100}=5,000 \mathrm{~A}\left(\text { Fig. 33.24); Here } I_{2 T}=I_{2 M}=I_{2}=5,000 \mathrm{~A}\right.
\end{aligned}
$$

As the two-phase load is balanced, the 3-phase side is also balanced.
Primary phase currents are $\quad=1.15 \mathrm{KI}_{2}=1.15 \times(1 / 33) \times 5,000=174.3 \mathrm{~A}$
Since for a star-connection, phase current is equal to line current,

$$
\therefore \text { Line current } \quad=174.3 \mathrm{~A}
$$

Note. We have made use of the fact that since secondary load is balanced, primary load is also balanced. If necessary, $I_{1 M}$ can also be found.
$I_{1 M}$ is the vector sum of (i) $K I_{2 M}$ and (ii) $\frac{1}{2} I_{1 T}$ or $0.58 K I_{2 T}$.

(a)

(b)

Fig. 33.24

$$
\begin{array}{ll}
\text { Now, } & K I_{2 M}=(1 / 33) \times 5,000=151 \mathrm{~A} \text { and } 0.58 K I_{2}=\frac{1}{2} I_{1 T}=174.3 / 2=87.1 \mathrm{~A} \\
\therefore & I_{1 M}=\sqrt{151^{2}+87.1^{2}}=\mathbf{1 7 4 . 3} \mathbf{A}
\end{array}
$$

Example 33.20. In a Scott-connection, calculate the values of line currents on the 3-phase side if the loads on the 2-phase side are 300 kW and 450 kW both at 100 V and $0.707 \mathrm{p} . f$. (lag) and the 3-phase line voltage is 3,300 V. The 300-kW load is on the leading phase on the 2-phase side. Neglect transformer losses.
(Elect. Technology, Allahabad Univ. 1991)
Solution. Connections are shown in Fig. 33.25 (a) and phasor diagram in Fig. 33.25 (b).


Fig. 33.25
Here,

$$
K=100 / 3,300=1 / 33
$$

Teaser secondary current is $I_{2 T}=450,000 / 100 \times 0.707=6360 \mathrm{~A}$
Teaser primary current is $\quad I_{1 T}=1.15 K I_{2 T}=1.5 \times(1 / 33) \times 6360=221.8 \mathrm{~A}$

As shown in Fig. 33.25 (b), main primary current $I_{1 M}$ has two rectangular components.
(i) $K I_{2 M}$ where $I_{2 M}$ is the secondary current of the main transformer and
(ii) Half of the teaser primary current $\frac{1}{2} I_{1 T}=\frac{1}{2} \times 1.15 K I_{2 T}=0.577 K I_{2 T}$

Now $K I_{2 M}=\frac{1}{33} \times \frac{300,000}{100 \times 0.707}=128.58 \mathrm{~A}$; Also $\frac{1}{2} I_{1 T}=\frac{1}{2} \times 221.8=110.9 \mathrm{~A}$
Main Primary current $\quad=\sqrt{128.58^{2}+110.9^{2}}=169.79 \mathrm{~A}$
Hence, the 3-phase line currents are 221.8 A in one line and 169.79 A in each of the other two.
Example 33.21. Two electric furnaces are supplied with 1-phase current at 80 V from a 3-phase, 11,000 V system by means of two single-phase Scott-connected transformers with similar secondary windings. When the load on one furnace is 500 kW (teaser secondary) and on the other 800 kW (secondary of main transformer) what current will flow in each of the 3-phase lines (a) at unity power factor and $(b)$ at 0.5 power factor? Neglect phase displacement in and efficiency of, the transformers.
(Electrical Engineering, Madras Univ. 1987)
Solution. The connections are shown in Fig. 33.26 and the phasor diagrams for unity and 0.5 p.f. are shown in Fig. 33.27 (a) and (b) respectively.


Fig. 33.27
Here,

$$
K=80 / 11,000=2 / 275
$$

(a) Unity p.f.

With reference to Fig. $33.27(a)$, we have $I_{2 T}=500,000 / 80 \times 1=6,250 \mathrm{~A}$
Teaser primary current $\quad I_{1 T}=1.15 K I_{2 T}=1.15 \times(2 / 275) \times 6,250=52.5 \mathrm{~A}$
For the main transformer primary
(i)

$$
K I_{2 M}=\frac{2}{275} \times \frac{800,000}{80 \times 1}=72.7 \mathrm{~A} \text { (ii) } \frac{1}{2} \times I_{1 T}=52.5 / 2=26.25 \mathrm{~A}
$$

Current in the primary of the main transformer is $=\sqrt{72.7^{2}+26.25^{2}}=77.1 \mathrm{~A}$
Hence, one 3-phase line carries 52.5 A whereas the other 2 carry 77.1 A each [Fig. 33.27 (a)].
(b) 0.5 p.f.

With reference to Fig. $33.27(b)$ we have $I_{2 T}=500,000 / 80 \times 0.5=12,500 \mathrm{~A}$
Teaser primary current $I_{1 T}=1.15 \times(2 / 275) \times 12,500=105 \mathrm{~A}$
For the main transformer primary
(i)

$$
K I_{2 M}=\frac{2}{275} \times \frac{800,000}{80 \times 0.5}=145.4 \mathrm{~A} \text { (ii) } \frac{1}{2} I_{1 T}=105 / 2=52.5 \mathrm{~A} .
$$

Current in the primary of the main transformer is $=\sqrt{145.4^{2}+52.5^{2}}=154.2 \mathrm{~A}$.
Hence, one 3-phase line carries 105 A and the other two carry 154.2 A each.
Note : Part (b) need not be worked out in full because at 0.5 p.f., each component current and hence the resultant are doubled. Hence, in the second case, answers can be found by multiplying by a factor of 2 the line currents found in (a).

Example 33.22. Two furnaces are supplied with 1-phase current at 50 V from a 3-phase, 4.6 kV system by means of two 1-phase, Scott-connected transformers with similar secondary windings. When the load on the main transformer is 350 kW and that on the other transformer is 200 kW at 0.8 p.f. lagging, what will be the current in each 3-phase line ? Neglect phase displacement and losses in transformers.
(Electrical Machinery-II, Bangalore Univ. 1991)
Solution. Connections and vector diagrams are shown in Fig. 33.28.

$$
\begin{aligned}
K & =50 / 4,600=1 / 92 ; I_{2 T}=200,000 / 50 \times 0.8=5,000 \mathrm{~A} \\
I_{1 T} & =1.15 K I_{2 T}=1.1 \times(1 / 92) \times 5,000=62.5 \mathrm{~A}
\end{aligned}
$$



Fig. 33.28
As shown in Fig. 33.28 (b), main primary current $I_{1 M}$ has two rectangular components.
(i) $K I_{2 M}$ where $I_{2 M}=350,000 / 50 \times 0.8=8,750 \mathrm{~A} \therefore K I_{2 M}=8,750 / 92=95.1 \mathrm{~A}$
(ii) $(1 / 2) I_{1 T}=62.5 / 2=31.3 \mathrm{~A} \quad \therefore \quad I_{l M}=\sqrt{95.1^{2}+31.3^{2}}=100 \mathrm{~A}$
$\therefore \quad$ Current in line $A=\mathbf{6 2 . 5} \mathrm{A}$; Current in line $B=100 \mathrm{~A}$; Current in line $C=100 \mathrm{~A}$.
Example 33.23. Two single-phase Scott-connected transformers supply a 3-phase four-wire distribution system with 231 volts between lines and the neutral. The h.v. windings are connected to a two-phase system with a phase voltage of 6,600 V. Determine the number of turns in each section of the h.v. and l.v. winding and the position of the neutral point if the induced voltage per turn is 8 volts.

Solution. As the volt/turn is 8 and the h.v. side voltage is $6,600 \mathrm{~V}$, the h.v. side turns are $=6,600 / 8=\mathbf{8 2 5}$ on both transformers.

Now, voltage across points $B$ and $C$ of main winding $=$ line voltage $=231 \times \sqrt{3}=400 \mathrm{~V}$

No. of turns on the l.v. side of the main transformer $=$ $400 / 8=50$

No. of turns on the 1.v. side of teaser transformer $=$ $\sqrt{(3 / 2)} \times$ mains turns
$=\sqrt{3} \times 50 / 2=43$ (whole number)
The neutral point on the 3-phase side divides teaser turns in the ratio $1: 2$.


Fig. 33.29
$\therefore \quad$ Number of turns between $A$ and $N=(2 / 3) \times A D$ $=(2 / 3) \times 43=29$

Hence, neutral point is located on the 29th turn from $A$ downwards (Fig. 33.29).
Example 33.24. A Scott-connected (2 to 3-phase) transformer links a 6,000 V, 2-phase system with a 440 V ; 3-phase system. The frequency is 50 Hz , the gross core area is $300 \mathrm{~cm}^{2}$, while the maximum flux density is to be about $1.2 \mathrm{~Wb} / \mathrm{m}^{2}$. Find the number of turns on each winding and the point to be tapped for the neutral wire on the 3-phase side. If the load is balanced on the one side of such a transformer, find whether it will also be balanced on the other side. (London Univ.)

Solution. Use the transformer voltage equation 1,
$E=4.44 f N \Phi_{x}$ volt
Gross core area $=300 \mathrm{~cm}^{2}$
Assuming net iron $=0.9$ of gross area, and considering the h.v. side, we have

$$
\begin{aligned}
& 6000=4.44 \times 50 \times N_{1} \times 1.2\left(300 \times 0.9 \times 10^{-4}\right) \\
& N_{1}=834
\end{aligned}
$$

Hence, h.v. sides of both transformers have 834 turns each.

Now $K=440 / 6000=11 / 150$
$\therefore$ Turns on the $l . v$. side of main transformer

$$
N_{2}=834 \times 11 / 150=61
$$



Fig. 33.30

Turns on the $l$.v. side of teaser $=(\sqrt{3} / 2) \times 61=53$
With reference to Fig. 33.30, number of turns in $A N=53 \times 2 / 3=35$
Example 33.25. A 2-phase, 4-wire, 250 V system is supplied to a plant which has a 3-phase motor load of 30 kVA . Two Scott-connected transformers supply the 250 V motors. Calculate (a) voltage (b) kVA rating of each transformer. Draw the wiring connection diagram.

Solution. (a) Both the main and the teaser have the same voltage rating as the supply voltage i.e. 250 V . The current in the main and the teaser coils is the same as the supply current and is

$$
=\frac{\text { Total kVA }}{2 \times \text { Line voltage }}=\frac{30,000}{2 \times 250}=60 \mathrm{~A} \arcsin \theta
$$

On the three-phase side, current is the same in all coils and is equal to the load line current $=30,000 / \sqrt{3} \times 250=69.3 \mathrm{~A}$

Load voltage on main secondary $=$ line voltage $=250 \mathrm{~V}$

Load voltage on teaser secondary $=0.866 \times 250=216.5 \mathrm{~V}$
Hence, voltage rating of main transformer is $\mathbf{2 5 0 / 2 5 0}$ whereas that of teaser transformer is 250/216.5.

The current rating of main transformer is $\mathbf{6 0 / 6 9 . 3}$ and it is the same for the teaser transformer.
(b) The volt-amp rating of the teaser primary as well as secondary is the same i.e. $60 \times 250 \times 10^{-3}$ $=69.3 \times 216.5 \times 10^{-3}=15 \mathbf{k V A}$

The main volt-ampere rating of secondary is $=250 \times 69.3 \times 10^{-3}=17.3 \mathbf{k V A}$
Incidentally, if two identical transformers are used for providing inter-changeability, then both must be rated at 17.3 kVA . In that case, a total capacity of 34.6 kVA would be required to provide a 30 kVA load.

The wiring connections are shown in Fig. 33.31.


Fig. 33.31

## Tutorial Problem No. 33.3

1. A Scott-connected transformer is fed from a $6,600-\mathrm{V}, 3$-phase network and supplies two single-phase furnaces at 100 V . Calculate the line currents on the 3-phase side when the furnaces take 400 kW and 700 kW respectively at 0.8 power factor lagging. (Elect. Machines II, Indore Univ. 1977)
[With 400 kW on teaser, line currents are $87.2 \mathrm{~A} ; 139 \mathrm{~A} ; 139 \mathrm{~A}$ ]
2. Two 220-V, 1-phase electrical furnaces take loads of 350 kW and 500 kW respectively at a power factor of 0.8 lagging. The main supply is at $11-\mathrm{kV}, 3$-phase, 50 Hz . Calculate current in the 3-phase lines which energise a Scott-connected transformer combination.
(Elect. Machines, Madras Univ. 1978)
[With 350 kW on teaser line currents are : $\mathbf{4 5 . 7} \mathrm{A}$; $61.2 \mathrm{~A} ; 61.2 \mathrm{~A}$ ]
3. Two electric furnaces are supplied with 1-phase current at 80 V from 3-phase, $11,000-\mathrm{V}$ supply mains by means of two Scott-connected transformers with similar secondary windings. Calculate the current flowing kW respectively in each of the 3 -phase lines at U.P.P. when the loads on the two transformers are 550 kW of 800 kW .
[With 550 kW on teaser, line currents are : 57.5 A; 78.2; 78.2 A] (Electrical Machines-I, Madras University, 1977)

### 33.11. Parallel Operation of 3-phase Transformers

All the conditions which apply to the parallel operation of single-phase transformers also apply to the parallel running of 3-phase transformers but with the following additions :

1. The voltage ratio must refer to the terminal voltage of primary and secondary. It is obvious that this ratio may not be equal to the ratio of the number of turns per phase. For example, if $V_{1}, V_{2}$ are the primary and secondary terminal voltages, then for $Y / \Delta$ connection, the turn ratio is $V_{2} /\left(V_{1} / \sqrt{3}\right)$ $=\sqrt{3} V_{2} / V_{1}$.
2. The phase displacement between primary and secondary voltages must be the same for all transformers which are to be connected for parallel operation.
3. The phase sequence must be the same.

4. All three transformers in the 3-phase transformer bank will be of the same construction either core or shell.

Note. (i) In dealing with 3-phase transformers, calculations are made for one phase only. The value of equivalent impedance used is the equivalent impedance per phase referred to secondary.
(ii) In case the impedances of primary and secondary windings are given separately, then primary impedance must be referred to secondary by multiplying it with (transformation ratio) ${ }^{2}$.
(iii) For $Y / \Delta$ or $\Delta / Y$ transformers, it should be remembered that the voltage ratios as given in the questions, refer to terminal voltages and are quite different from turn ratio.

Example 33.26. A load of 500 kVA at 0.8 power factor lagging is to be shared by two threephase transformers $A$ and $B$ of equal ratings. If the equivalent delta impedances as referred to secondary are $(2+j 6) \Omega$ for $A$ and $(2+j 5) \Omega$ for $B$, calculate the load supplied by each transformer.

$$
\begin{aligned}
& \text { Solution. } \quad \mathbf{S}_{\mathbf{A}}=\mathbf{S} \frac{\mathbf{Z}_{\mathbf{B}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}}=\mathbf{S} \frac{\mathbf{1}}{\left.\mathbf{1 + ( \mathbf { Z } _ { \mathbf { A } }} / \mathbf{Z}_{\mathbf{B}}\right)} \\
& \text { Now } \quad \begin{aligned}
& \\
& S=500(0.8-j 0.6)=(400-j 300) \\
& \mathbf{Z}_{\mathrm{A}} / \mathbf{Z}_{\mathbf{B}}=(2+j 6) /(2+j 5)=1.17+j 0.07 ; Z_{B} / \mathbf{Z}_{A}=(2+j 5) /(2+j 6)=0.85-j 0.05 \\
& \mathbf{S}_{\mathbf{A}}=(400-j 300) /(2.17+j 0.07)=180-j 144.2=230.7 \angle-38.7^{\circ} \\
& \cos \phi_{A}=0.78 \text { lagging } \\
& \mathbf{S}_{\mathbf{B}}=(400-j 300) /(1.85-j 0.05)=220.1-j 156=270 \angle-40^{\circ} 28^{\prime} \therefore \cos \Phi_{B}=\mathbf{0} .76 \text { lagging. }
\end{aligned}
\end{aligned}
$$

Example 33.27. State (i) the essential and (ii) the desirable conditions to be satisfied so that two 3-phase transformers may operate successfully in parallel.

A 2,000-kVA transformer (A) is connected in parallel with a 4,000 kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage voltage drops in the windings at their rated loads are as follows :

| Transformer $A$ | resistance $2 \% ;$ | reactance $8 \%$ |
| :--- | :--- | :--- |
| Transformer B | resistance $1.6 \% ;$ | reactance $3 \%$ |

(Elect. Engineering-II, Bombay Univ. 1987)
Solution. On the basis of $4,000 \mathrm{kVA}$

$$
\begin{aligned}
\% Z_{A} & =(4,000 / 2,000)(2+j 8)=(4+j 16)=16.5 \angle 76^{\circ} \\
\% Z_{B} & =(1.6+j 3) ; \% Z_{A}+\% Z_{B}=(5.6+j 16)=19.8 \angle 73.6^{\circ} \\
\mathbf{S} & =5,000 \angle-36.9^{\circ}=(4,000-j 3,000) \\
S_{B} & =\mathbf{S} \cdot \frac{\mathbf{Z}_{\mathbf{A}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}}=5,000 \angle-36.9^{\circ} \times \frac{16.5 \angle 76^{\circ}}{19.8 \angle 73.6^{\circ}} \\
& =5,000 \angle-36.9^{\circ} \times 0.832 \angle 2.4^{\circ}=4,160 \angle-34.5^{\circ}=(3,425-j 2,355) \\
S_{A} & =\mathbf{S}-\mathbf{S}_{\mathrm{B}}=(4,000-j 3,000)-(3,425-j 2355) \\
& =(575-j 645)=864 \angle-48.3^{\circ} \\
\cos \phi_{B} & =\cos 34.5^{\circ}=0.824(\mathrm{lag}) ; \cos \phi_{A}=\cos 48.3^{\circ}=\mathbf{0 . 6 6 5} \text { (lag). }
\end{aligned}
$$

Now

Example 33.28. A load of $1,400 \mathrm{kVA}$ at 0.866 p.f. lagging is supplied by two 3-phase transformers of $1,000 \mathrm{kVA}$ and 500 kVA capacity operating in parallel. The ratio of transformation is the same in both : 6,600/400 delta-star. If the equivalent secondary impedances are ( $0.001+j 0.003$ ) ohm and $(0.0028+j 0.005)$ ohm per phase respectively, calculate the load and power factor of each transformer.
(Elect. Engg-I, Nagpur Univ. 1993)

Solution. On the basis of $1000 \mathrm{kVA}, \mathbf{Z}_{\mathbf{A}}=(0.001+j 0.003) \Omega$

$$
\begin{aligned}
\mathbf{Z}_{\mathbf{B}} & =(1000 / 500)(0.0028+j 0.005)=(0.0056+j 0.01) \Omega \\
\frac{\mathbf{Z}_{\mathbf{A}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}} & =\frac{(0.001+j 0.003)}{(0.0066+j 0.013)}=\frac{3.162 \times 10^{-3} \angle 71.6^{\circ}}{14.57 \times 10^{-3} \angle 63.1^{\circ}}=0.2032 \angle 8.5^{\circ} \\
\mathbf{S} & =1400 \angle \cos ^{-1}(0.866)=1400 \angle-30^{\circ}=(1212-j 700) \\
\mathbf{S}_{\mathbf{B}} & =\mathbf{S} \cdot \frac{\mathbf{Z}_{\mathbf{A}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}}=1400 \angle-30^{\circ} \times 0.2032 \angle 8.5^{\circ} \\
& =284.5 \angle-21.5^{\circ}=265-j 104 \\
\mathbf{S}_{\mathbf{A}} & =\mathbf{S}-\mathbf{S}_{\mathbf{B}}=(1212-j 700)-(265-j 104)=(947-j 596)=1145 \angle-32.2^{\circ} \\
\cos \phi_{A} & =\cos 32.3^{\circ}=0.846(\mathrm{lag}) ; \cos \phi_{B}=\cos 21.5^{\circ}=\mathbf{0 . 9 3}(\mathrm{lag}) .
\end{aligned}
$$

Example 33.29. Two 3-phase transformers $A$ and $B$ having the same no-load line voltage ratio 3,300/400-V supply a load of 750 kVA at 0.707 lagging when operating in parallel. The rating of A is 500 kVA , its resistance is $2 \%$ and reactance $3 \%$. The corresponding values for B are 250 kVA ; $1.5 \%$ and $4 \%$ respectively. Assuming that both transformers have star-connected secondary windings, calculate
(a) the load supplied by each transformer,
(b) the power factor at which each transformer is working,
(c) the secondary line voltage of the parallel circuit.

Solution. On the basis of 500 kVA ,

$$
\begin{aligned}
\% \mathbf{Z}_{\mathrm{A}} & =2+j 3, \% \mathbf{Z}_{\mathrm{B}}=(500 / 200)(1.5+j 4)=(3+j 8) \\
\frac{\mathbf{Z}_{\mathbf{A}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}} & =\frac{2+j 3}{5+j 1}=0.3 \angle-9.3^{\circ} ; \frac{\mathbf{Z}_{\mathbf{B}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}}=\frac{3+j 8}{5+j 11}=0.711 \angle 3.8^{\circ}
\end{aligned}
$$

Now, $\quad S=750 \angle-45^{\circ}$
(a)

$$
\begin{aligned}
\mathbf{S}_{\mathbf{A}} & =\mathbf{S} \frac{\mathbf{Z}_{\mathbf{B}}}{\mathbf{Z}_{\mathbf{A}}+\mathbf{Z}_{\mathbf{B}}}=750 \angle-45^{\circ} \times 0.711 \angle 3.8^{\circ} \\
& =533 \angle-41.2^{\circ}=(400-j 351) \\
\mathbf{S}_{\mathbf{B}} & =750 \angle-45^{\circ} \times 0.3 \angle 09.3^{\circ}=225 \angle-54.3^{\circ} \\
\cos \phi_{A} & =\cos 41.2^{\circ}=0.752(\mathrm{lag}) ; \cos \phi_{B}=\cos 54.3^{\circ}=\mathbf{0 . 5 8 3 5}(\mathrm{lag})
\end{aligned}
$$

(c) Since voltage drop of each transformer is the same, its value in the case of transformer $A$ would only be calculated. Now, for transformer $A, \mathrm{~kW}=400$ for the active component of the current and kVAR $=351$ for the reactive component.
$\therefore \quad \%$ resistive drop $=2 \times 400 / 500$
$=1.6 \% ; \%$ reactive drop $=3 \times 351 / 500=2.1 \%$
Total percentage drop $=1.6+2.1=3.7$
Secondary line voltage $=400-(3.7 \times 400 / 100)=385.2 \mathrm{~V}$.

### 33.12. Instrument Transformers

In d.c. circuit when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with a high resistance connected
in series with them. But it is not convenient to use this methods with alternating current and voltage instruments. For this purpose, specially constructed accurate ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. These instrument transformers are of two kinds : (i) current transformers for measuring large alternating currents and (ii) potential transformers for measuring high alternating voltages.

### 33.13. Current Transformers

These transformers are used with low-range ammeters to measure currents in high-voltage alternating-current circuits where it is not practicable to connect instruments and meters directly to the lines. In addition to insulating the instrument from the high voltage line, they step down the current in a known ratio. The current (or series) transformer has a primary coil of one or more turns of thick wire connected in series with the line whose current is to be measured as shown in Fig. 33.32. The secondary consists of a large number of turns of fine wire and is connected across the ammeter terminals (usually of 5-ampere bracket should be removed or 1-ampere range).


As regards voltage, the transformers is of step-up variety but it is obvious that current will be stepped down. Thus, if the current transformer has primary to secondary current ratio of $100: 5$, then it steps up the voltage 20 times whereas it steps down the current to $1 / 20$ th of its actual value. Hence, if we know current ratio $\left(I_{1} / I_{2}\right)$ of the transformer and the reading of the a.c. ammeter, the line current can be calculated. In fact, line current is given by the current transformation ratio times the reading on the ammeter. One of the most commonly used current transformer is the one known as clamp-on or clip-on type. It has a laminated core which is so arranged that it can be opened out at hinged section by merely pressing a triggr-like projection (Fig. 33.33). When the core is thus opened, it permits the admission of very heavy currentcarrying bus bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current carrying conductor or feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle.

It should be noted that, since the ammeter resistance is very low, the current transformer normally works short circuited. If for any reason, the ammeter is taken out of the secondary winding, then this winding

must be short-circuited with the help of short-circulating switch $S$. If this is not done, then due to the absence of counter amp-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating and a high voltage across the secondary terminals. This is not the case with ordinary constant-potential transformers, because their primary current is determined by the load in their secondary whereas in a current transformer, the primay current is determined entirely by the load on the system and not by the load on its own secondary.

Hence, the secondary of a current transformer should never be left open under any circumstances.
Example 33.30. A $100: 5$ transformer is used in conjunction with a 5-amp ammeter. If the latter reads 3.5 A , find the line current.

Solution. Here, the ratio $100: 5$ stands for the ratio of primary-to-secondary currents i.e. $I_{1} / I_{2}=100 / 5$
$\therefore \quad$ Primary (or line) current $=3.5 \times(100 / 5)=70 \mathrm{~A}$
Example 33.31. It is desired to measure a line current of the order of 2,000 A to 2,500 A. If a standard 5-amp ammeter is to be used along with a current transformer, what should be the turn ratio of the latter? By what factor should the ammeter reading be multiplied to get the line current in each case?

Solution. $I_{1} / I_{2}=2000 / 5=400$ or $2500 / 5=500$. Since $I_{1} / I_{2}=N_{2} / N_{1}$ hence $N_{2} / N_{1}=400$ in the first case and 500 in the second case. It means that $N_{1}: N_{2}:: 1: 400$ or $1: 500$.

Ratio or multiplication factor in the first case is 400 and in the second in the second case 500.

### 33.14. Potential Transformers

These transformers are extremely accurate-ratio step-down transformers and are used in conjunction with standard low-range voltmeters (usually $150-\mathrm{V}$ ) whose deflection when divided by voltage transformation ratio, gives the true voltage on the high voltage side. In general, they are of the shell-type and do not differ much from the ordinary two-winding transformers discussed so far, except that their power rating is extremely small. Upto voltages of 5,000, potential transformers are usually of the dry type, between 5,000 and 13,800 volts, they may be either dry type or oil immersed type, although for voltages above 13,800 they are always oil immersed type. Since their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100 W . For safety, the secondary should be completely insulated from


Small Potential transformer the high-voltage primary and should be, in addition, grounded for affording protection to the operator. Fig. 33.34 shows the connections of such a transformer.


Fig. 33.34


Fig. 33.35

Fig. 33.35 shows the connections of instrument transformers to a wattmeter. While connecting the wattmeter, the relative polarities of the secondary terminals of the transformers with respect to their primary terminals must be known for connections of the instruments.

## OBJECTIVE TEST - 33

1. Which of the following connections is best suited for 3-phase, 4 -wire service ?
(a) $\Delta-\Delta$
(b) $\mathrm{Y}-\mathrm{Y}$
(c) $\Delta-\mathrm{Y}$
(d) $\mathrm{Y}-\Delta$
2. In a three-phase $\mathrm{Y}-\mathrm{Y}$ transformer connection, neutral is fundamental to the
(a) suppression of harmonics
(b) passage of unbalanced currents due to unbalanced loads
(c) provision of dual electric service
(d) balancing of phase voltages with respect to line voltages.
3. As compared to $\Delta-\Delta$ bank, the capacity of the $V-V$ bank of transformers is $\qquad$ percent.
(a) 57.7
(b) 66.7
(c) 50
(d) 86.6
4. If three transformers in a $\Delta-\Delta$ are delivering their rated load and one transformer is removed, then overload on each of the remaining transformers is $\qquad$ percent.
(a) 66.7
(b) 173.2
(c) 73.2
(d) 58
5. When a $\mathrm{V}-\mathrm{V}$ system is converted into a $\Delta-\Delta$ system, increase in capacity of the system is ......... percent.
(a) 86.6
(b) 66.7
(c) 73.2
(d) 50
6. For supplying a balanced $3-\phi$ load of $40-\mathrm{kVA}$, rating of each transformer in $\mathrm{V}-\mathrm{V}$ bank should be nearly $\qquad$ kVA.
(a) 20
(b) 23
(c) 34.6
(d) 25
7. When a closed - $\Delta$ bank is converted into an open $-\Delta$ bank, each of the two remaining transformers supplies $\qquad$ percent of the original load.
(a) 66.7
(b) 57.7
(c) 50
(d) 73.2
8. If the load p.f. is 0.866 , then the average p.f. of the $\mathrm{V}-\mathrm{V}$ bank is
(a) 0.886
(b) 0.75
(c) 0.51
(d) 0.65
9. $\mathrm{AT}-\mathrm{T}$ connection has higher ratio of utilization that $\mathrm{a} \mathrm{V}-\mathrm{V}$ connection only when
(a) identical transformers are used
(b) load power factor is leading
(c) load power factor is unity
(d) non-identical transformers are used.
10. The biggest advantage of $\mathrm{T}-\mathrm{T}$ connection over the $\mathrm{V}-\mathrm{V}$ connection for 3-phase power transformation is that it provides
(a) a set of balanced voltages under load
(b) a true 3-phase, 4-wire system
(c) a higher ratio of utilization
(d) more voltages.
11. Of the following statements concerning parallel operation of transformers, the one which is not correct is
(a) transformers must have equal voltage ratings
(b) transformers must have same ratio of transformation
(c) transformers must be operated at the same frequency
(d) transformers must have equal kVA ratings.
12. Statement

An auto-transformer is more efficient in transferring energy from primary to secondary circuit.

## Reason

Because it does so both inductively and conductively.
Key
(a) statement is false, reason is correct and relevant
(b) statement is correct, reason is correct but irrelevant
(c) both statement and reason are correct and are connected to each other as cause and effect
(d) both statement and reason are false.
13. Out of the following given choices for poly phase transformer connections which one will you select for three-to-two phase conversion ?
(a) Scott
(b) $\mathrm{star} / \mathrm{star}$
(c) double Scott
(d) star/double-delta
14. A T - T transformer cannot be paralleled with ......... transformer.
(a) $\mathrm{V}-\mathrm{V}$
(b) $\mathrm{Y}-\Delta$
(c) $\mathrm{Y}-\mathrm{Y}$
(d) $\Delta-\Delta$
15. Instrument transformers are used on a.c. circuits for extending the range of
(a) ammeters
(b) voltmeters
(c) wattmeters
(d) all of the above.
16. Before removing the ammeter from a current transformer, its secondary must be shortcircuited in order to avoid
(a) excessive heating of the core
(b) high secondary e.m.f.
(c) increase in iron losses
(d) all of the above.

## ANSWERS




[^0]:    * $\overline{\text { In Ex. }} \overline{33.13}$, the three transformers are not supplying their rated load of $20 \times 3=60 \mathrm{kVA}$ but only 40 kVA .

[^1]:    * Overloading becomes $73.2 \%$ only when full rated load is supplied by the $\Delta-\Delta$ bank (i.e. $3 \times 20=60 \mathrm{kVA}$ in this case) before it becomes $V-V$ bank.

