CONTENTS

CHAPTER 30

SPEED CONTROL OF D.C. MOTORS

Learning Objectives

- Factors Controlling Motor Speed
- Speed Control of Shunt Motors
- Speed Control of Series Motors
- Merits and Demerits of Rheostatic Control Method
- Series-Parallel Control
- Electric Braking
- Electric Braking of Shunt Motor
- Electric Braking of Series Motors
- Electronic Speed control Method for D.C. Motors
- Uncontrolled Rectifiers
- Controlled Rectifiers
- Thyristor Choppers
- Thyristor Inverters
- Thyristor Speed Control of Separately-excited D.C. Motor
- Thyristor Speed Control of D.C. Series Motor
- Full-wave Speed Control of a Shunt Motor
- Thyristor Control of a Shunt Motor
- Thyristor Speed Control of a Series D.C. Motor
- Necessity of a Starter
- Shunt Motor Starter
- Three-point Starter
- Four-point Starter
- Starting and Speed Control of Series Motors
- Grading of Starting Resistance
- Shunt Motors
- Series Motor Starters
- Thyristor Controller Starters

DC motor speed controller control the speed of any common dc motor rated upto 100 V. It operates on 5V to 15 V.

CONTENTS

30.1. Factors Controlling Motor Speed

It has been shown earlier that the speed of a motor is given by the relation

$$N = \frac{V - I_a R_a}{Z\Phi} \cdot \left(\frac{A}{P}\right) = K\frac{V - I_a R_a}{\Phi} \text{ r.p.s.}$$

where R_a = armature circuit resistance.

It is obvious that the speed can be controlled by varying (*i*) flux/pole, Φ (Flux Control) (*ii*) resistance R_a of armature circuit (Rheostatic Control) and (*iii*) applied voltage V (Voltage Control). These methods as applied to shunt, compound and series motors will be discussed below.

30.2. Speed Control of Shunt motors

(*i*) Variation of Flux or Flux Control Method

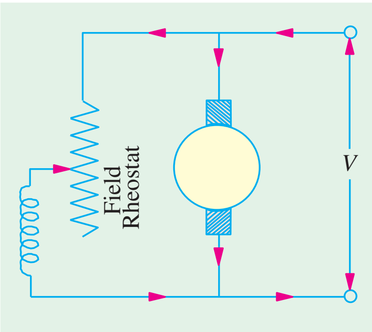


Fig. 30.1

It is seen from above that $N \propto 1/\Phi$. By decreasing the flux, the speed can be increased and *vice versa*. Hence, the name *flux* or *field control* method. The flux of a d.c. motor can be changed by changing I_{sh} with help of a shunt field rheostat (Fig. 30.1). Since I_{sh} is relatively small, shunt field rheostat has to carry only a small current, which means I^2R loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpolar machine, the speed can be increased by this method in the ratio 2 : 1. Any further weakening of flux Φ adversely affects the communication and hence puts a limit to the maximum speed obtainable with the method. In machines fitted with interpoles, a ratio of maximum to minimum speed of 6 : 1 is fairly common.

Example 30.1. *A 500 V shunt motor runs at its normal speed of 250 r.p.m. when the armature current is 200 A. The resistance of armature is 0.12 ohm. Calculate the speed when a resistance is inserted in the field reducing the shunt field to 80% of normal value and the armature current is 100 ampere.* **(Elect. Engg. A.M.A.E. S.I. June 1992)**

Solution. $E_{b1} = 500 - 200 \times 0.12 = 476$ V; $E_{b2} = 500 - 100 \times 0.12 = 488$ V

$$\Phi_2 = 0.8\,\Phi_1 \,;\, N_1 = 250 \text{ rpm} \,;\, N_2 = ?$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{250} = \frac{488}{476} \times \frac{\Phi_1}{0.8\Phi_1},\ N_2 = \textbf{320 r.p.m.}$$

Example 30.2. *A 250 volt d.c. shunt motor has armature resistance of 0.25 ohm, on load it takes an armature current of 50 A and runs at 750 r.p.m. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor.* **(Elect. Eng-II, Pune Univ. 1987)**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, $T_a \propto \Phi\, I_a$. Hence $T_{a1} \propto \Phi_1 I_{a1}$ and $T_{a2} \propto \Phi_2 I_{a2}$.

Since $T_{a1} = T_{a2}$ $\therefore$ $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

Now, $\Phi_2 = 0.9\,\Phi_1$ $\therefore$ $50\,\Phi_1 = 0.9\,\Phi,\ I_{a2} = 55.6$ A

$\therefore$ $E_{b1} = 250 - (50 \times 0.25) = 237.5$ V ; $E_{b2} = 250 - (55.6 \times 0.25) = 231.1$ V

$\therefore$ $$\frac{N_2}{750} = \frac{231.1}{237.5} \times \frac{\Phi_1}{0.9\Phi_1}\ ;\ N_2 = \textbf{811 r.p.m.}$$

Example 30.3. *Describe briefly the method of speed control available for dc motors.*

A 230 V d.c. shunt motor runs at 800 r.p.m. and takes armature current of 50 A. Find resistance *to be added to the field circuit to increase speed to 1000 r.p.m. at an armature current of 80 A. Assume flux proportional to field current. Armature resistance = 0.15 Ω and field winding resistance = 250 Ω.* **(Elect. Technology, Hyderabad Univ. 1991)**

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ since flux ∝ field current

$E_{b1} = 230 - (50 \times 0.15) = 222.5$ V ; $E_{b2} = 230 - (80 \times 0.15) = 218$V

Let R_t = total shunt resistance = $(250 + R)$ where R is the additional resistance

$I_{sh1} = 230/250 = 0.92$ A, $I_{sh2} = 230/R_t$; $N_1 = 800$ r.p.m. ; $N = 1000$ r.p.m.

∴ $\frac{1000}{800} = \frac{218}{222.5} \times \frac{0.92}{230/R_t}$; $R_t = 319\ \Omega$ ∴ $R = 319 - 250 =$ **69 Ω**,

$I_{sh2} = \frac{230}{319} = 0.721$

Ratio of torque in two cases $= \frac{T_2}{T_1} = \frac{I_{sh2}\,I_{a2}}{I_{sh1}\,I_{a1}} = \frac{0.721 \times 80}{0.92 \times 50} =$ **1.254**

Example 30.4. *A 250 V, d.c. shunt motor has shunt field resistance of 250 Ω and an armature resistance of 0.25 Ω. For a given load torque and no additional resistance included in the shunt field circuit, the motor runs at 1500 r.p.m. drawing an armature current of 20 A. If a resistance of 250 Ω is inserted in series with the field, the load torque remaining the same, find out the new speed and armature current. Assume the magnetisation curve to be linear.*

(Electrical Engineering-I, Bombay Univ. 1987)

Solution. In this case, the motor speed is changed by changing the flux.

Now, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since it is given that magnetisation curve is linear, it means that flux is directly proportional to shunt current. Hence $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ where $E_{b2} = V - I_{a2}\,R_a$ and $E_{b1} = V - I_{a1}\,R_a$.

Since load torque remains the same ∴ $T_a \propto \Phi_1\,I_{a1} \propto \Phi_2\,I_{a2}$ or $\Phi_1\,I_{a1} = \Phi_2\,Ia_2$

∴ $I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$

Now, $I_{sh1} = 250/250 = 1$ A; $I_{sh2} = 250/(250 + 250) = 1/2$ A

∴ $I_{a2} = 20 \times \frac{1}{1/2} =$ **40 A** ∴ $E_{b2} = 250 - (40 \times 0.25) = 240$ V and

$E_{b1} = 250 - (20 \times 0.25) = 245$ V ∴ $\frac{N_2}{1500} = \frac{240}{245} \times \frac{1}{1/2}$

∴ N_2 = **2, 930 r.p.m.**

Example 30.5. *A 250 V, d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω. When driving a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance should be inserted in the shunt field circuit ? Assume that the magnetic circuit is unsaturated.*

(Elect. Engg. AMIETE, June 1992)

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since the magnetic circuit is unsaturated, it means that flux is directly proportional to the shunt current.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}} \quad \text{where } E_{b2} = V - I_{a2} R_a \text{ and } E_{b1} = V - I_{a1} R_a$$

Since motor is driving at load of constant torque,

$$T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \therefore \quad \Phi_2 Ia_2 = \Phi_1 I_{a1} \quad \text{or} \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$$

Now, $I_{sh1} = 250/250 = 1\text{ A}$; $I_{sh2} = 250/R_t$

where R_t is the total resistance of the shunt field circuit.

$$\therefore \quad I_{a2} = 20 \times \frac{1}{250/R_t} = \frac{2R_t}{25} \text{ ; } E_{b1} = 250 - (20 \times 0.5) = 240 \text{ V}$$

$$E_{b2} = 250 - \left(\frac{2R_t}{25} \times 0.5\right) = 250 - (R_t/25) \quad \therefore \quad \frac{800}{600} = \frac{250 - (R_t/25)}{240} \times \frac{1}{250/R_t}$$

$$0.04 R_t^2 - 250 R_t + 80{,}000 = 0$$

$$\text{or} \quad R_t = \frac{250 \pm \sqrt{62{,}500 - 12{,}800}}{0.08} = \frac{27}{0.08} = 337.5\,\Omega$$

Additional resistance required in the shunt field circuit = 337.5 – 250 = **87.5 Ω.**

Example 30.6. *A 220 V shunt motor has an armature resistance of 0.5 Ω and takes a current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ?* **(Elect. Machines, AMIE, Sec B, 1991)**

Solution. Formula used is $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$. Since torque remains constant,

$$\text{Hence} \quad \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \therefore \quad I_{a2} = I_{a1} \cdot \frac{\Phi_1}{\Phi_2} = 40x \text{ where } x = \frac{\Phi_1}{\Phi_2}$$

$$E_{b1} = 220 - (40 \times 0.5) = 200 \text{ V} \text{ ; } E_{b2} = 220 - (40x \times 0.5) = (220 - 20x) \text{ V}$$

$$\frac{N_2}{N_1} = \frac{3}{2} \quad \text{... given} \quad \therefore \quad \frac{3}{2} = \frac{(220 - 20x)}{200} \times x \quad \therefore \quad x^2 - 11x + 15 = 0$$

$$\text{or} \quad x = \frac{11 \pm \sqrt{121 - 60}}{2} = \frac{11 \pm 7.81}{2} = 9.4^* \quad \text{or} \quad 1.6$$

$$\therefore \quad \frac{\Phi_1}{\Phi_2} = 1.6 \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{1}{1.6}$$

$$\therefore \quad \frac{\Phi_1 - \Phi_2}{\Phi_1} = \frac{1.6 - 1}{1.6} = \frac{3}{8} \quad \therefore \quad \text{percentage change in flux} = \frac{3}{8} \times 100 = \mathbf{37.5\%}$$

Example 30.7. *A 220-V, 10-kW, 2500 r.p.m. shunt motor draws 41 A when operating at rated conditions. The resistances of the armature, compensating winding, interpole winding and shunt field winding are respectively 0.2 Ω, 0.05 Ω, 0.1 Ω and 110 Ω. Calculate the steady-state values of armature current and motor speed if pole flux is reduced by 25%, a 1 Ω resistance is placed in series with the armature and the load torque is reduced by 50%.*

Solution. $I_{sh} = 220/110 = 2$ A; $I_{a1} = 41 - 2 = 39$ A (Fig. 30.2)

$$T_1 \propto \Phi_1 I_{a1} \text{ and } T_2 \propto \Phi_2 I_{a2}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_{a1}}{I_{a2}}$$

* This figure is rejected as it does not give the necessay increase in speed.

or $\dfrac{1}{2} = \dfrac{3}{4} \times \dfrac{I_{a2}}{39}$ $\therefore$ $I_{a2} = 26$ A

$E_{b1} = 220 - 39\,(0.2 + 0.1 + 0.05) = 206.35$ V

$E_{b2} = 220 - 26\,(1 + 0.35) = 184.9$ V

Now, $\dfrac{N_2}{2500} = \dfrac{184.9}{206.35} \times \dfrac{4}{3}$; $N_2 =$ **2987 r.p.m.**

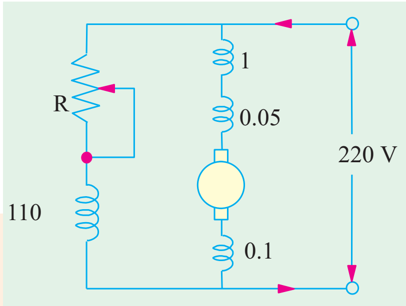


Fig. 30.2

Example 30.8. *A 220 V, 15 kW, 850 r.p.m. shunt motor draws 72.2 A when operating at rated condition. The resistances of the armature and shunt field are 0.25 Ω and 100 Ω respectively. Determine the percentage reduction in field flux in order to obtain a speed of 1650 r.p.m. when armature current drawn is 40 A.*

Solution. $I_{sh} = 220/100 = 2.2$ A; $I_{a1} = 72.2 - 2.2 = 70$ A

$E_{b1} = 220 - 70 \times 0.25 = 202.5$ V, $E_{b2} = 220 - 40 \times 0.25 = 210$ V.

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$ or $\dfrac{1650}{850} = \dfrac{210}{202.5} \times \dfrac{\Phi_1}{\Phi_2}$

$\therefore$ $\Phi_2 = 0.534\,\Phi_1$;

$\therefore$ Reduction in field flux $= \dfrac{\Phi_1 - 0.534\,\Phi_1}{\Phi_1} \times 100 =$ **46.6%.**

Example 30.9. *A 220 V shunt motor has an armature resistance of 0.5 ohm and takes an armature current of 40 A on a certain load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ? Neglect saturation and armature reaction.*

(Elect. Machines, AMIE, Sec B, 1991)

Solution. $T_1 \propto \Phi_1 \times \Phi_1, \rightarrow \Phi_1 I_{a1}$, and $T_2 \propto \Phi_2\, I_{a2}$ Since, $T_1 = T_2$

$\therefore$ $\Phi_2\, I_{a2} = \Phi_1\, I_{a1}$ or $\Phi_2/\Phi_1 = I_{a1}/I_{a2} = 40/I_{a2}$

$E_{b1} = 220 - 40 \times 0.5 = 200$ V ; $E_{b2} = 220 - 0.5\, I_{a2}$

Now, $\dfrac{E_{b2}}{E_{b1}} = \dfrac{N_2\,\Phi_2}{N_1\,\Phi_1}$ or $\dfrac{220 - 0.5\, I_{a2}}{200} = \dfrac{1.5\, N_1}{N_1} \times \dfrac{40}{I_{a2}}$

$\therefore$ $I_{a2} - 40\, I_{a2} + 24{,}000 = 0$ or $I_{a2} = 63.8$ A

$\therefore$ $\dfrac{\Phi_2}{\Phi_1} = \dfrac{40}{63.8} = 0.627$ or $\Phi_2 = 0.627\,\Phi_1 =$ **62.7% of Φ_1**

Example 30.10. *A d.c. shunt motor takes an armature current of 20 A from a 220 V supply. Armature circuit resistance is 0.5 ohm. For reducing the speed by 50%, calculate the resistance required in the series, with the armature, if*

(a) *the load torque is constant*

(b) *the load torque is proportional to the square of the speed.* **(Sambalpur Univ., 1998)**

Solution. $E_{b1} = V - I_a\, r_a = 220 - 20 \times 0.5 = 210$ V

$210 \propto N_1$

(a) Constant Load torque

In a shunt motor, flux remains constant unless there is a change in terminal voltage or there is a change in the field-circuit resistance.

If torque is constant, armature-current then must remain constant. $I_a = 20$ amp

With an external armature-circuit resistor of R ohms, $20 \times (R + 0.5) = 220 - E_{b2}$

The speed required now is $0.5\, N_1$.

With constant flux, $E_b \propto$ speed. Hence, $210 \propto N_1$

$$E_{b2} \propto 0.5\,N_1,\; E_{b2} = 105$$

$$R + 0.5 = (220 - 105)/20 = 5.75, \text{ giving } R = \mathbf{5.25\ ohms}$$

(b) Load torque is proportional to the square of speed.

With constant flux, Developed Torque at N_1 r.p.m. $\propto I_{a1}$

$$T_{m1} \propto 20$$

From the Load Side, $T_{L1} \propto N_1^2$

Since $T_{m1} = T_{L1}$

$$20 \propto N_1^2 \qquad \text{...(a)}$$

At 50% speed, Load Torque, $T_{L2} \propto (0.5\,N_1)^2$

For motor torque, $T_{m2} \propto I_{a2}$

Since $T_{L2} = T_{m2},\; I_{a2} \propto (0.5\,N_1)^2$ ...(b)

From eqn. (a) and (b) above

$$\frac{I_{a2}}{I_{a1}} = 0.25,\; I_{a2} = 5 \text{ amp}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{220 - I_{a2}(R + 0.5)}{210} = 0.5\,N_1$$

$$220 - I_{a2}(R + 0.5) = 0.5 \times 210$$

$$R + 0.5 = \frac{220 - 105}{5} = 23,\; R = \mathbf{22.5\ ohms}$$

Check : With the concept of armature power output : (applied here for part (b) only as an illustration).

$$\text{Armature power–output} = E_b \times I_a = T \times \omega$$

When $T \propto (\text{speed})^2$, $E_b\,I_a = k_1\,T_\omega = K_2\,\omega^3$

At N_1 r.p.m. $210 \times 20 = K_3\,N_1^3$ ...(c)

With constant flux, at half speed

$$E_{b2} = 105$$

$$105 \times I_{a2} = K_3\,(0.5\,N_1)^3 \qquad \text{...(d)}$$

From eqs. (c) and (d),

$$\frac{105 \times I_{a2}}{210 \times 20} = \frac{0.125 \times N_1^3}{N_1^3}, \text{ giving } I_{a2} = 5 \text{ amp}$$

This gives $R = \mathbf{22.5\ ohms.}$

Example 30.11. *A 250 V shunt motor runs at 1000 r.p.m. at no-load and takes 8 A. The total armature and shunt field resistances are respectively 0.2 ohm, 250 ohm. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.*

(Nagpur Univ. Summer 2000)

Solution. The current distribution is shown in Fig. 30.3.

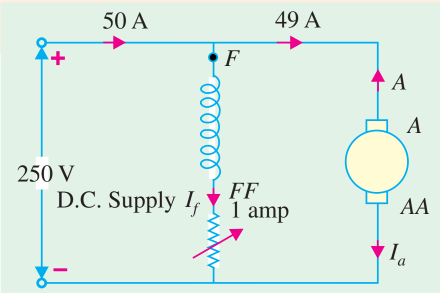


Fig. 30.3

At no load, $I_L = 8$ amp, $I_f = 1$ amp,

Hence, $I_a = 7$ amp

$$E_{b0} = 250 - 7 \times 0.2 = 248.6 \text{ volts},$$

$$= K\,\phi \times 1000$$

$$\therefore \quad K\,\phi = 0.2486$$

At load, I_a = 49 amp

$$E_{b1} = 250 - 49 \times 0.2 = 240.2$$

$$N_1 = \frac{240.2}{0.2486} = 966.2 \text{ r.p.m.}$$

Notes : (*i*) The assumption of constant flux has simplified the issue. Generally, armature reaction tends to weaken the flux and then the speed tends to increase slightly.

(*ii*) The no load armature current of 7 amp is required to overcome the mechanical losses of motor as well as driven load, at about 1000 r.p.m.

Example 30.12. *A 240 V d.c. shunt motor has an armature-resistance of 0.25 ohm, and runs at 1000 r.p.m., taking an armature current 40 A. It is desired to reduce the speed to 800 r.p.m.*

(*i*) *If the armature current remains the same, find the additional resistance to be connected in series with the armature-circuit.*

(*ii*) *If, with the above additional resistance in the circuit, armature current decreases to 20 A, find the speed of the motor.* **(Bhartiar Univ., November 1997)**

Solution. $E_b = 240 - 0.25 \times 40 = 230$ V, $230 \propto 1000$ ...(*a*)

(*i*) For 800 r.p.m., $E_{b2} \propto 800$ ...(*b*)

From **(*a*)** and **(*b*)**, $E_{b2} = \frac{800}{1000} \times 230 = 184$

$$240 - (R + 0.25) \times 40 = 184, \quad R = \mathbf{1.15 \text{ ohm}}$$

(*ii*) $E_{b3} = 240 - 20\,(1.40) = 212$

$$E_{b3} = \frac{N_3}{1000} \times 230 = 212, \quad N_3 = \frac{212}{230} \times 1000 = \mathbf{922 \text{ r.p.m.}}$$

Example 30.13. *A 7.48 kW, 220 V, 990 r.p.m. shunt motor has a full load efficiency of 88%, the armature resistance is 0.08 ohm and shunt field current is 2 A. If the speed of this motor is reduced to 450 r.p.m. by inserting a resistance in the armature circuit, find the motor output, the armature current, external resistance to be inserted in the armature circuit and overall efficiency. Assume the load torque to remain constant.* **(Nagpur Univ., November 1998)**

Solution. With an output of 7.48 kW and an efficiency of 88%, the input power is 8.50 kW. Losses are 1.02 kW.

Input Current = 85000/220 = 38.64 A

Armature-Current = 38.64 – 2.00 = **36.64 A**

Power Loss in the shunt field circuit = $220 \times 2 = 440$ W

Copper-Loss in armature-circuit = $36.64^2 \times 0.08 = 107.4$ W

No-Load-Loss at 900 r.p.m. = 1020 – 107.4 – 440 = 473 W

At 900 r.p.m. Back-emf = $E_{b1} = 200 - (36.64 \times 0.08) = 217.3$ V

Motor will run at 450 r.p.m. with flux per pole kept constant, provided the back-emf = E_{b2}

$$= (459/900) \times 217.1\text{V} = 108.5 \text{ V}$$

There are two simplifying assumptions in this case, which must be stated before further calculations :

1. Load-torque is constant,

2. No Load Losses are constant.

(These statements can be different which leads to variations in the next steps of calculations.)

For constant load-torque, the condition of constant flux per pole results into constant armature current, which is 36.64 A.

With an armature current of 36.64 A, let the external resistance required for this purpose be R.

$$36.64\,R = 217.1 - 108.5 = 108.6 \text{ V}, \quad R = 2.964 \text{ ohms}$$

Total Losses

Total i^2r - loss in armature = $36.64^2 \times (2.964 + 0.08) = 4086$ W

Field-copper-loss + No load loss = 440 + 473 = 913

$$\text{Total Loss} = 4999 \text{ W}$$

$$\text{Total Output} = 8500 - 4999 = 3501 \text{ W}$$

Hence, Efficiency = (3501/8500) × 100 = **41.2%**

(Note : Because of missing data and clarification while making the statements in the question, there can be variations in the assumption and hence in the final solutions.**)**

Example 30.14. *A d.c. shunt motor supplied at 230 V runs at 990 r.p.m. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 r.p.m. assuming that armature current is 25 amp.* **(Nagpur Univ., November 1997)**

Solution. It is assumed that armature resistance is to

(*a*) At 900 r.p.m. : $E_{b1} = 230 = K \times 900$

(*b*) At 500 r.p.m. : $E_{b2} = K \times 500$

Therefore, $E_{b2} = E_{b1} \times 500/900 = 127.8$ volts

The difference between E_{b1} and E_{b2} must be the drop in the external resistance to be added to the armature circuit for the purpose of reducing the speed to 500 r.p.m.

$$E_{b1} - E_{b2} = 25 \times R$$

$$R = (230 - 127.8)/25 = \textbf{4.088 ohms.}$$

Example 30.15. *A 220 V d.c. shunt motor has an armature resistance of 0.4 ohm and a field circuit resistance of 200 ohms. When the motor is driving a constant-torque load, the armature-current is 20 A, the speed being 600 r.p.m. It is desired to run the motor at 900 r.p.m. by inserting a resistance in the field circuit. Find its value, assuming that the magnetic circuit is not saturated.*

(Nagpur Univ., November 1996)

Solution. **(*i*)** At 600 r.p.m. $i_{f1} = 220/200 = 1.1$ amp

$$E_{b1} = 220 - (20 \times 0.4) = 212 \text{ volts}$$

$$T_L = K_1 \times 1.1 \times 20$$

$$\text{The back e.m.f.} = 212 = K_2 \times 1.1 \times 600$$

or $K_2 = 212/660 = 0.3212$

(*ii*) At 900 r.p.m. : $T_L = K_1 \times i_{f2} \times I_{a2}$

Due to constant load torque,

$$i_{f2} \times I_{a2} = 1.1 \times 20 = 22$$

$$E_{b2} = 220 - (0.4\, I_{a2}) = K_2 \times i_{f2} \times 900 = 289\, i_{f2}$$

$$220 - (0.4 \times 22/i_{f2}) = 289\, i_{f2}$$

Guess for approximate value of i_{f2} : Neglecting armature -resistance drop and saturation, 50% rise in speed is obtained with proportional decrease in i_{f2} related by

$$\frac{600}{900} \cong \frac{i_{f2}}{i_{f1}} \text{ giving } i_{f2} \cong 0.73 \text{ amp}$$

[In place of i_{f2}, I_{a2} can be evaluated first. Its guess-work will give $I_{a2} \cong 1.5 \times 20 \cong 30$ amp]

Continuing with the solution of the equation to evaluate a value of i_{f2}, accepting that value which is near 0.73 amp, we have $i_{f2} = 0.71865$. [Note that other value of i_{f2}, which is 0.04235, is not acceptable.]. Corresponding $I_{a2} = 30.6$ amp. Previous shunt field current, $i_{f1} = 1.1$, $R_{f1} = 200\ \Omega$. New shunt field current, $i_{f2} = 0.71865$, $R_{f2} = 220/0.71865 = 306\ \Omega$. Final answer is that a resistor of 106 ohms is to be added to the field circuit to run the motor at 900 r.p.m. at constant torque.

Example 30.16. *A 220 V d.c. shunt motor has an armature resistance of 0.40 ohm and field-resistance of 200 ohms. It takes an armature current of 22 A and runs at 600 r.p.m. It drives a load whose torque is constant. Suggest a suitable method to raise the speed to 900 r.p.m. Calculate the value of the controllable parameter.* **(Nagpur Univ., April 1998)**

Solution. At 600 r.p.m. I_{a1} = 22 amp, N_1 = 600 r.p.m., i_{f1} = 220/200 = 1.1 amp.

$$E_{b1} = 220 - (22 \times 0.40) = 211.2 \text{ volts.}$$

Let the Load-torque be denoted by T_2, k_1 and k_2 in the equations below represent machine constants appearing in the usual emf-equation and torque-equation for the d.c. shunt motor.

$$E_{b1} = 211.2 = k_1 \times i_{f1} \times N_1 = k_1 \times 1.10 \times 600 \quad \text{or} \quad k_1 = 211.2/660 = 0.32$$

$$T_L = k_2 \times i_{f1} \times I_{a1} = k_2 \times 1.10 \times 22$$

Since the load torque will remain constant at 900 r.p.m. also, the corresponding field current (= i_{f2}) and armature current (= i_{a2}) must satisfy the following relationships :

$$T_L = k_2\, i_{f2}\, I_{a2} = k_2 \times 1.10 \times 22$$

or $$i_{f2} \times I_{a2} = 24.2$$

And $$E_{b2} = 220 - (I_{a2} \times 0.40) = k_1 \times i_{f2} \times 900$$

$$220 - (I_{a2} \times 0.40) = 0.32 \times (24.2 \times I_{a2}) \times 900$$

(Alternatively, the above equation can also lead to a quadratic in i_{f2}.)

This leads to a quadratic equation in I_{a2}.

Guess for i_{f2} : Approximately, speed of a d.c. shunt motor is inversely proportional to the field current. Comparing the two speeds of 600 and 900 r.p.m., the value of i_{f2} should be approximately given by

$$i_{f2} \cong i_{f1} \times (600/900) = 0.733 \text{ amp}$$

Guess for I_{a2} : For approximate conclusions, armature-resistance drop can be ignored. With constant load-torque, armature-power must be proportional to the speed.

$$\frac{\text{Armature-power at 900 r.p.m.}}{\text{Armature-power at 600 r.p.m.}} = \frac{900}{600} = 1.5$$

$$E_{b2}\, I_{a2} = 1.5 \times E_{b1}\, I_{a1}$$

Neglecting armature-resistance drops, $E_{b1} - V$ and $E_{b2} - V$.

This gives $$I_{a2} = 1.5 \times 22 = 33 \text{ amps}$$

Thus, out of the two roots for i_{f2}, that which is close to 0.733 is acceptable. If quadratic equation for I_{a2} is being handled, that root which is near 33 amp is acceptable.

Continuing with the solution to quadratic equation for I_{a2}, we have

$$220 - 0.40\, I_{a2} = 0.32 \times (24.2 \times I_{a2}) \times 900$$

$$220 - 0.40\, I_{a2} = 6969/I_{a2}$$

$$I_{a2}^2 - 550\, I_{a2} + 17425 = 0$$

This gives I_{a2} as ***either*** 33.75 amp *or* 791.25 amp.

From the reasoning given above, acceptable root corresponds to I_{a2} = 33.75 amp.

Corresponding field current, i_{f2} = 24.2/33.75 = 0.717 amp

Previous field circuit resistance = 200 ohms

New field circuit resistance = 220/0.717 = 307 ohms

Hence, additional resistance of 107 ohms must be added to the shunt field circuit to run the motor at 900 r.p.m. under the stated condition of constant Load torque.

Additional Check : Exact calculations for proportions of armature-power in two cases will give the necessary check.

$$E_{b2} = 220 - (33.75 \times 0.40) = 206.5$$

As mentioned above, while guessing the value of I_{a2}, the proportion of armature-power should be 1.5.

$$\frac{E_{b2}\,I_{a2}}{E_{b1}\,I_{a1}} = \frac{206.56 \times 33.75}{211.2 \times 22} = 1.50$$

Thus, the results obtained are confirmed.

Example 30.17. *A 250 V, 25 kW d.c. shunt motor has an efficiency of 85% when running at 1000 r.p.m. on full load. The armature resistance is 0.1 ohm and field resistance is 125 ohms. Find the starting resistance required to limit the starting current to 150% of the rated current.*

(Amravati Univ., 1999)

Solution.

Output power = 25 kW, at full-load.

$$\text{Input power} = \frac{25{,}000}{0.85} = 29412 \text{ watts}$$

At Full load, Input Current = 29412/250 = 117.65 amp

Field Current = 250/125 = 2 amp

F.L. Armature Current = 117.65 – 2 = 115.65 amp

Limit of starting current = 1.50 × 115.65 = 173.5 amp

Total resistance in armature circuit at starting

$$= \frac{250}{173.5} = 1.441 \text{ ohms}$$

External resistance to be added to armature circuit

= 1.441 – 0.1 = 1.341 ohm.

Tutorial Problems 30.1

1. A d.c. shunt motor runs at 900 r.p.m. from a 460 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230-V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 460 V. **[595 r.p.m.]**
2. A 250 V shunt motor has an armature resistance of 0.5 Ω and runs at 1200 r.p.m. when the armature current is 80 A. If the torque remains unchanged, find the speed and armature current when the field is strengthened by 25%. **[998 r.p.m. ; 64 A]**
3. When on normal full-load, a 500 V, d.c. shunt motor runs at 800 r.p.m. and takes an armature current 42 A. The flux per pole is reduced to 75% of its normal value by suitably increasing the field circuit resistance. Calculate the speed of the motor if the total torque exerted on the armature is (*a*) unchanged (*b*) reduced by 20%.

   The armature resistance is 0.6 Ω and the total voltage loss at the brushes is 2 V.

   [(*a*) 1,042 r.p.m. (*b*) 1,061 r.p.m.]
4. The following data apply to d.c. shunt motor.

   Supply voltage = 460 V ; armature current = 28 A ; speed = 1000 r.p.m. ; armature resistance = 0.72 Ω. Calculate (*i*) the armature current and (*ii*) the speed when the flux per pole is increased to 120% of the initial value, given that the total torque developed by the armature is unchanged. **[(*i*) 23.33 A (*ii*) 840 r.p.m.]**
5. A 100-V shunt motor, with a field resistance of 50 Ω and armature resistance of 0.5 Ω runs at a speed of 1,000 r.p.m. and takes a current of 10 A from the supply. If the total resistance of the field circuit is reduced to three quarters of its original value, find the new speed and the current taken from the supply. Assume that flux is directly proportional to field current. **[1,089 r.p.m. ; 8.33 A]**

6. A 250 V d.c. shunt motor has armature circuit resistance of 0.5 Ω and a field circuit resistance of 125 Ω. It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to 150 Ω. If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current. **[1186 r.p.m. 33.6 A]**
7. A 250 V, shunt motor with an armature resistance of 0.5 Ω and a shunt field resistance of 250 Ω drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be affected in the shunt field resistance ? Assume that the magnetization curve of the motor is a straight line. **[88 Ω]**
8. A 240 V, d.c. shunt motor runs at 800 r.p.m. with no extra resistance in the field or armature circuit, on no-load. Determine the resistance to be placed in series with the field so that the motor may run at 950 r.p.m. when taking an armature current of 20 A. Field resistance = 160 Ω. Armature resistance = 0.4 Ω. It may be assumed that flux per pole is proportional to field current. **[33.6 Ω]**
9. A shunt-wound motor has a field resistance of 400 Ω and an armature resistance of 0.1 Ω and runs off 240 V supply. The armature current is 60 A and the motor speed is 900 r.p.m.; Assuming a straight line magnetization curve, calculate (*a*) the additional resistance in the field to increase the speed to 1000 r.p.m. for the same armature current and (*b*) the speed with the original field current of 200 A. **[(*a*) 44.4 Ω (*b*) 842.5 r.p.m.]**
10. A 230 V d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of $76\ ^2/_3$ Ω. The motor draws a line current of 13 A while running light at 1000 r.p.m. At a certain load, the field circuit resistance is increased by $38\ ^1/_3$ Ω. What is the new speed of the motor if the line current at this load is 42 A ? **[1400 r.p.m.] (*Electrical Engg. ; Grad I.E.T.E. Dec. 1986*)**
11. A 250 V d.c. shunt motor runs at 1000 r.p.m. and takes an armature current of 25 amp. Its armature resistance is 0.40 ohm. Calculate the speed with increased load with the armature current of 50 amp. Assume that the increased load results into flux-weakening by 3%, with respect to the flux in previous loading condition. **(*Nagpur Univ., April 1996*)**

Hint : (*i*) First Loading condition :

$$E_{b1} = 250 - 25 \times 0.40 = K_1 \times 1000$$

(*ii*) Second Loading condition :

$$E_{b2} = 250 - 50 \times 0.40 = 230\ K_1 \times (0.97\ \phi) \times N_2.$$ This gives N_2. **[988 r.p.m.]**

(*ii*) Armature or Rheostatic Control Method

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 30.4 (*a*). As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed. For a load constant torque, speed is approximately proportional to the p.d. across the armature. From the speed/armature current characteristic [Fig. 30.4 (*b*)], it is seen that greater the resistance in the armature circuit, greater is the fall in the speed.

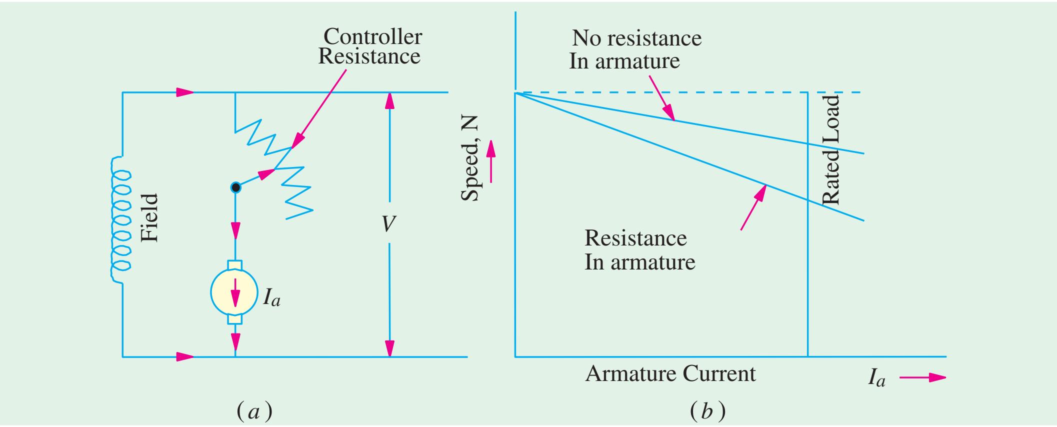


Fig. 30.4

Let I_{a1} = armature current in the first case

I_{a2} = armature current in the second case

(If I_{a1} = I_{a2}, then the load is of constant torque.)

N_1, N_2 = corresponding speeds, V = supply voltage

Then $N_1 \propto V - I_{a1} R_a \propto E_{b1}$

Let some controller resistance of value R be added to the armature circuit resistance so that its value becomes $(R + R_a) = R_t$.

Then $N_2 \propto V - I_{a2} R_t \propto E_{b2} \quad \therefore \quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$

(In fact, it is a simplified form of relation given in Art. 27.9 because here $\Phi_1 = \Phi_2$.)

Considering no-load speed, we have $\dfrac{N}{N_0} = \dfrac{V - I_a R_t}{V - I_{a0} R_a}$

Neglecting $I_{a0} R_a$ with respect to V, we get

$$N = N_0 \left(1 - \frac{I_a R_t}{V}\right)$$

It is seen that for a given resistance R_t the speed is a linear function of armature current I_a as shown in Fig. 30.5 (*a*).

The load current for which the speed would be zero is found by putting $N = 0$ in the above relation.

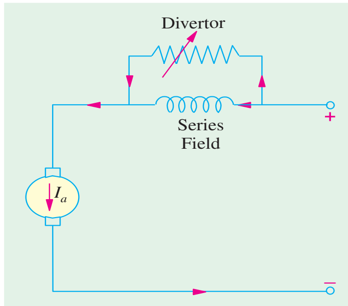


Fig. 30.5 (*a*)

$$\therefore \quad 0 = N_0 \left(1 - \frac{I_a R_t}{V}\right) \quad \text{or} \quad I_a = \frac{V}{R_t}$$

This is the maximum current and is known as ***stalling*** current.

As will be shown in Art. 30.5 (*a*), this method is very wasteful, expensive and unsuitable for rapidly changing loads because for a given value of R_t, speed will change with load. A more stable operation can be obtained by using a divertor across the armature in addition to armature control resistance (Fig. 30.5 (*b*)). Now, the changes in armature current (due to changes in the load torque) will not be so effective in changing the p.d. across the armature (and hence the armature speed).

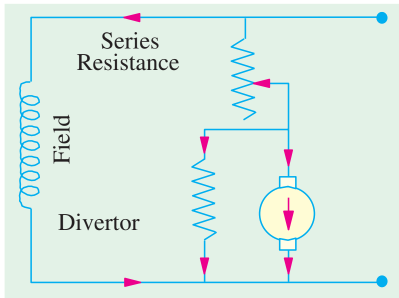


Fig. 30.5 (*b*)

Example 30.18. *A 200 V d.c. shunt motor running at 1000 r.p.m. takes an armature current of 17.5 A. It is required to reduce the speed to 600 r.p.m. What must be the value of resistance to be inserted in the armature circuit if the original armature resistance is 0.4 Ω ? Take armature current to be constant during this process.* **(Elect. Engg. I Nagpur Univ. 1993)**

Solution. N_1 = 1000 r.p.m. ; $E_{b1} = 200 - 17.5 \times 0.4 = 193$ V

R_t = total arm. circuit resistance ; N_2 = 600 r.p.m. ; $E_{b2} = (200 - 17.5\, R_t)$

Since I_{sh} remains constant ; $\Phi_1 = \Phi_2$

$$\therefore \quad \frac{600}{1000} = \frac{(200 - 17.5\, R_t)}{193}\,; \quad R_t = 4.8\ \Omega$$

∴ Additional resistance reqd. $R = R_t - R_a = 4.8 - 0.4 = \mathbf{4.4\ \Omega}$.

It may be noted that brush voltage drop has not been considered.

Example 30.19. *A 500 V d.c. shunt motor has armature and field resistances of 1.2 Ω and 500 Ω respectively. When running on no-load, the current taken is 4 A and the speed is 1000 r.p.m. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 26 A. Estimate the speed at this load if* **(a)** *a resistance of 2.3 Ω is connected in series with the armature and* **(b)** *the shunt field current is reduced by 15%.* **(Electrical Engg. I, Sd. Patel Univ. 1985)**

Solution. $I_{sh} = 500/500 = 1$ A ; $I_{a1} = 4 - 1 = 3$ A

$E_{b1} = 500 - (3 \times 1.2) = 496.4$ V ; $I_{a2} = 26 - 1 = 25$ A

$E_{b2} = 500 - (25 \times 1.2) = 470$ V $\quad \therefore \quad \frac{N_2}{1000} = \frac{470}{496.4}$; $N_2 =$ **947 r.p.m.**

(*a*) In this case, total armature circuit resistance = 1.2 + 2.3 = 3.5 Ω

$\therefore \quad E_{b2} = 500 - (25 \times 3.5) = 412.5$ V $\quad \therefore \quad \frac{N_2}{1000} = \frac{412.5}{496.4}$; $N_2 =$ **831 r.p.m.**

(*b*) When shunt field is reduced by 15%, $\Phi_2 = 0.85\ \Phi_1$ assuming straight magnetisation curve.

$$\frac{N_2}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85} \; ; \; N_2 = \textbf{977.6 r.p.m.}$$

Example 30.20. *A 250-V shunt motor (Fig. 30.6) has an armature current of 20 A when running at 1000 r.p.m. against full load torque. The armature resistance is 0.5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 r.p.m. at the same torque and what will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout and neglect brush contact drop.*

(Elect. Machines AMIE Sec. B Summer 1991)

Solution. $E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240$ V

Let R_t to be total resistance in the armature circuit *i.e.* $R_t = R_a + R$, where R is the additional resistance.

$\therefore \quad E_{b2} = V - I_{a2} R_t = 250 - 20\ R_t$

It should be noted that $I_{a1} = I_{a2} = 20$ A because torque remains the same and $\Phi_1 = \Phi_2$ in both cases.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{500}{1000} = \frac{250 - 20\ R_t}{240}$$

$\therefore \quad R_t = 6.5$ Ω ; hence, $R = 6.5 - 0.5 =$ **6 Ω**

Since the load is halved, armature current is also halved because flux remains constant. Hence, $I_{a3} = 10$ A.

$$\therefore \quad \frac{N_3}{1000} = \frac{250 - 10 \times 6.5}{240} \quad \text{or} \quad N_3 = \textbf{771 r.p.m.}$$

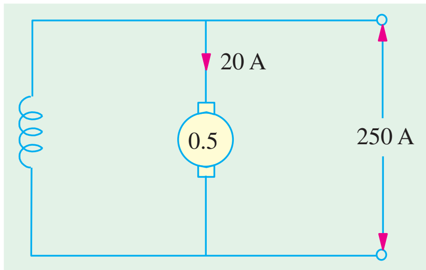


Fig. 30.6

Example 30.21. *A 250-V shunt motor with armature resistance of 0.5 ohm runs at 600 r.p.m. on full-load and takes an armature current of 20 A. If resistance of 1.0 ohm is placed in the armature circuit, find the speed at* (*i*) *full-load torque* (*ii*) *half full-load torque.*

(Electrical Machines-II, Punjab Univ. May 1991)

Solution. Since flux remains constant, the speed formula becomes $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$.

(*i*) In the first case, full-load torque is developed.

$N_1 = 600$ r.p.m.; $E_{b1} = V - I_{a1} R_{a1} = 250 - 20 \times 0.5 = 240$ V

Now, $T \propto \Phi I_a \propto I_a$ (∵ Φ is constant)

$\therefore \quad \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$ Since $T_2 = T_1$; $I_{a2} = I_{a1} = 20$ A

$E_{b2} = V - I_{a2} R_{a2} = 250 - 20 \times 1.5 = 220$ V,

$$N_2 = \frac{N_2}{600} = \frac{220}{240} \; ; \; N_2 = \frac{600 \times 220}{240} = \textbf{550 r.p.m.}$$

(*ii*) In this case, the torque developed is half the full-load torque.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{T_1/2}{T_1} = \frac{I_{a2}}{20} \; ; I_{a2} = 10 \text{ A} \; ; E_{b2} = 250 - 10 \times 1.5 = 235 \text{ V}$$

$$\frac{N_2}{600} = \frac{235}{240} \; ; \; N_2 = 600 \times \frac{235}{240} = \textbf{587.5 r.p.m.}$$

Example 30.22. *A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs of 500 rev. per minute and takes an armature current of 30 A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at* **(a)** *full-load torque* **(b)** *double full-load torque.* **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Since flux remains constant, the speed formula becomes $N_2/N_1 = E_{b2}/E_{b1}$.

(a) Full-load torque

With no additional resistance in the armature circuit,

$$N_1 = 500 \text{ r.p.m.} \; ; I_{a1} = 30 \text{ A} \; ; E_{b1} = 220 - 30 \times 0.5 = 205 \text{ V}$$

Now, $T \propto I_a$ (since Φ is constant.) $\therefore \; \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$ Since $T_2 = T_1$; $I_{a2} = I_{a1} = 30$ A

When additional resistance of 1 Ω is introduced in the armature circuit,

$$E_{b2} = 220 - 30\,(1 + 0.5) = 175 \text{ V} \; ; N_2 = ? \quad \frac{N_2}{500} = \frac{175}{205} \; ; N_2 = \textbf{427 r.p.m.}$$

(b) Double Full-load Torque

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{2T_1}{T_1} = \frac{I_{a2}}{30} \; ; I_{a2} = 60 \text{ A}$$

$$\therefore \quad E_{b2} = 220 - 60\,(1 + 0.5) = 130 \text{ V}$$

$$\therefore \quad \frac{N_2}{500} = \frac{130}{205} \; ; \; N_2 = \textbf{317 r.p.m.}$$

Example 30.23. *The speed of a 50 h.p (37.3 kW) series motor working on 500 V supply is 750 r.p.m. at full-load and 90 per cent efficiency. If the load torque is made 350 N-m and a 5 ohm resistance is connected in series with the machine, calculate the speed at which the machine will run. Assume the magnetic circuit to be unsaturated and the armature and field resistance to be 0.5 ohm.* **(Electrical Machinery I, Madras Univ. 1986)**

Solution. Load torque in the first case is given by

$$T_1 = 37{,}300/2\pi\,(750/60) = 474.6 \text{ N-m}$$

Input current, $I_{a1} = 37{,}300/0.9 \times 500 = 82.9$ A

Now, $T_2 = 250$ N-m ; $I_{a2} = ?$

In a series motor, before magnetic saturation,

$$T \propto \Phi I_a \propto I_{a2} \quad \therefore \quad T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2$$

$$\therefore \quad \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \frac{T_2}{T_1} \quad \therefore \quad I_{a2} = 82.9 \times \sqrt{250/474.6} = 60.2 \text{ A}$$

Now, $E_{b1} = 500 - (82.9 \times 0.5) = 458.5$ V ;

$$E_{b2} = 500 - 60.2\,(5 + 0.5) = 168.9 \text{ V}$$

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a2}}{I_{a1}}$, we get $\frac{N_2}{750} = \frac{168.9}{458.5} \times \frac{82.9}{60.2} \quad \therefore \; N_2 = \textbf{381 r.p.m.}$

Example 30.24. *A 7.46 kW, 220 V, 900 r.p.m. shunt motor has a full-load efficiency of 88 per cent, an armature resistance of 0.08 Ω and shunt field current of 2 A. If the speed of this motor is*

reduced to 450 r.p.m. by inserting a resistance in the armature circuit, the load torque remaining constant, find the motor output, the armature current, the external resistance and the overall efficiency.
(Elect. Machines, Nagpur Univ. 1993)

Solution. Full-load motor input current $I = 7460/220 \times 0.88 = 38.5$ A

$\therefore \quad I_{a1} = 38.5 - 2 = 36.5$ A

Now, $T \propto \Phi I_a$. Since flux remains constant.

$\therefore \quad T \propto I_a \quad \therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2}$ or $\dfrac{T_{a2}}{T_{a1}} = \dfrac{I_{a2}}{I_{a1}}$

It is given that $T_{a1} = T_{a2}$; hence $I_{a1} = I_{a2} =$ **36.5 A**

$E_{b1} = 220 - (36.5 \times 0.08) = 217.1\ \Omega$

$E_{b2} = 220 - 36.5\ R_t$; $N_1 = 900$ r.p.m.; $N_2 = 450$ r.p.m.

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \qquad (\because \Phi_1 = \Phi_2)$$

$$\therefore \quad \frac{450}{900} = \frac{200 - 36.5\ R_t}{217.1}\ ;\ R_t = 3.05\ \Omega$$

$\therefore$ External resistance $R = 3.05 - 0.08 =$ **2.97 Ω**

For calculating the motor output, it will be assumed that all losses except copper losses vary directly with speed.

Since motor speed is halved, stray losses are also halved in the second case. Let us find their value.

In the first case, motor input = $200 \times 38.5 = 8{,}470$ W; Motor output = 7,460 W

Total Cu losses + stray losses = 8470 – 7460 = 1010 W

Arm. Cu loss = $I_{a1}^2 R_a = 36.5^2 \times 0.08 = 107$ W ; Field Cu loss = $220 \times 2 = 440$ W

Total Cu loss = 107 + 440 = 547 W $\quad\therefore$ Stray losses in first case = 1010 – 547 = 463 W

Stray losses in the second case = $463 \times 450/900 = 231$ W

Field Cu loss = 440 W, as before ; Arm. Cu loss = $36.5^2 \times 3.05 = 4{,}064$ W

Total losses in the 2nd case = 231 + 440 + 4,064 = 4,735 W

Input = 8,470 W – as before

Output in the second case = 8,470 – 4,735 = **3,735 W**

$\therefore$ Overall η = 3,735/8,470 = 0.441 or **44.1 per cent***

Example 30.25. *A 240 V shunt motor has an armature current of 15 A when running at 800 r.p.m. against F.L. torque. The arm. resistance is 0.6 ohms. What resistance must be inserted in series with the armature to reduce the speed to 400 r.p.m., at the same torque ?*

What will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout. **(Elect. Machines-I Nagpur Univ. 1993)**

Solution. Here, $N_1 = 800$ r.p.m., $E_{b1} = 240 - 15 \times 0.6 = 231$ V

Flux remaining constant, $T \propto I_a$. Since torque is the same in both cases, $I_{a2} = I_{a1} = 15$ A. Let R be the additional resistance inserted in series with the armature. $E_{b2} = 240 - 15\,(R + 0.6)$; $N_2 = 400$ r.p.m.

$$\therefore \quad \frac{400}{800} = \frac{240 - 15\,(R + 0.6)}{231}\ ;\ R = 7.7\ \Omega$$

* It may be noted that efficiency is reduced almost in the ratio of the two speeds.

When load torque is halved :

With constant flux when load torque is halved, I_a is also halved. Hence, $I_{a3} = I_{a1}/2 = 15/2 = 7.5$ A

$$\therefore \quad E_{b3} = 240 - 7.5\,(7.7 + 0.6) = 177.75 \text{ V} \; ; N_3 = ?$$

$$\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b1}} \text{ or } \frac{N_3}{800} = \frac{177.75}{231} \; ; N_3 = \textbf{614.7 r.p.m.}$$

Example 30.26. **(*a*)** *A 400 V shunt connected d.c. motor takes a total current of 3.5 A on no load and 59.5 A at full load. The field circuit resistance is 267 ohms and the armature circuit resistance is 0.2 ohms (excluding brushes where the drop may be taken as 2 V). If the armature reaction effect at 'full-load' weakens the flux per pole by 2 percentage change in speed from no-load to full-load.*

(*b*) *What resistant must be placed in series with the armature in the machine of (a) if the full-load speed is to be reduced by 50 per cent with the gross torque remaining constant ? Assume no change in the flux.* **(Electrical Machines, AMIE Sec. B, 1989)**

Solution. **(*a*)** Shunt current $I_{sh} = 400/267 = 1.5$ A. At no load, $I_{a1} = 3.5 - 1.5 = 2$ A, $E_{b1} = V - I_{a1} R_{ai}$ – brush drop $= 400 - 2 \times 0.2 - 2 = 397.6$ V. On full-load, $I_{a2} = 59.5 - 1.5 = 58$ A, $E_{b2} = 400 - 58 \times 0.2 - 2 = 386.4$ V.

$$\therefore \quad \frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \text{ or } \frac{397.6}{386.4} = \frac{\Phi_1 N_1}{0.98\,\Phi_1 N_2} ; \frac{N_1}{N_2} = 1.0084$$

$$\% \text{ change in speed} = \frac{N_1 - N_2}{N_1} \times 100$$

$$= \left(1 - \frac{1}{1.0084}\right) \times 100 = 0.833$$

(*b*) Since torque remains the same, I_a remains the same, hence $I_{a3} = I_{a2}$. Let R be the resistance connected in series with the armature.

$$E_{b3} = V - I_{a2}(R_a + R) - \text{brush drop}$$

$$= 400 - 58\,(0.2 + R) - 2 = 386.4 - 58\,R$$

$$\therefore \quad \frac{E_{b2}}{E_{b3}} = \frac{\Phi_2 N_2}{\Phi_3 N_3} = \frac{N_2}{N_3} \qquad (\text{फ} \quad \Phi_2 = \Phi_3)$$

$$\frac{386.4}{386.4 - 58\,R} = \frac{1}{0.5} \; ; \quad R = \textbf{3.338 }\boldsymbol{\Omega}$$

Example 30.27. *A d.c. shunt drives a centrifugal pump whose torque varies as the square of the speed. The motor is fed from a 200 V supply and takes 50 A when running at 1000 r.p.m. What resistance must be inserted in the armature circuit in order to reduce the speed to 800 r.p.m. ? The armature and field resistance of the motor are 0.1 Ω and 100 Ω respectively.*

(Elect. Machines, Allahabad Univ. 1992)

Solution. In general, $T \propto \Phi I_a$

For shunt motors whose excitation is constant,

$$T \propto I_a \propto N^2, \text{ as given.}$$

$$\therefore \quad I_a \propto N^2. \text{ Now } I_{sh} = 200/100 = 2 \text{ A} \quad \therefore \quad I_{a1} = 50 - 2 = 48 \text{ A}$$

Let I_{a2} = new armature current at 800 r.p.m.

then $48 \propto N_1^2 \propto 1000^2$ and $I_{a2} \propto N_2^2 \propto 800^2$

$$\therefore \quad \frac{I_{a2}}{48} = \left(\frac{800}{1000}\right)^2 = 0.8^2 \quad \therefore \quad I_{a2} = 48 \times 0.64 = 30.72 \text{ A}$$

$\therefore \quad E_{b1} = 200 - (48 \times 0.1) = 195.2 \text{ V} \; ; E_{b2} = (200 - 30.72\, R_t) \text{ V}$

Now, $\quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \quad \therefore \quad \dfrac{800}{1000} = \dfrac{200 - 30.72\, R_t}{195.2}, \; R_t = 1.42\ \Omega$

Additional resistance $= 1.42 - 0.1 =$ **1.32 Ω**

Example 30.28. *A 250 V, 50 h.p. (373 kW) d.c. shunt motor has an efficiency of 90% when running at 1,000 r.p.m. on full-load. The armature and field resistances are 0.1 Ω and 115 Ω respectively. Find*

(*a*) *the net and developed torque on full-load.*

(*b*) *the starting resistance to have the line start current equal to 1.5 times the full-load current.*

(*c*) *the torque developed at starting.* **(Elect. Machinery-I, Kerala Univ. 1987)**

Solution. (*a*) $\quad T_{sh} = 9.55 \times 37{,}300/1000 =$ **356.2 N-m**

Input current $= \dfrac{37{,}300}{250 \times 0.9} = 165.8 \text{ A} \; ; I_{sh} = \dfrac{250}{125} = 2 \text{ A}$

$\therefore \quad I_a = 165.8 - 2 = 163.8 \text{ A} \; ; E_b = 250 - (163.8 \times 0.1) = 233.6 \text{ V}$

$\therefore \quad T_a = 9.55 \dfrac{233.6 \times 163.8}{1000} =$ **365.4 N-m**

(*b*) F.L. input line $I = 165.8$ A ; Permissible input $I = 165.8 \times 1.5 = 248.7$ A

Permissible armature current $= 248.7 - 2 = 246.7$ A

Total armature resistance $= 250/246.7 = 1.014\ \Omega$

$\therefore$ Starting resistance required $= 1.014 - 0.1 =$ **0.914 Ω**

(*c*) Torque developed with 1.5 times the F.L. current would be practically 1.5 times the F.L. torque.

i.e. $\quad 1.5 \times 365.4 =$ **548.1 N-m.**

Example 30.29. *A 200 V shunt motor with a shunt resistance of 40 Ω and armature resistance of 0.02 Ω takes a current of 55 A and runs at 595 r.p.m. when there is a resistance of 0.58 Ω in series with armature. Torque remaining the same, what change should be made in the armature circuit resistance to raise the speed to 630 r.p.m. ? Also find*

(*i*) *At what speed will the motor run if the load torque is reduced such that armature current is 15 A.*

(*ii*) *Now, suppose that a divertor of resistance 5 Ω is connected across the armature and series resistance is so adjusted that motor speed is again 595 r.p.m., when armature current is 50 A. What is the value of this series resistance ? Also, find the speed when motor current falls of 15 A again.*

Solution. The circuit is shown in Fig. 30.7.

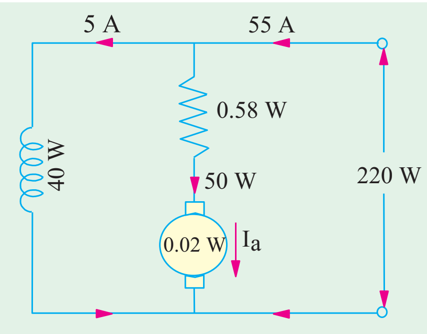


Fig. 30.7

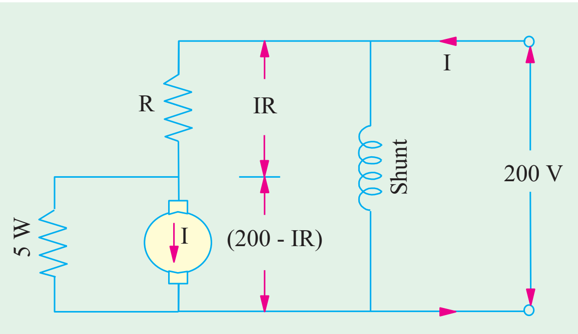


Fig. 30.8

$$I_{sh} = 200/40 = 5\text{ A} \quad \therefore \quad I_{a1} = 55 - 5 = 50\text{ A}$$

$$\text{Armature circuit resistance} = 0.58 + 0.02 = 0.6\ \Omega$$

$$\therefore \quad E_{b1} = 200 - (50 \times 0.6) = 170\text{ V}$$

Since torque is the same in both cases, $I_{a1}\,\Phi_1 = I_{a2}\,\Phi_2$

Moreover, $\Phi_1 = \Phi_2 \quad \therefore \quad I_{a1} = I_{a2} \quad \therefore \quad I_{a2} = 50\text{ A}$

Now $E_{b1} = 170\text{ V}, \quad N_1 = 595\text{ r.p.m.}, \quad N_2 = 630\text{ r.p.m.}, \quad E_{b2} = ?$

Using $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$ $(\because \quad \Phi_1 = \Phi_2)$

we get $E_{b2} = 170 \times (630/595) = 180\text{ V}$

Let R_2 be the new value of armature circuit resistance for raising the speed from 595 r.p.m. to 630 r.p.m.

$$\therefore \quad 180 = 200 - 50\,R_2 \quad \therefore \quad R = 0.4\ \Omega$$

Hence, armature circuit resistance should be reduced by 0.6 – 0.4 = **0.2 Ω.**

(*i*) We have seen above that

$$I_{a1} = 50\text{ A}, E_{b1} = 170\text{ V}, N_1 = 595\text{ r.p.m.}$$

If $I_{a2} = 15\text{ A}, \quad E_{b2} = 200 - (15 \times 0.6) = 191\text{ V}$

$$\therefore \quad \frac{N_2}{595} = \frac{191}{170} \quad \therefore \quad N_2 = \mathbf{668.5\ r.p.m.}$$

(*ii*) When armature divertor is used (Fig. 30.8).

Let R be the new value of series resistance

$$\therefore \quad E_{b3} = 200 - IR - (50 \times 0.02) = 199 - IR$$

Since speed is 595 r.p.m., E_{b3} must be equal to 170 V

$$\therefore \quad 170 = 199 - IR \quad \therefore \quad IR = 29\text{ V; P.D. across divertor} = 200 - 29 = 171\text{ V}$$

$$\text{Current through divertor } I_d = 171/5 = 34.2\text{ A} \quad \therefore \quad I = 50 + 34.2 = 84.2\text{ A}$$

As $IR = 29\text{ V} \quad \therefore \quad R = 29/84.2 = $ **0.344 W**

When $I_a = 15\text{ A, then } I_d = (I - 15)\text{ A}$

$$\text{P.D. across divertor} = 5\,(I - 15) = 200 - 0.344\,I \quad \therefore \quad I = 51.46\text{ A}$$

$$E_{b4} = 200 - 0.344\,I - (15 \times 0.02)$$
$$= 200 - (0.344 \times 51.46) - 0.3 = 182\text{ V}$$

$$\therefore \quad \frac{N_4}{N_1} = \frac{E_{b4}}{E_{b1}} \quad \text{or} \quad \frac{N_4}{595} = \frac{182}{170} \quad \therefore \quad N_4 = \mathbf{637\ r.p.m.}$$

The effect of armature divertor is obvious. The speed without divertor is 668.5 r.p.m. and with armature divertor, it is 637 r.p.m.

(*iii*) Voltage Control Method

(*a*) Multiple Voltage Control

In this method, the shunt field of the motor is connected permanently to a fixed exciting voltage, but the armature is supplied with different voltages by connecting it across one of the several different voltages by means of suitable switchgear. The armature speed will be approximately proportional to these different voltages. The intermediate speeds can be obtained by adjusting the shunt field regulator. The method is not much used, however.

(*b*) Ward-Leonard System

This system is used where an unusually wide (upto 10 : 1) and very sensitive speed control is required as for colliery winders, electric excavators, elevators and the main drives in steel mills and blooming and paper mills. The arrangement is illustrated in Fig. 30.9.

M_1 is the main motor whose speed control is required. The field of this motor is permanently connected across the d.c. supply lines. By applying a variable voltage across its armature, any desired speed can be obtained. This variable voltage is supplied by a motor-generator set which consists of either a d.c. or an a.c. motor M_2 directly coupled to generator G.

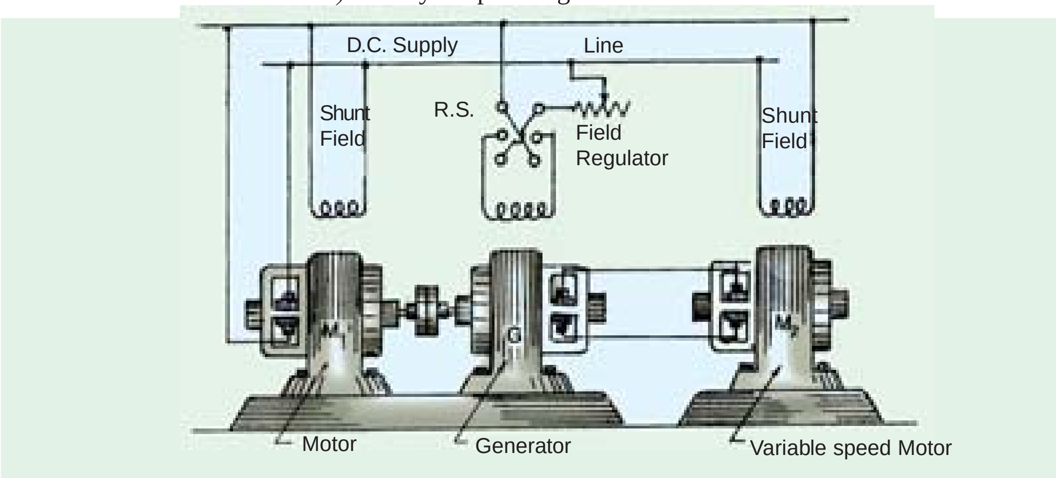


Fig. 30.9

The motor M_2 runs at an approximately constant speed. The output voltage of G is directly fed to the main motor M_1. The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator. By reversing the direction of the field current of G by means of the reversing switch *RS*, generated voltage can be reversed and hence the direction of rotation of M_1. It should be remembered that motor generator set always runs in the same direction.

Despite the fact that capital outlay involved in this system is high because ***(i)*** a large output machine must be used for the motor generator set and ***(ii)*** that two extra machines are employed, still it is used extensively for elevators, hoist control and for main drive in steel mills where motor of ratings 750 kW to 3750 kW are required. The reason for this is that the almost unlimited speed control in either direction of rotation can be achieved entirely by field control of the generator and the resultant economies in steel production outwiegh the extra expenditure on the motor generator set.

A modification of the Ward-Leonard system is known as Ward-Leonard-Ilgner system which uses a smaller motor-generator set with the addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit. When main motor M_1 becomes suddenly overloaded, the driving motor M_2 of the motor generator set slows down, thus allowing the inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor M_1, then M_2 speeds up, thereby again storing energy in the flywheel.

When the Ilgner system is driven by means of an a.c. motor (whether induction or synchronous) another refinement in the form of a 'slip regulator' can be usefully employed, thus giving an additional control.

The chief disadvantage of this system is its low overall efficiency especially at light loads. But as said earlier, it has the outstanding merit of giving wide speed control from maximum in one direction through zero to the maximum in the opposite direction and of giving a smooth acceleration.

Example 30.30. *The O.C.C. of the generator of a Ward-Leonard set is*

Field amps :	*1.4*	*2.2*	*3*	*4*	*5*	*6*	*7*	*8*
Armature volts :	*212*	*320*	*397*	*472*	*522*	*560*	*586*	*609*

The generator is connected to a shunt motor, the field of which is separately-excited at 550 V. If the speed of motor is 300 r.p.m. at 550 V, when giving 485 kW at 95.5% efficiency, determine the excitation of the generator to give a speed of 180 r.p.m. at the same torque. Resistance of the motor *armature circuit = 0.01 Ω, resistance of the motor field = 60 Ω, resistance of generator armature circuit = 0.01 Ω. Ignore the effect of armature reaction and variation of the core factor and the windage losses of the motor.*

Solution. Motor input $= 485 \times 10^3/0.955 = 509{,}300$ W

Motor to motor field $= 550/60 = 55/6$ A

Input to motor field $= 550 \times 55/6 = 5{,}040$ W

$\therefore$ Motor armature input $= 509{,}300 - 5{,}040 = 504{,}260$ W

$\therefore$ Armature current $= 504{,}260/550 = 917$ A

Back e.m.f. E_{b1} at 300 r.p.m. $= 550 - (917 \times 0.01) = 540.83$ V

Back e.m.f. E_{b2} at 180 r.p.m. $= 540.83 \times 180/300 = 324.5$ V

Since torque is the same, the armature current of the main motor is also the same *i.e.* 917 A because its excitation is independent of its speed.

$\therefore$ $V = 324.5 + (917 \times 0.01) = 333.67$ V

Generated e.m.f. $= V + I_a R_a$ ...for generator

$333.67 + (917 \times 0.011) = 343.77$ V.

If O.C.C. is plotted from the above given data, then it would be found that the excitation required to give 343.77 V is 2.42 A.

$\therefore$ Generator exciting current = **2.42 A**

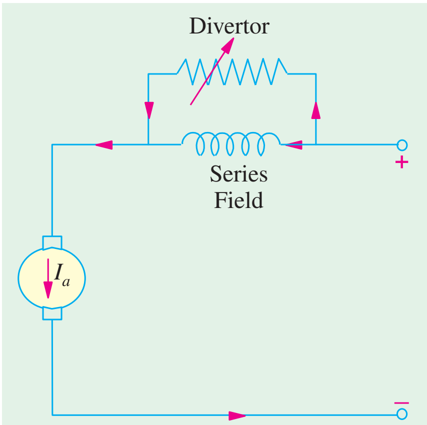


Fig. 30.10

30.3. Speed Control of Series Motors

1. Flux Control Method

Variations in the flux of a series motor can be brought about in any one of the following ways :

(*a*) Field Divertors

The series winding are shunted by a variable resistance known as field divertor (Fig. 30.10). Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.

(*b*) Armature Divertor

A divertor across the armature can be used for giving speeds lower than the normal speed (Fig. 30.11). For a ***given constant load torque***, if I_a is reduced due to armature divertor, the Φ must increase.

($\because$ $T_a \propto \Phi I_a$). This results in an increase in current taken from the supply (which increases the flux and a fall in speed ($N \propto I/\Phi$)). The variation in speed can be controlled by varying the divertor resistance.

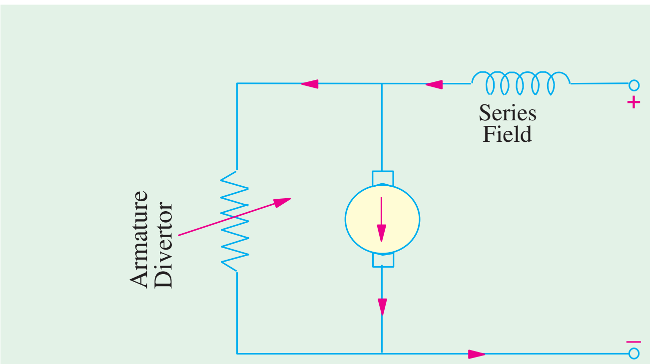


Fig. 30.11

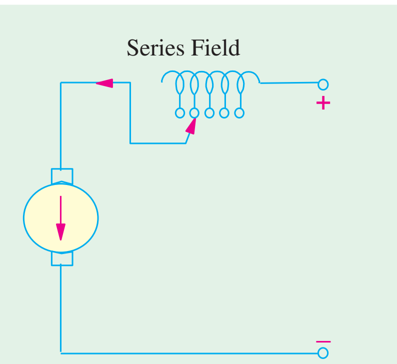


Fig. 30.12

(c) Trapped Field Control Field

This method is often used in electric traction and is shown in Fig. 30.12.

The number of series filed turns in the circuit can be changed at will as shown. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

(d) Paralleling Field coils

In this method, used for fan motors, several speeds can be obtained by regrouping the field coils as shown in Fig. 30.13. It is seen that for a 4-pole motor, three speeds can be obtained easily. (Ex.30.35)

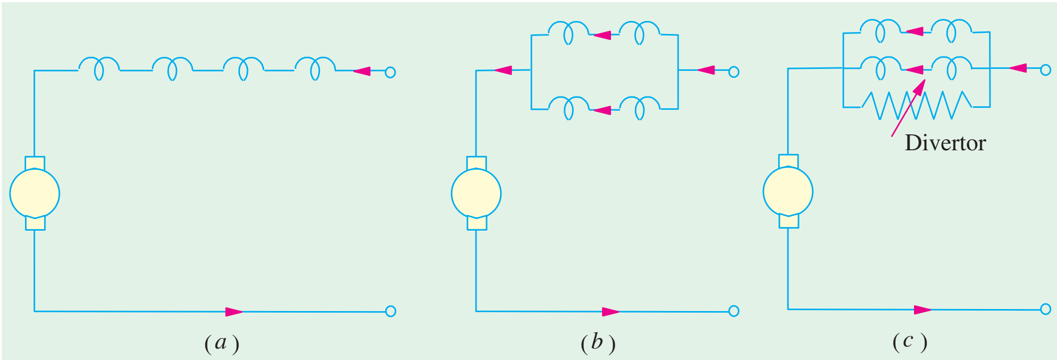


Fig. 30.13

2. Variable Resistance in Series with Motor

By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

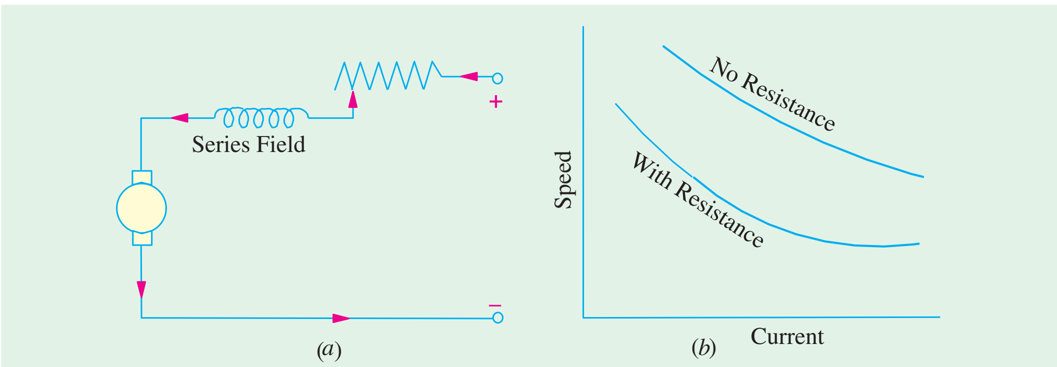


Fig. 30.14

Example 30.31. *A d.c. series motor drives a load the torque of which varies as the square of the speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and the current when the motor field winding is shunted by a divertor of the same resistance as that of the field winding. Mention the assumptions made, if any.* **(Elect. Machines, AMIE Sec B, 1993)**

Solution.

$$T_{a1} \propto N_1^2,\ T_{a2} \propto N_2^2 \quad \therefore \quad T_{a2}/T_{a1} = N_2^2/N_1^2$$

Also, $T_{a1} \propto \Phi_1 I_{a1} \propto I_{a1}^2,\ T_{a2} \propto \Phi_2 I_{a2} \propto (I_{a2}/2)\, I_{a2} \propto I_{a2}^2/2$

It is so because in the second case, field current is half the armature current.

$$\therefore \quad \frac{N_2^2}{N_1^2} = \frac{I_{a2}^2/2}{I_{a1}^2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{I_{a2}}{\sqrt{2}\, I_{a1}} \qquad \ldots (i)$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

If we neglect the armature and series winding drops as well as brush drop, then $E_{b1} = E_{b2} = V$

$$\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}/2} = \frac{2 I_{a1}}{I_{a2}} \qquad \text{...}(ii)$$

From (*i*) and (*ii*), $\frac{I_{a2}}{\sqrt{2}\, I_{a1}} = \frac{2 I_{a1}}{I_{a2}}$ or $I_{a2}^2 = 2\sqrt{2}\, I_{a1}^2 = 2\sqrt{2} \times 15^2$ or $I_{a2} = 25.2$ A

From (*ii*), we get, $N_2 = 600 \times 2 \times 15/252 =$ **714 r.p.m.**

Example 30.32. *A 2-pole series motor runs at 707 r.p.m. when taking 100 A at 85 V and with the field coils in series. The resistance of each field coil is 0.03 Ω and that of the armature 0.04 Ω. If the field coils are connected in parallel and load torque remains constant, find* (*a*) *speed* (*b*) *the additional resistance to be inserted in series with the motor to restore the speed to 707 r.p.m.*

Solution. Total armature circuit resistance = 0.04 + (2 × 0.03) = 0.1 Ω

$$I_{a1} = 100 \text{ A} ; E_{b1} = 85 - (100 \times 0.1) = 75 \text{ V}$$

When series field windings are placed in parallel, the current through each is half the armature current.

If I_{a2} = new armature current ; then $\Phi_2 \propto I_{a2}/2$.

As torque is the same in the two cases,

$$\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \text{or} \quad I_{a1}^{\,2} = \frac{I_{a2}}{2} \times I_{a2} = \frac{I_{a2}^{\,2}}{2}$$

$$\therefore \quad 100^2 = I_{a2}^{\,2} \qquad \therefore \quad I_{a2} = 100\sqrt{2} = 141.4 \text{ A}$$

In this case, series field resistance = 0.03/2 = 0.015 Ω

$$\therefore \quad E_{b2} = 85 - 141.4\,(0.04 + 0.015) = 77.22 \text{ V}$$

$$\frac{N_2}{707} = \frac{77.22}{75} \times \frac{100}{141.4/2} \qquad (\because \quad \Phi_2 \propto I_{a2}/2)$$

(*a*) $\therefore \quad N_2 = 707 \times \frac{77.22}{75} \times \frac{200}{141.4} =$ **1029 r.p.m.**

(*b*) Let the total resistance of series circuit be R_t.

Now, $E_{b1} = 77.22$ V, $N_1 = 1029$ r.p.m. ; $E_{b2} = 85 - 141.4\, R_t$, $N_2 = 707$ r.p.m.

$$\frac{707}{1029} = \frac{85 - 141.4\, R_t}{77.22} \qquad \therefore \quad R_t = 0.226\ \Omega$$

∴ Additional resistance = 0.226 – 0.04 – 0.015 = **0.171 Ω**

Example 30.33. *A 240 V series motor takes 40 amperes when giving its rated output at 1500 r.p.m. Its resistance is 0.3 ohm. Find what resistance must be added to obtain rated torque* (*i*) *at starting* (*ii*) *at 1000 r.p.m.* **(Elect. Engg., Madras Univ. 1987)**

Solution. Since torque remains the same in both cases, it is obvious that current drawn by the motor remains constant at 40 A. $(\because \quad T_a \propto I_a^2)$

(*i*) If R is the series resistance added, then $40 = 240/(R + 0.3)$ ∴ $R = 5.7$ Ω

(*ii*) Current remaining constant, $T_a \propto E_b/N$ – Art. 29.7

$$\frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2}$$

Now, $E_{b1} = 240 - 40 \times 0.3 = 228$ V ; $N_1 = 1500$ r.p.m.

$E_{b2} = 240 - 40\,(R + 0.3)$ V ; $N_2 = 1000$ r.p.m.

$$\therefore \quad \frac{228}{1500} = \frac{240 - 40\,(R + 0.3)}{1000} \; ; \; R = \mathbf{1.9\ \Omega}$$

Example 30.34. *A 4-pole, series-wound fan motor runs normally at 600 r.p.m. on a 250 V d.c. supply taking 20 A. The field coils are connected at in series. Estimate the speed and current taken by the motor if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.* **(Elect. Machines, AMIE, Sec B. 1990)**

Solution. When coils are connected in two parallel groups, current through each becomes $I_{a2}/2$ where I_{a2} is the new armature current.

Hence, $\quad \Phi_2 \propto I_{a2}/2$

Now $\quad T_a \propto \Phi\, I_a$

$\quad \propto N^2$ – given

$$\therefore \quad \Phi_1 I_{a1} \propto N_1^2 \text{ and } \Phi_2 I_{a2} \propto N_2^2 \quad \therefore \quad \left(\frac{N_2}{N_1}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \text{... }(i)$$

Since losses are negligible, field coil resistance as well as armature resistance are negligible. It means that armature and series field voltage drops are negligible. Hence, back e.m.f. in each case equals the supply voltage.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ becomes } \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} \quad \text{... }(ii)$$

Putting this value in **(*i*)** above, we get

$$\left(\frac{\Phi_1}{\Phi_2}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \text{or} \quad \frac{I_{a2}}{I_{a1}} = \left(\frac{\Phi_1}{\Phi_2}\right)^3$$

Now, $\Phi_1 \propto 20$ and $\Phi_2 \propto I_{a2}/2 \quad \therefore \quad \dfrac{I_{a2}}{20} = \left(\dfrac{20}{I_{a2}/2}\right)^3$ or $I_{a2} = 20 \times 2^{3/4} = \mathbf{33.64\ A}$

From **(*ii*)** above, we get $\dfrac{N_2}{N_1} = \dfrac{\Phi_1}{\Phi_2} = \dfrac{I_{a1}}{I_{a2}/2} = \dfrac{2\,I_{a1}}{I_{a2}}$; $N_2 = 600 \times 2 \times 20/33.64 = \mathbf{714\ r.p.m.}$

Example 30.35. *A d.c. series motor having a resistance of 1 Ω drives a fan for which the torque varies as the square of the speed. At 220 V, the set runs at 350 r.p.m. and takes 25 A. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated.*

(Electrical Engg., Banaras Hindu Univ. 1998)

Solution. Since $\Phi \propto I_a$, hence $T_a \propto \Phi\, I_a \propto I_a^2$. Also $T_a \propto N^2$ ...(given)

$$\therefore \quad I_a^2 \propto N^2 \quad \text{or} \quad I_a \propto N \quad \text{or} \quad I_{a1} \propto N_1 \text{ and } I_{a2} \propto N_2$$

$$\therefore \quad \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{500}{350}\,; \; I_{a2} = 25 \times \frac{500}{350} = \frac{250}{7}\ \text{A}$$

$$E_{b1} = 220 - 25 \times 1 = 195\ \text{V} \; ; \; E_{b1} = \text{V} - (250/7) \times 1, \; \frac{\Phi_1}{\Phi_2} = \frac{25}{250/7} = \frac{7}{10}$$

$$\text{Now} \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}\,; \; \frac{500}{350} = \frac{V - (250/7)}{195} \times \frac{7}{10}\,; \; V = \mathbf{433.7\ V}$$

Example 30.36. *A d.c. series motor runs at 1000 r.p.m. when taking 20 A at 200 V. Armature resistance is 0.5 Ω. Series field resistance is 0.2 Ω. Find the speed for a total current of 20 A when a divertor of 0.2 Ω resistance is used across the series field. Flux for a field current of 10 A is 70 per cent of that for 20 A.*

Solution. $E_{b1} = 200 - (0.5 + 0.2) \times 20 = 186$ V ; $N_1 = 1000$ r.p.m.

Since divertor resistance equals series field resistance, the motor current of 20 A is divided equally between the two. Hence, a current of 10 A flows through series field and produces flux which is 70% of that corresponding to 20 A. In other words, $\Phi_2 = 0.7$ or $\Phi_1/\Phi_2 = 1/0.7$

Moreover, their combined resistance = 0.2/2 = 0.1 Ω

Total motor resistance becomes = 0.5 + 0.1 = 0.6 Ω

$$\therefore \quad E_{b2} = 200 - 0.6 \times 20 = 188 \text{ V} ; N_2 = ?$$

$$\therefore \quad \frac{N_2}{1000} = \frac{188}{186} \times \frac{1}{0.7} ; \; N_2 = \textbf{1444 r.p.m.}$$

Example 30.37. *A 200 V, d.c. series motor takes 40 A when running at 700 r.p.m. Calculate the speed at which the motor will run and the current taken from the supply if the field is shunted by a resistance equal to the field resistance and the load torque is increased by 50%.*

Armature resistance = 0.15 Ω, field resistance = 0.1 Ω

It may be assumed that flux per pole is proportional to the field.

Solution. In a series motor, prior to magnetic saturation

$$T \propto \Phi I_a \propto I_a^2 \quad \therefore \quad T_1 \propto I_{a1}^2 \propto 40^2 \qquad \text{...}(i)$$

If I_{a2} is the armature current (or motor current) in the second case when divertor is used, then only $I_{a2}/2$ passes through the series field winding.

$$\therefore \quad \Phi_2 \propto I_{a2}/2 \text{ and } T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) \times I_{a2} \propto I_{a2}^2/2 \qquad \text{...}(ii)$$

From (*i*) and (*ii*), we get $\dfrac{T_2}{T_1} = \dfrac{I_{a2}^2}{2 \times 40^2}$

Also $T_2/T_1 = 1.5 \quad \therefore \quad 1.5 = I_{a2}/2 \times 40^2$

$$\therefore \quad I_{a2} = \sqrt{1.5 \times 2 \times 40^2} = 69.3 \text{ A} ;$$

Now $E_{b1} = 220 - (40 \times 0.25) = 210$ V

$$E_{b2} = 220 - (69.3 \times 0.2^{*}) = 206.14 \text{ V} ; N_1 = 700 \text{ r.p.m.} ; N_2 = ?$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \therefore \quad \frac{N_2}{700} = \frac{206.14}{210} \times \frac{40}{69.3/2} \quad \therefore \quad N_2 = \textbf{794 r.p.m.}$$

Example 30.38. *A 4-pole, 250 V d.c. series motor takes 20 A and runs at 900 r.p.m. Each field coil has resistance of 0.025 ohm and the resistance of the armature is 0.1 ohm. At what speed will the motor run developing the same torque if :*

(*i*) *a divertor of 0.2 ohm is connected in parallel with the series field*

(*ii*) *rearranging the field coils in two series and parallel groups*

Assume unsaturated magnetic operation.

(Electric Drives and their Control, Nagpur Univ. 1993)

Solution. The motor with its field coils all connected in series is shown in Fig. 30.15 (*a*). Here, $N_1 = 900$ r.p.m., $E_{b1} = 250 - 20 \times (0.1 + 4 \times 0.025) = 246$ V.

In Fig. 30.15 (*b*), a divertor of resistance 0.2 Ω has been connected in parallel with the series field coils. Let I_{a2} be the current drawn by the motor under this condition. As per current-divider rule, part of the current passing through the series fields is $I_{a2} \times 0.2/(0.1 + 0.2) = 2\, I_{a2}/3$. Obviously, $\Phi_2 \propto 2\, I_{a2}/3$.

* The combined resistance of series field winding and the divertor is 0.1/2 = 0.05 Ω. Hence, the total resistance = 0.15 + 0.05 = 0.2 Ω, in example 30.37.

Now, $T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2$; $T_2 \propto \Phi_2 I_{a2} \propto (2 I_{a2}/3) I_{a2} \propto 2 I_{a2}^2/3$

Since $T_1 = T_2$; $\therefore I_{a1}^2 = 2 I_{a2}^2/3$ or $20^2 = 2 I_{a2}^2/3$; $\therefore I_{a2} = 24.5$ A.

Combined resistance of the field and divertor = 0.2 × 0.1/0.3 = 0.667 Ω ; Arm. circuit resistance = 0.1 + 0.0667 = 0.1667 Ω; E_{b2} = 250 – 24.5 × 0.1667 = 250 – 4.1 = 245.9 V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} ; \frac{N_2}{900} = \frac{245.9}{246} \times \frac{20}{(2/3)\,24.5} ; N_2 = 1102 \text{ r.p.m.} \qquad ...(\because \Phi_2 \propto 2 I_{a2}/3)$$

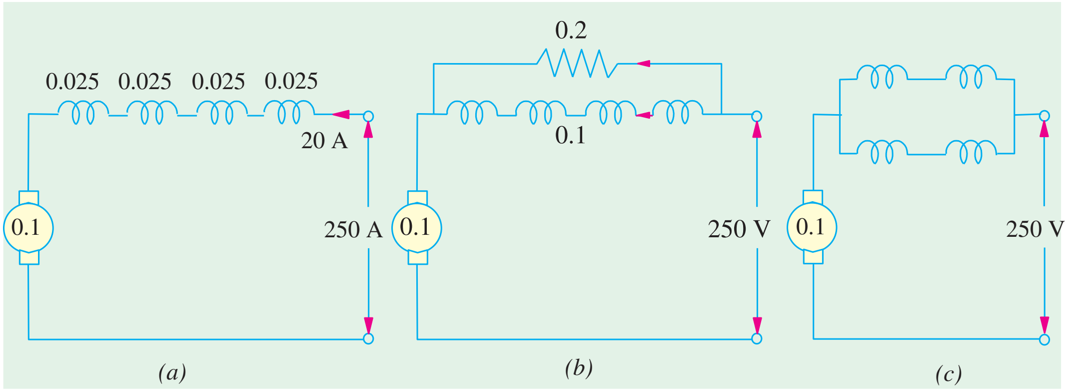


Fig. 30.15

(ii) In Fig. 30.15 (*c*), the series field coils have been arranged in two parallel groups. If the motor current is I_{a2}, then it is divided equally between the two parallel paths. Hence, $\Phi_2 \propto I_{a2}/2$.

Since torque remains the same,

$T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \propto 20^2$; $T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$

Since $T_1 = T_2$; $\therefore 20^2 = I_{a2}^2/2$; $I_{a2} = 28.28$ A

Combined resistance of the two parallel paths = 0.05/2 = 0.025 Ω

Total arm. circuit resistance = 0.1 + 0.025 = 0.125 Ω

$\therefore E_{b2}$ = 250 – 28.28 × 0.125 = 246.5 V

$$\frac{N_2}{900} = \frac{246.5}{246} \times \frac{20}{28.28/2} ; N_2 = \mathbf{1275 \text{ r.p.m.}}$$

Example 30.39. *A 4-pole, 230 V series motor runs at 1000 r.p.m., when the load current is 12 A. The series field resistance is 0.8 Ω and the armature resistance is 1.0 Ω. The series field coils are now regrouped from all in series to two in series with two parallel paths. The line current is now 20 A. If the corresponding weakening of field is 15%, calculate the speed of the motor.*

(Electrotechnology-I, Guwahati Univ. 1987)

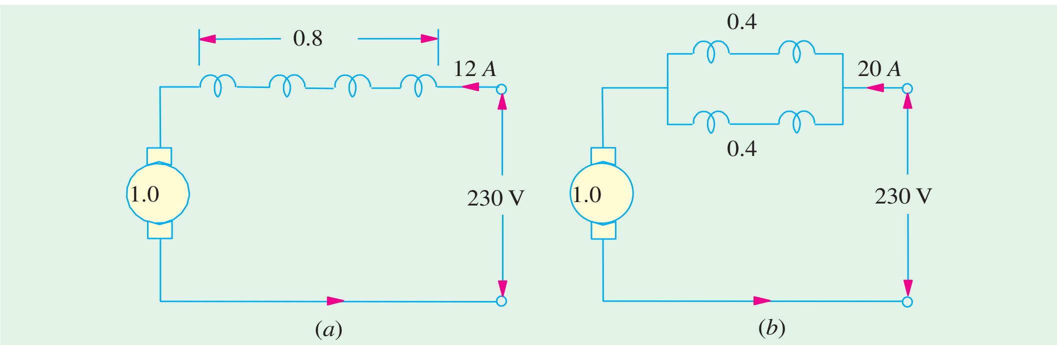


Fig. 30.16

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}, E_{b1} = 230 - 12 \times 1.8 = 208.4 \text{ V, as in Fig. 30.16}(a)$$

For circuit in Fig. 30.16 (*b*),

$$E_{b2} = 230 - 20\,(1 + 0.4/2) = 206 \text{ V};$$

$$\Phi_2 = 0.85\,\Phi_1 \text{ or } \Phi_1/\Phi_2 = 1/0.85$$

$$\therefore \quad \frac{N_2}{1000} = \frac{206}{208.4} \times \frac{1}{0.85} \quad \therefore \quad N_2 = \textbf{1163 r.p.m.}$$

Example 30.40. *A 200 V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0.2 Ω and that of the series field 0.6 Ω. At what speed will it run when developing the same torque when armature divertor of 10 Ω is used ? Assume a straight line magnetisation curve.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Resistance of motor = 0.2 + 0.6 = 0.8 Ω

$$\therefore \quad E_{b1} = 200 - (25 \times 0.8) = 180 \text{ V}$$

Let the motor input current be I_2, when armature divertor is used, as shown in Fig. 30.17.

Series field voltage drop $= 0.6\, I_2$

$$\therefore \quad \text{P.D. at brushes} = 200 - 0.6\, I_2$$

$$\therefore \quad \text{Arm. divertor current} = \left(\frac{200 - 0.6\, I_2}{10}\right) \text{A}$$

$$\therefore \quad \text{Armature current} = I_2 \left(\frac{200 - 0.6\, I_2}{10}\right)$$

$$\therefore \quad I_{a2} = \frac{10.6\, I_2 - 200}{10}$$

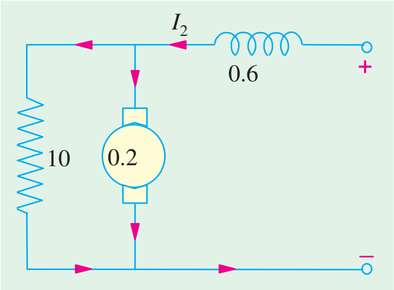


Fig. 30.17

As torque in both cases is the same, $\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$\therefore \quad 25 \times 25 = I_2 \left(\frac{10.6\, I_2 - 200}{10}\right) \quad \text{or} \quad 6{,}250 = 10.6\, I_2^2 - 200\, I_2$$

or $10.6\, I_2^2 - 200\, I_2 - 6250 = 0$ or $I_2 = 35.6$ A

P.D. at brushes in this case $= 200 - (35.6 \times 0.6) = 178.6$ V

$$\therefore \quad I_{a2} = \frac{10.6 \times 35.6 - 200}{10} = 17.74 \text{ A};$$

$$E_{b2} = 178.6 - (17.74 \times 0.2) = 175 \text{ V}$$

Now $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_1}{I_2} \qquad (\because \quad \Phi \propto I)$$

$$\therefore \quad \frac{N_2}{500} = \frac{175}{180} \times \frac{25}{35.6} \quad \therefore \quad N_2 = \textbf{314 r.p.m.}$$

Example 30.41. *A series motor is running on a 440 V circuit with a regulating resistance of R Ω for speed adjustment. The armature and field coils have a total resistance of 0.3 Ω. On a certain load with R = zero, the current is 20 A and speed is 1200 r.p.m. With another load and R = 3 Ω, the current is 15 A. Find the new speed and also the ratio of the two values of the power outputs of the motor. Assume the field strength at 15 A to be 80% of that at 20 A.*

Solution. $$I_{a1} = 20 \text{ A}, R_a = 0.3\,\Omega;\; E_{b1} = 440 - (20 \times 0.3) = 434 \text{ V}$$

$$I_{a2} = 15 \text{ A}, R_a = 3 + 0.3 = 3.3\,\Omega \;\therefore E_{b2} = 440 - (3.3 \times 15) = 390.5 \text{ V}$$

$$\Phi_2 = 0.8\,\Phi_1, \; N_1 = 1200 \text{ r.p.m.}$$

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$, we get $N_2 = 1200 \times \frac{390.5}{434} \times \frac{1}{0.8}$ = **1350 r.p.m.**

Now, in a series motor,

$$T \propto \Phi I_a \text{ and power } P \propto T \times N \quad \text{or} \quad P \propto \Phi N I_a$$

$$\therefore \quad P_1 \propto \Phi_1 \times 1200 \times 20 \quad \text{and} \quad P_2 \propto 0.8\ \Phi_1 \times 1350 \times 15$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{1200 \times 20\ \Phi_1}{1350 \times 15 \times 0.8\ \Phi_1} = \mathbf{1.48}$$

Hence, power in the first case is 1.48 times the power in the second case.

Example 30.42. *A d.c. series with an unsaturated field and negligible resistance, when running at a certain speed on a given load takes 50 A at 460 V. If the load-torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.*

(Nagpur, Univ. November 1999, Madras Univ. 1987)

Solution. Let the speed be ω radians/sec and the torque, T_{em} Nw-m developed by the motor. Hence power handled = T. ω watts

Let load torque be T_L, $\quad T_L \propto \omega^3$

$$T_L = T_{em}, \quad T_{em} \propto (50)^2$$

Hence $\quad$ Load power = $T_L\ \omega$

Since no losses have to be taken into account, $50^2 \propto \omega^3$

Armature power, $460 \times 50 \propto \omega^4$

Based on back e.m.f. relationship, $E_b \propto \omega I_a$

$$460 \propto \omega \times 50$$

To reduce the speed by 25%, operating speed = 0.75 ω rad/sec

Let the new current be I.

From Load side torque $\propto (0.75\ \omega)^3$

From electro-mech side, $\quad T \propto I^2$

$$I^2 \propto (0.75\ \omega)^3$$

Comparing similar relationship in previous case,

$$\frac{I^2}{50^2} = \frac{(0.75\ \omega)^3}{\omega^3} = 0.75^3$$

$$I^2 = 50^2 \times 0.422 = 1055$$

$$I = 32.48 \text{ amp}$$

$$E_{b2} \propto I \times \text{speed}$$

$$\propto I \times 0.75\ \omega$$

$$\propto 32.48 \times 0.75\ \omega$$

$$\frac{E_{b2}}{460} = \frac{32.48 \times 0.75}{50}$$

$$E_{b2} = 224 \text{ volts}$$

If R is the resistance externally connected in series with the motor,

$$E_{b2} = 460 - 32.48 \times R = 224$$

$$\therefore \quad R = 7.266 \text{ ohms}$$

Previous armature power = $460 \times 50 \times 10^{-3} = 23$ kW

New armature power should be

$$23 \times (0.75)^4 = 7.28 \text{ kW}$$

With E_b as 224 V and current as 32.48 amp

$$\text{Armature power} = 224 \times 32.48 \text{ watts} = 7.27 \text{ kW}$$

Thus, the final answer is checked by this step, since the results agree.

Example 30.43. *A d.c. series motor drives a load, the torque of which varies as the square of speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and current when the motor-field-winding is shunted by a divertor of equal resistance as that of the field winding. Neglect all motor losses and assume the magnetic circuit as unsaturated.*

(Bharathithasan Univ. April 1997)

Solution. Let the equations governing the characteristics of series motor be expressed as follows, with no losses and with magnetic circuit unsaturated.

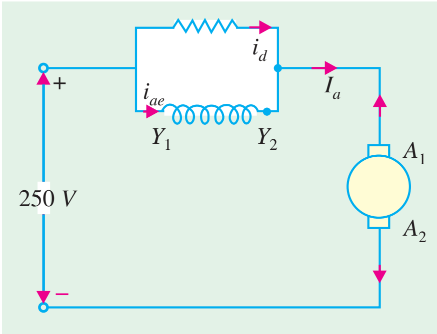


Fig. 30.18 (Divertor for speed control

Torque developed by motor = Load torque

$$k_m \times i_{se} \times I_a = k_L \times (600)^2$$

where k_m, k_L are constants

I_{se} = series field current

$$I_a = \text{Armature current}$$

In the first case $\quad i_{se} = I_a$

With divertor, $\quad i_{se} = i_d = 0.5\, I_a,$

Since, the resistances of divertor and series field are equal.

In the first case, $k_m \times 15 \times 15 = k_L \times (600)^2$ ...(*a*)

Let the supply voltage be V_L volts.

Since Losses are to be neglected, armature receives a power of ($V_L \times 15$) watts.

Case (*ii*) Let the new speed = N_2 r.p.m. and the new armature current = I_{a2} amp

So that new series-field current = 0.5 I_{a2}

Torque developed by motor = Load torque

$$k_m \times (0.5\, I_{a2}) \times I_{a2} = k_L \times (N_2)^2 \quad \text{...(b)}$$

From equations (*a*) and (*b*) above,

$$\frac{0.5\, I_{a2}^2}{225} = \frac{N_2^2}{600^2}$$

or $$\left(\frac{N_2^2}{I_{a2}^2}\right) = \frac{600^2}{450} = 800 \quad \text{...(c)}$$

or $$N_2/I_{a2} = 28.28 \quad \text{...(d)}$$

Now armature receives a power of $I_{a2}\, V_L$ watts. Mechanical outputs in the two cases have to be related with these electrical-power-terms.

$$k_L = (600)^2 \times 2600/60 = 15\, V_L \quad \text{...(e)}$$

$$k_L = N^2 \times 2\, N/60 = I_{a2}\, V_L \quad \text{...(f)}$$

From these two equations,

$$N^3/600^3 = I_{a2}/15 \quad \text{...(g)}$$

From (*c*) and (*g*), $\quad N_2\, I_{a2} = 18{,}000$ ...(*h*)

From **(*d*)** and **(*h*)**, N_2 = 713.5 r.p.m.

And I_{a2} = 25.23 amp

Additional Correlation : Since the load-torque is proportional to the square of the speed, the mechanical output power is proportional to the cube of the speed. Since losses are ignored, electrical power (input) must satisfy this proportion.

$$(15\ V_L) / (25.23\ V_L) = (600/713.5)^3$$

$$\text{L.H.S.} = 0.5945,\ \text{R.H.S.} = 0.5947$$

Hence, correlated and checked.

30.4. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also. This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.

2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.

3. Maximum power developed is diminished in the same ratio as speed.

4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.

5. It gives speeds ***below*** the normal, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method is, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where motor is continually started and stopped.

Advantages of Field Control Method

This method is economical, more efficient and convenient though it can give speeds *above* (not below) the normal speed. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

30.5. Series-parallel Control

In this system of speed control, which is widely used in electric traction, two or more similar mechanically-coupled series motors are employed. At low speeds, the motors are joined in series Fig. 30.19 (*a*) and for high speeds, are joined in parallel Fig. 30.19 (*b*).

When in series, the two motors have the same current passing through them, although the voltage across each motor is $V/2$ *i.e.*, half the supply voltage. When joined in parallel, voltage across each machines is V, though current drawn by each motor is $I/2$.

When in Parallel

Now speed $\propto E_b / \phi \propto E_b$ / current (being series motors)

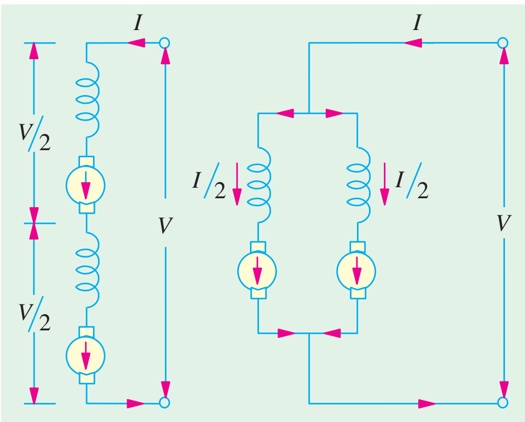


Fig. 30.19 (*a*) **Fig. 30.19 (*b*)**

Since E_b is approximately equal to the applied voltage V :

$$\therefore \qquad \text{speed} \propto \frac{V}{I/2} \propto \frac{2V}{I}$$

$$\text{Also, } \qquad \text{torque} \propto \Phi I \propto I^2 \qquad (\because \quad \Phi \propto I) \qquad ...(i)$$

$$\therefore \qquad T \propto (I/2)^2 \propto I^2/4 \qquad ...(ii)$$

When in Series

$$\text{Here} \qquad \text{speed} \propto \frac{E_b}{\Phi} \propto \frac{V/2}{I} \propto \frac{V}{2I} \qquad ...(iii)$$

This speed is one-fourth of the speed of the motors when in parallel.

$$\text{Similarly} \qquad T \propto \Phi I \propto I^2$$

The torque is four times that produced by motors when in parallel.

This system of speed control is usually combined with the variable resistance method of control described in Art. 30.3 (2).

The two motors are started up in series with each other and with variable resistance which is cut out in sections to increase the speed. When all the variable series resistance is cut out, the motors are connected in parallel and at the same time, the series resistance is reinserted. The resistance is again reduced gradually till full speed is attained by the motors. The switching sequence is shown in Fig.30.20. As the variable series controller resistance is not continuously rated, it has to be cut out of the circuit fairly quickly although in the four running positions A, B,C and D, it may be left in circuit for any length of time.

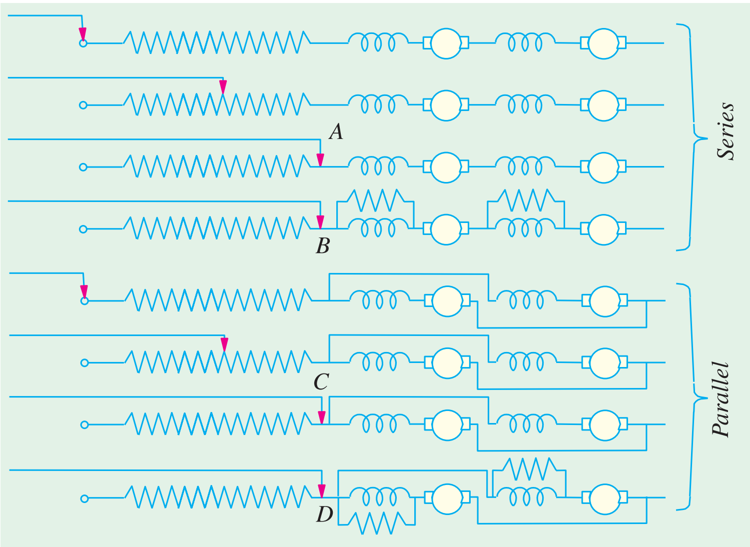


Fig. 30.20

Example 30.44. *Two series motors run at a speed of 500 r.p.m. and 550 r.p.m. respectively when taking 50 A at 500 V. The terminal resistance of each motor is 0.5 Ω. Calculate the speed of the combination when connected in series and coupled mechanically. The combination is taking 50 A on 500 V supply.* **(Electrical Machinery-I, Mysore Univ. 1985)**

Solution. First Motor

$$E_{b1} = 500 - (50 \times 0.5) = 475 \text{ V} ; I = 50 \text{ A}$$

$$\text{Now,} \qquad N_1 \propto E_{b1}/\Phi_1 \quad \text{or} \quad E_{b1} \propto N_1 \Phi_1 \quad \text{or} \quad E_{b1} = k N_1 \Phi_1$$

$$\therefore \qquad 475 = k \times 500 \times \Phi_1 \quad \therefore \quad k\Phi_1 = 475/500$$

Second Motor $\qquad E_{b2} = 500 - (50 \times 0.5) = 475$ V. Similarly, $k\,\Phi_2 = 475/550$

When both motors are in series

$$E_b' = 500 - (50 \times 2 \times 0.5) = 450 \text{ V}$$

Now, $E_b' = E_{b1} + E_{b2} = k\,\Phi_1 N + k\,\Phi_2 N$

where N is the common speed when joined in series.

$\therefore \quad 450 = \dfrac{475}{500} N + \dfrac{475}{550} N \quad \therefore \quad N =$ **248 r.p.m.**

Example 30.45. *Two similar 20 h.p. (14.92 kW), 250 V, 1000 r.p.m. series motors are connected in series with each other across a 250 V supply. The two motors drive the same shaft through a reduction gearing 5 : 1 and 4 : 1 respectively. If the total load torque on the shaft is 882 N-m, calculate (i) the current taken from the supply main (ii) the speed of the shaft and (iii) the voltage across each motor. Neglect all losses and assume the magnetic circuits to be unsaturated.*

(Elect. Machines, Punjab Univ., 1991)

Solution. ***(i)*** Rated current of each motor = 14,920/250 = 59.68 A

Back e.m.f. $E_b = 250$ V (neglecting $I_a R_a$ drop)

Now, $E_b \propto N\Phi$ As $\Phi \propto I \;\therefore\; E_b \propto NI$ or $E_b = kNI$

$250 = k \times (1000/60) \times 59.68 \quad \therefore \quad k = 0.25$

Let N_{sh} be the speed of the shaft.

Speed of the first motor $N_1 = 5\,N_{sh}$; Speed of the second motor $N_2 = 4\,N_{sh}$

Let I be the new current drawn by the series set, then

$$E_b' = E_{b1} + E_{b2} = kI \times 5\,N_1 + kI \times N_2 = kI \times 5\,N_{sh} + kI \times 4\,N_{sh}$$

$$250 = 9 \times kI\,N_{sh} \qquad \text{...}(i)$$

Now, torque $T = 0.159 \dfrac{E_b I}{N} = 0.159 \times \dfrac{kI\,N_{sh} \times I}{N_{sh}} = 0.159\,k\,I^2$

Shaft torque due to gears of 1st motor = $5 \times 0.159\,kI^2$

Shaft torque due to gears of 2nd motor = $4 \times 0.159\,kI^2$

$\therefore \quad 882 = kI^2\,(5 \times 0.159 + 4 \times 0.159) = 1.431\,kI^2$

$\therefore \quad I^2 = 882/1.431 \times 0.25 = 2{,}449$ A $\quad \therefore \quad I =$ **49.5 A**

(ii) From equation (*i*), we get

$250 = 9 \times 0.25 \times 49.5 \times N_{sh} \quad \therefore \quad N_{sh} = 2.233$ r.p.s. = **134 r.p.m.**

(iii) Voltage across the armature of Ist motor is

$E'_{b1} = 5\,kI\,N_{sh} = 5 \times 0.25 \times 49.5 \times 2.233 =$ **139 V**

Voltage across the armature of 2nd motor

$E_{b2} = 4\,kI\,N_{sh} = 4 \times 0.25 \times 49.5 \times 2.233 =$ **111 V**

Note that E_{b1} and E_{b2} are respectively equal to the applied voltage across each motor because $I_a R_a$ drops are negligible.

30.6. Electric Braking

A motor and its load may be brought to rest quickly by using either ***(i)*** Friction Braking or ***(ii)*** Electric Braking. The commonly-used mechanical brake has one drawback : it is difficult to achieve a smooth stop because it depends on the condition of the braking surface as well as on the skill of the operator.

Strong electric brake for airwheels

The excellent electric braking methods are available which eliminate the need of brake lining levers and other mechanical gadgets. Electric braking, both for shunt and series motors, is of the following three types: ***(i)*** rheostatic or dynamic braking

(ii) plugging *i.e.*, reversal of torque so that armature tends to rotate in the opposite direction and
(iii) regenerative braking.

Obviously, friction brake is necessary for holding the motor even after it has been brought to rest.

30.7. Electric Braking of Shunt Motors

(*a*) Rheostatic or Dynamic Braking

In this method, the armature of the shunt motor is disconnected from the supply and is connected across a variable resistance R as shown in Fig. 30.21 (*b*). The field winding is, however, left connected across the supply undisturbed. The braking effect is controlled by varying the series resistance R. Obviously, this method makes use of generator action in a motor to bring it to rest.* As seen from Fig. 30.21 (*b*), armature current is given by

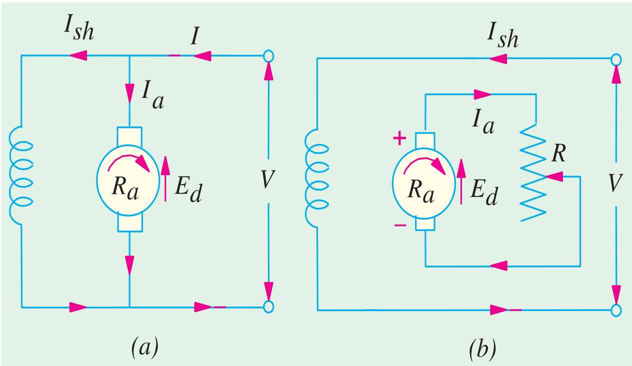


Fig. 30.21

$$I_a = \frac{E_b}{R + R_a} = \frac{\Phi ZN(P/A)}{R + R_a}$$

$$= \frac{k_1 \Phi N}{R + R_a}$$

Braking torque is given by

$$T_B = \frac{1}{2\pi} \Phi Z I_a \left(\frac{P}{A}\right) \text{N-m}$$

$$= \frac{1}{2\pi} \Phi Z \left(\frac{P}{A}\right) . \frac{\Phi ZN(P/A)}{R + R_a} = \frac{1}{2\pi} \left(\frac{ZP}{A}\right)^2 \frac{\Phi^2 N}{R + R_a} = k_2 \Phi^2 N \quad \therefore\ T_B \propto N$$

Obviously, T_B decreases as motor slows down and disappear altogether when it comes to a stop.

(*b*) Plugging or Reverse Current Braking

This method is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

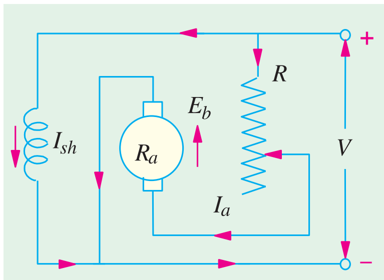


Fig. 30.22

In this method, connections to the armature terminals are reversed so that motor tends to run in the opposite direction (Fig. 30.22). Due to the reversal of armature connections, applied voltage V and E_b start acting ***in the same direction*** around the circuit. In order to limit the armature current to a reasonable value, it is necessary to insert a resistor in the circuit while reversing armature connections.

$$I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{E_b}{R + R_a}$$

$$= \frac{V}{R + R_a} + \frac{\Phi ZN\,(P/A)}{R + R_a} = \frac{V}{R + R_a} + \frac{k_1 \Phi Z}{R + R_a}$$

$$T_B = \frac{1}{2\pi} . \Phi Z I_a \left(\frac{P}{A}\right) = \frac{1}{2\pi} . \left(\frac{\Phi ZP}{A}\right) . I_a = \frac{1}{2\pi} \left(\frac{\Phi ZP}{A}\right) \left[\frac{V}{R + R_a} + \frac{\Phi ZN\,(P/A)}{R + R_a}\right]$$

* The motor while acting as a generator feeds current to the resistor dissipating heat at the rate of $I^2 R$. The current I_a produced by dynamic braking flow in the opposite direction, thereby producing a counter torque that slows down the machine.

$$= \frac{1}{2\pi}\left(\frac{ZP}{A}\right)\left(\frac{V}{R+R_a}\right).\Phi + \frac{1}{2\pi}.\left(\frac{ZP}{A}\right)^2.\frac{\Phi^2 N}{R+R_a} = k_2\Phi + k_3\Phi^2 N$$

or $$T_b = k_4 + k_5 N, \text{ where } k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R+R_a}\right)$$

and $$k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \times \frac{1}{(R+R_a)}.$$

Plugging gives greater braking torque than rheostatic braking. Obviously, during plugging, power is drawn from the supply and is dissipated by R in the form of heat. It may be noted that even when motor is reaching zero speed, there is some braking torque $T_B = k_4$ (see Ex. 30.47).

(*c*) Regenerative Braking

This method is used when the load on the motor has overhauling characteristic as in the lowering of the cage of a hoist or the downgrade motion of an electric train. Regeneration takes place when E_b becomes grater than V. This happens when the overhauling load acts as a prime mover and so drives the machines as a generator. Consequently, direction of I_a and hence of armature torque is reversed and speed falls until E_b becomes lower than V. It is obvious that during the slowing down of the motor, power is returned to the line which may be used for supplying another train on an upgrade, thereby relieving the powerhouse of part of its load (Fig. 30.23).

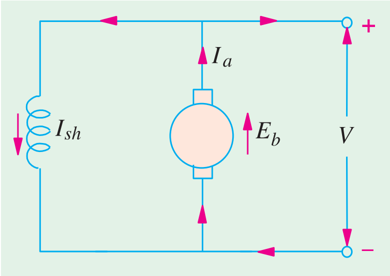


Fig. 30.23


Regenerative braking demonstrations

For protective purposes, it is necessary to have some type of mechanical brake in order to hold the load in the event of a power failure.

Example 30.46. *A 220 V compensated shunt motor drives a 700 N-m torque load when running at 1200 r.p.m. The combined armature compensating winding and interpole resistance is 0.008 Ω and shunt field resistance is 55 Ω. The motor efficiency is 90%. Calculate the value of the dynamic braking resistor that will be capable of 375 N-m torque at 1050 r.p.m. The windage and friction losses may be assumed to remain constant at both speeds.*

Solution. Motor output $= \omega T = 2\pi NT = 2\pi\,(1200/60) \times 700 = 87{,}965$ W

Power drawn by the motor $= 87{,}965/0.9 = 97{,}740$ W

Current drawn by the motor $= 97{,}740/220 = 444$ A.

$I_{sh} = 220/55 = 4$ A ; $I_{a1} = 444 - 4 = 440$ A

$E_{b1} = 220 - 440 \times 0.008 = 216.5$ V

Since field flux remains constant, T_1 is proportional to I_{a1} and T_2 to I_{a2}.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad I_{a2} = 440 \times \frac{375}{100} = 2650 \text{ A}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{1050}{1200} = \frac{E_{b2}}{216.5}\,;\; E_{b2} = 189.4 \text{ V}$$

With reference to Fig. 30.23, we have

$$189.4 = 2650\,(0.008 + R)\,;\; R = 0.794\ \Omega$$

30.8. Electric Braking of Series Motor

The above-discussed three methods as applied to series motors are as follows :

(*a*) **Rheostatic (or Dynamic) Braking**

The motor is disconnected from the supply, the field connections ***are reversed*** and the motor is connected in series with a variable resistance R as shown in Fig. 30.24. Obviously, now, the machine is running as a generator. The field connections are reversed to make sure that current through field winding flows in the *same* direction as before (*i.e.*, from M to N) in order to assist residual magnetism. In practice, the variable resistance employed for starting purpose is itself used for braking purposes. As in the case of shunt motors,

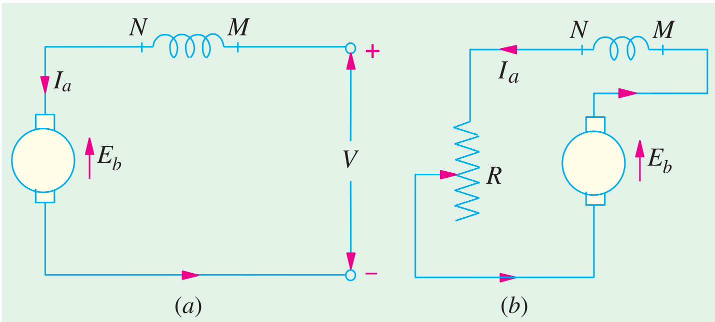


Fig. 30.24

$$T_B = k_2\,\Phi^2\,N = k_3\,I_{a2}\,N \qquad \text{... prior to saturation}$$

(*b*) **Plugging or Reverse Current Braking**

As in the case of shunt motors, in this case also the connections of the armature are reversed and a variable resistance R is put in series with the armature as shown in Fig. 30.25. As found in Art. 30.7 (*b*),

$$T_B = k_2\,\Phi + k_3\Phi^2 N$$

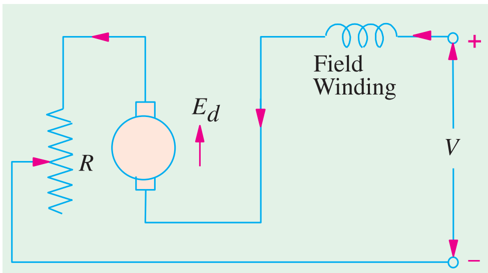


Fig. 30.25

(*c*) **Regenerative Braking**

This type of braking of a sereis motor is not possible without modification because reversal of I_a would also mean reversal of the field and hence of E_b. However, this method is sometimes used with traction motors, special arrangements being necessary for the purpose.

Example 30.47. *A 400 V, 25 h.p. (18.65 kW), 45 r.p.m., d.c. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full-load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2 Ω.* **(Electrical Technology, M.S. Univ. Baroda, 1988)**

Solution.

F.L. Motor input current $I = 18{,}650/0.746 \times 400 = 62.5$ A

$I_a = 62.5$ A (neglecting I_{sh}); $E_b = 400 - 62.5 \times 0.2 = 387.5$ V

Total voltage around the circuit is $= 400 + 387.5 = 787.5$ V

Max. braking current $= 2 \times 62.5 = 125$ A

Total resistance required in the circuit $= 787.5/125 = 6.3\ \Omega$

Braking resistance $R = 6.3 - 0.2 =$ **6.1 Ω**

Maximum braking torque will be produced initially when the motor speed is maximum *i.e.*, 450 r.p.m. or 7.5 r.p.s.

$$\text{Maximum value of } T_B = k_4 + k_5\,N \qquad \text{– Art. 25.7}(b)$$

Now, $$k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R+R_a}\right) \text{ and } k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \cdot \frac{1}{(R+R_a)}$$

Now, $E_b = \Phi ZN\,(P/A)$; also $N = 450/60 = 7.5$ r.p.s.

$\therefore \quad 387.5 = 7.5\,(\Phi ZP/A)$ or $(\Phi ZP/A) = 51.66$

$$\therefore \quad k_4 = \frac{1}{2\pi} \times 51.66 \times \frac{400}{6.3} = 522 \text{ and } k_5 = \frac{1}{2\pi} \times (51.66)^2 \times \frac{1}{6.3} = 67.4$$

$\therefore$ Maximum $T_B = 522 + 67.4 \times 7.5 =$ **1028 N-m**

When speed is also zero *i.e.*, $N = 0$, the value of torque is $T_B = K_4 =$ **522 N-m.**

30.9. Electronic Speed Control Method for DC Motors

Of late, solid-state circuits using semiconductor diodes and thyristors have become very popular for controlling the speed of a.c. and d.c. motors and are progressively replacing the traditional electric power control circuits based on thyratrons, ignitrons, mercury arc rectifiers, magnetic amplifiers and motor-generator sets etc. As compared to the electric and electromechanical systems of speed control, the electronic methods have higher accuracy, greater reliability, quick response and also higher efficiency as there are no I^2R losses and moving parts. Moreover, full 4-quadrant speed control is possible to meet precise high-speed standards.

All electronic circuits control the motor speed by adjusting either ***(i)*** the voltage applied to the motor armature or ***(ii)*** the field current or ***(iii)*** both.

DC motors can be run from d.c. supply if available or from a.c. supply after it has been converted into d.c. supply with the help of rectifiers which can be either half-wave or full-wave and either controlled (by varying the conduction angle of the thyristors used) or uncontrolled.

AC motors can be run on the a.c. supply or from d.c. supply after it has been converted into a.c. supply with the help of inverters (opposite of rectifiers).

As stated above, the average output voltage of a thyristor-controlled rectifier can be changed by changing its conduction angle and hence the armature voltage of the d.c. motor can be adjusted to control its speed.

When run on a d.c. supply, the armature d.c. voltage can be changed with the help of a thyristor chopper circuit which can be made to interrupt d.c. supply at different rates to give different average values of the d.c. voltage. If d.c. supply is not available, it can be obtained from the available a.c. supply with the help of uncontrolled rectifiers (using only diodes and not thyristors). The d.c. voltages so obtained can be then chopped with the help of a thyristor chopper circuit.

A brief description of rectifiers, inverters* and d.c. choppers would now be given before discussing the motor speed control circuits.

30.10. Uncontrolled Rectifiers

As stated earlier, rectifiers are used for a.c. to d.c. conversion *i.e.*, when the supply is alternating but the motor to be controlled is a d.c. machine.

Fig. 30.26 (*a*) shows a half-wave uncontrolled rectifier. The diode *D* conducts only during positive half-cycles of the single-phase a.c. input *i.e.*, when its anode *A* is positive with respect to its cathode *K*. As shown, the average voltage available across the load (or motor) is 0.45 V where *V* is the r.m.s. value of the a.c. voltage (in fact, $V = V_m/\sqrt{2}$). As seen it is a pulsating d.c. voltage.

In Fig. 30.26 (*b*) a single-phase, full-wave bridge rectifier which uses four semiconductor diodes and provides double the voltage *i.e.*, 0.9 V is shown. During positive input half-cycles when end *A* is positive with respect to end *B*, diodes D_1 and D_4 conduct (*i.e.* opposite diodes) whereas during negative input half-cycles, D_2 and D_3 conduct. Hence, current flows through the load during both half-cycles in the *same* direction. As seen, the d.c. voltage supplied by a bridge rectifier is much less pulsating than the one supplied by the half-wave rectifier.

* Rectifiers convert a.c. power into d.c power, whereas inverters convert d.c. power into a.c. power. However, converter is a general term embracing both rectifiers and inverters.

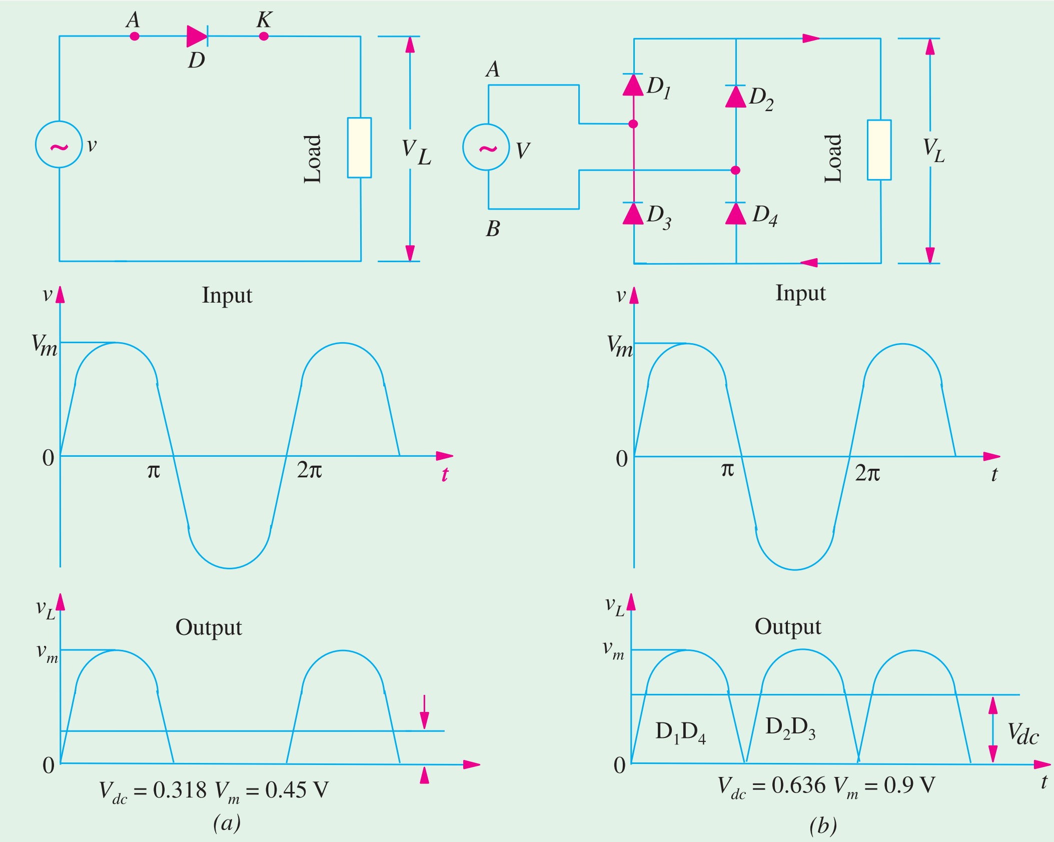


Fig. 30.26

30.11. Controlled Rectifiers

In these rectifiers, output load current (or voltage) can be varied by controlling the point in the input a.c. cycle at which the thyristor is turned ON with the application of a suitable low-power gate pulse. Once triggered (or fired) into conduction, the thyristor remains in the conducting state for the rest of the half-cycle *i.e.*, upto 180º. The firing angle α can be adjusted with the help of a control circuit. When conducting, it offers no resistance *i.e.*, it acts like a short-circuit.

Fig. 30.27 (*a*) shows an elementary half-wave rectifier in which thyristor triggering is delayed by angle α with the help of a phase-control circuit. As shown, the thyristor starts conducting at point *A* and not at point *O* because its gate pulse is applied after a delay of α. Obviously, the conduction angle is reduced from 180º to (180º – α) with a consequent decrease in output voltage whose value is given by

$$V_L = \frac{V_m}{2\pi}(1 + \cos\alpha) = 0.16\ V_m(1 + \cos\alpha) = 0.32\ V_m \cos^2\frac{\alpha}{2}$$

where V_m is the peak value of a.c. input voltage. Obviously, V_L is maximum when α = 0 and is zero when α = 180º.

Fig. 30.27 (*b*) shows the arrangement where a thyristor is used to control current through a load connected in series with the a.c. supply line.

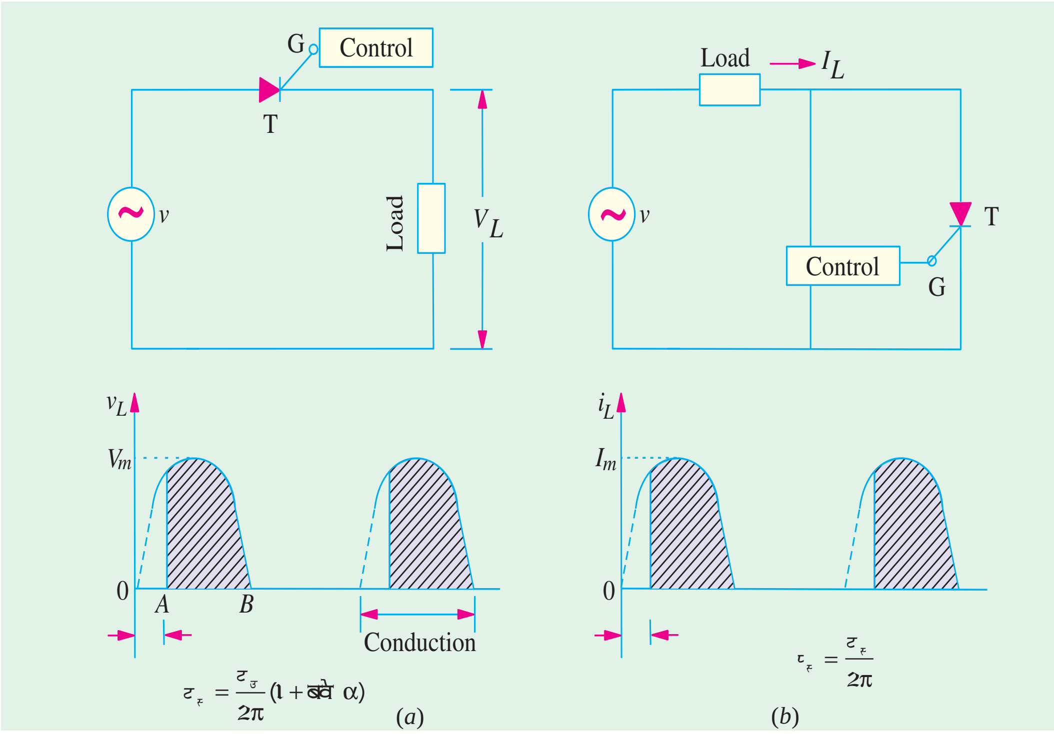


Fig. 30.27

The load current is given by

$$I_L = \frac{V_L}{R_L} = \frac{V_m}{2\pi R_L}(1+\cos\alpha) = \frac{V_m}{\pi R_L}\cos^2\frac{\alpha}{2}$$

Fig. 30.28 (*a*) shows a single-phase, full-wave half-controlled rectifier. It is called half-controlled because it uses two thyristors and two diodes instead of four thyristors. During positive input half-cycle when *A* is positive, conduction takes place via T_1, load and D_1. During the negative half-cycle when *B* becomes positive, conduction route is via T_2, load and D_2.

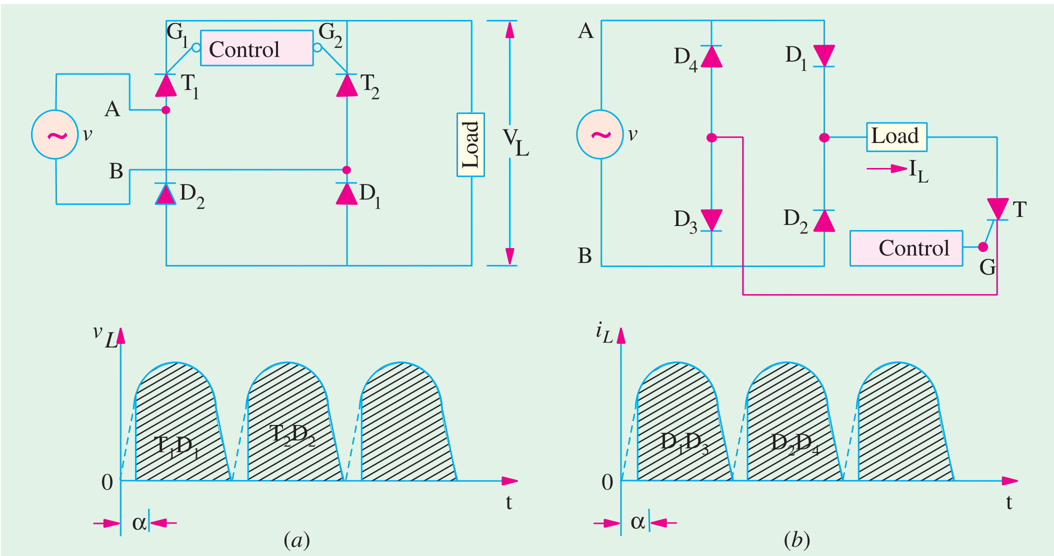


Fig. 30.28

The average output voltage V_L or V_{dc} is given by $V_L = 2 \times$ half-wave rectifier output

$$\therefore \quad V_L = 2 \times \frac{V_m}{2\pi}(1 + \cos\alpha) = \frac{V_m}{\pi}(1 + \cos\alpha)^* = \frac{2V_m}{\pi}\cos^2\frac{\alpha}{2}$$

Similarly, Fig. 30.28 (*b*) shows a 4-diode bridge rectifier controlled by a single thyristor. The average load current through the series load is given by

$$I_L = \frac{V_m}{\pi R_L}(1 + \cos\alpha) = \frac{2V_m}{\pi R_L}\cos^2\frac{\alpha}{2}$$

As seen from the figure, when *A* is positive, D_1 and D_3 conduct provided *T* has been fired. In the negative half-cycle, D_2 and D_4 conduct via the load.

30.12. Thyristor Choppers

Since thyristors can be switched ON and OFF very rapidly, they are used to interrupt a d.c. supply at a regular frequency in order to produce a lower (mean) d.c. voltage supply. In simple words, they can produce low-level d.c. voltage from a high-voltage d.c. supply as shown in Fig. 30.29.

The mean value of the output voltage is given by

$$V_{dc} = V_L = V\frac{T_{ON}}{T_{ON} + T_{OFF}} = V\frac{T_{ON}}{T}$$

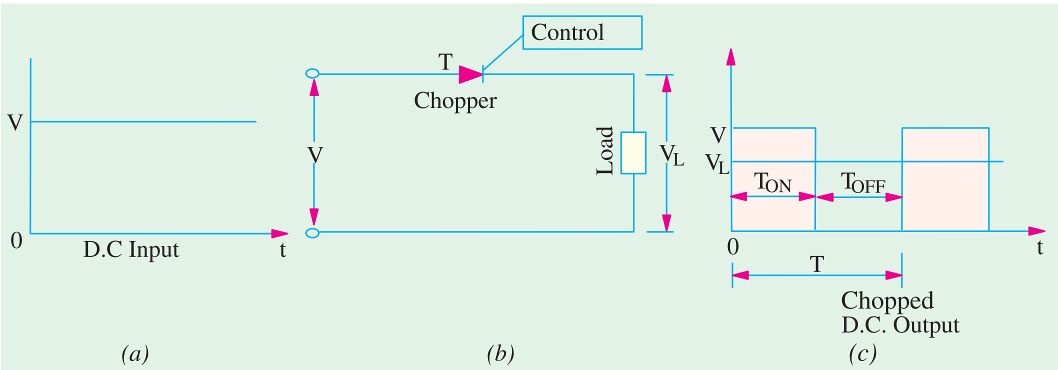


Fig. 30.29

Fig. 30.30 (*a*) shows a simple thyristor chopper circuit alongwith extra commutating circuitry for switching T_1 OFF. As seen, T_1 is used for d.c. chopping, whereas *R*, T_2 and *C* are used for commutation purposes as explained below.

When T_1 is fired into conduction by its control circuit (not shown), current is set up through the load and commutation capacitor *C* gets charged via *R* with the polarity shown in the figure during this ON period.

For switching T_1 OFF, second thyristor T_2 is triggered into conduction allowing *C* to discharge through it (since it acts as a short-circuit while conducting) which reverse-biases T_1 thus turning it OFF. The discharge from *C* leaves T_2 with reverse polarity so that it is turned OFF, whereas T_1 is triggered into conduction again.

Depending upon the frequency of switching ON and OFF, the input d.c. voltage is cut into d.c. pulses as shown in Fig. 30.30 (*c*).

* For a fully-controlled bridge rectifier, its value is

$$V_L = \frac{2V_m}{\pi}\cos\alpha\,.$$

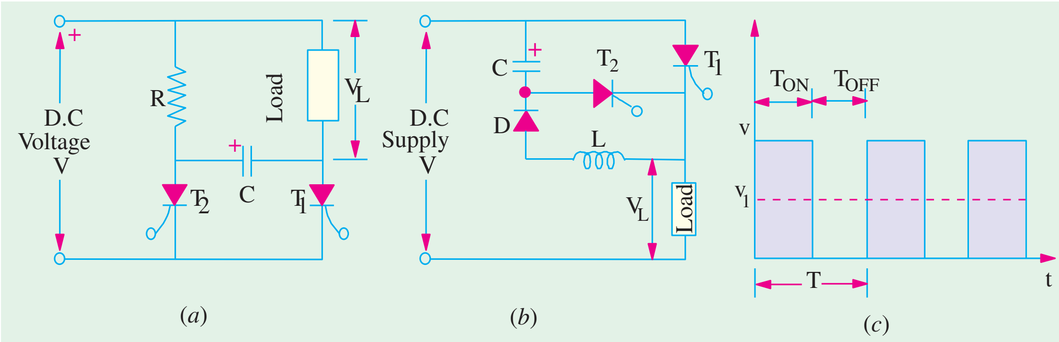


Fig. 30.30

In Fig. 30.30 (*b*), T_1 is the chopping thyristor, whereas *C*, *D*, T_2 and *L* constitute the commutation circuitry for switching T_1 OFF and ON at regular intervals.

When T_2 is fired, *C* becomes charged via the load with the polarity as shown. Next, when T_1 is fired, *C* reverse-biases T_2 to OFF by discharging via T_1, *L* and *D* and then recharges with reverse polarity. T_2 is again fired and the charge on *C* reverse-biases T_1 to non-conducting state.

It is seen that output (or load) voltage is present only when T_1 is ON and is absent during the interval it is OFF. The mean value of output d.c. voltage depends on the relative values of T_{ON} and T_{OFF}. In fact, output d.c. voltage is given by

$$V_{dc} = V_L \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

Obviously, by varying thyristor ON/OFF ratio, V_L can be made any percentage of the input d.c. voltage *V*.

Example 30.48. *The speed of a separately excited d.c. motor is controlled by a chopper. The supply voltage is 120 V, armature circuit resistance = 0.5 ohm, armature circuit inductance = 20 mH and motor constant = 0.05 V/r.p.m. The motor drives a constant load torque requiring an average current of 20 A. Assume motor current is continuous. Calculate* **(a)** *the range of speed control* **(b)** *the range of duty cycle.* **(Power Electronics-I, Punjab Univ. Nov. 1990)**

Solution. The minimum speed is zero when $E_b = 0$

$$V_t = E_b + I_a R_a = I_a \times R_a = 200 \times 0.5 = 10 \text{ V}$$

Now, $$V_t = \frac{T_{ON}}{T} V = \alpha V \therefore 10 = 120\alpha \therefore \alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ when $V_t = V = 120$ V

$\therefore$ $$E_b = 120 - 20 \times 0.5 = 110 \text{ V}$$

Now, $$N = E_b/K_a \Phi = 110/0.05 = 2200 \text{ r.p.m.}$$

(*a*) Hence, speed range is from 0 to 2200 r.p.m.

(*b*) Range of duty cycle is from $\frac{1}{12}$ to 1.

30.13. Thyristor Inverters

Such inverters provide a very efficient and economical way of converting direct current (or voltage) into alternating current (or voltage). In this application, a thyristor serves as a controlled switch alternately opening and closing a d.c. circuit. Fig. 30.31 (*a*), shows a basic inverter circuit where an a.c. output is obtained by alternately opening and closing switches S_1 and S_2. When we replace the mechanical switches by two thyristors (with their gate triggering circuits), we get the thyristor inverter in Fig. 30.31 (*b*).

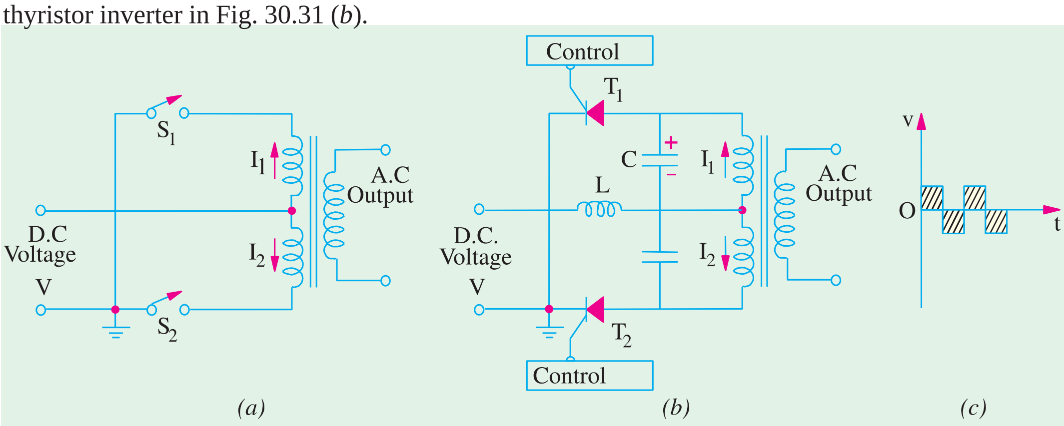


Fig. 30.31

Before discussing the actual circuit, it is worthwhile to recall that thyristor is a latching device which means that once it starts conducting, gate loses control over it and cannot switch it OFF whatever the gate signal. A separate ***commutating*** circuitry is used to switch the thyristor OFF and thus enable it to perform ON-OFF switching function.

Second generation thyristor inverter

Suppose T_1 is fired while T_2 is still OFF. Immediately I_1 is set up which flows through *L*, one half of transformer primary and T_1. At the same time, *C* is charged with the polarity as shown.

Next when T_2 is fired into condition, I_2 is set up and *C* starts discharging through T_1 thereby reverse-biasing it to CUT-OFF.

When T_1 is again pulsed into condition, I_1 is set up and *C* starts discharging thereby reverse-biasing T_2 to *OFF* and the process just described repeats. As shown in Fig. 30.31 (*c*), the output is an alternating voltage whose frequency depends on the switching frequency to thyristors T_1 and T_2.

30.14. Thyristor Speed Control of Separately-excited D.C. Motor

In Fig. 30.32, the bridge rectifier converts a voltage into d.c. voltage which is then applied to the armature of the separately-excited d.c. motor *M*.

As we know, speed of a motor is given by

$$N = \frac{V - I_a R_a}{\Phi}\left(\frac{A}{ZP}\right)$$

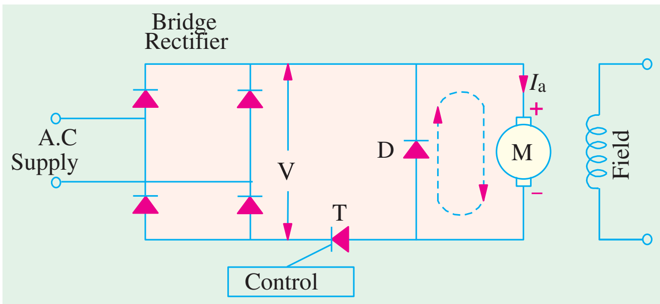


Fig. 30.32

If Φ is kept constant and also if $I_a R_a$ is neglected, then, $N \propto V \propto$ voltage across the armature. The value of this voltage furnished by the rectifier can be changed by varying the firing angle α of the thyristor *T* with the help of its contol circuit. As α is increased *i.e.*, thyristor firing is delayed more, its conduction period is reduced and, hence, armature voltage is decreased which, in turn, decreases the motor speed. When α is decreased *i.e.*, thyristor is fired earlier, conduction period is increased which increases the mean value of the voltage applied across the motor armature. Consequently, motor speed is increased. In short, as α increases, *V* decreases and hence *N* decreases. Conversely, as α decreases, *V* increases and so, *N* increases. The free-wheeling diode *D* connected across the motor provides a circulating current path (shown dotted) for the energy stored in the inductance of the armature winding at the time *T* turns OFF. Without *D*, current will flow through *T* and bridge rectifier, prohibiting *T* from turning OFF.

30.15. Thyristor Speed Control of a D.C. Series Motor

In the speed control circuit of Fig. 30.33, an *RC* network is used to control the diac voltage that triggers the gate of a thyristor. As the a.c. supply is switched ON, thyristor *T* remains OFF but the capacitor *C* is charged through motor armature and *R* towards the peak value of the applied a.c. voltage. The time it takes for the capacitor voltage V_C to reach the breakover voltage of the diac* depends on the setting of the variable resistor *T*. When V_C becomes equal to the breakover voltage of diac, it conducts and a triggering pulse is applied to the thyristor gate *G*. Hence, *T* is turned ON and allows current to pass through the motor. Increasing *R* delays the rise of V_C and hence the breakover of diac so that thyristor is fired later in each positive half cycle of the a.c. supply. It reduces the conduction angle of the thyristor which, consequently, delivers less power to the motor. Hence, motor speed is reduced.

If *R* is reduced, time-constant of the *RC* network is decreased which allows V_C to rise to the breakover voltage of diac more quickly. Hence, it makes the thyristor fire early in each positive input half-cycle of the supply. Due to increase in the conduction angle of the thyristor, power delivered to the motor is increased with a subsequent increase in its speed. As before *D* is the free-wheeling diode which provides circulating current path for the energy stored in the inductance of the armature winding.

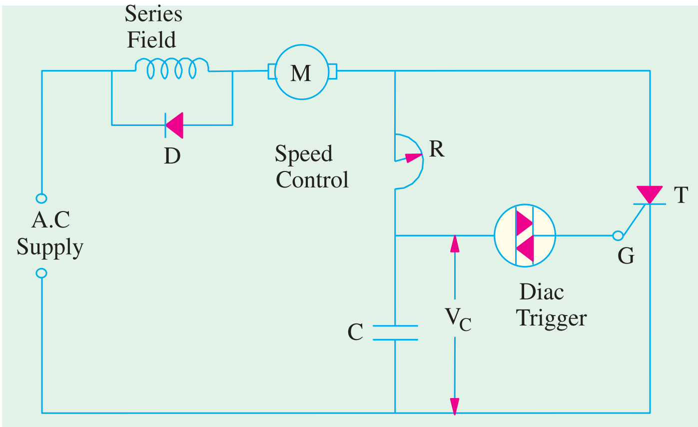


Fig. 30.33

30.16. Full-wave Speed Control of a Shunt Motor

Fig. 30.34 shows a circuit which provides a wide range of speed control for a fractional kW shunt d.c. motor. The circuit uses a bridge circuit for full-wave rectification of the a.c. supply. The shunt field winding is permanently connected across the d.c. output of the bridge circuit. The armature voltage is supplied through thyristor *T*. The magnitude of this

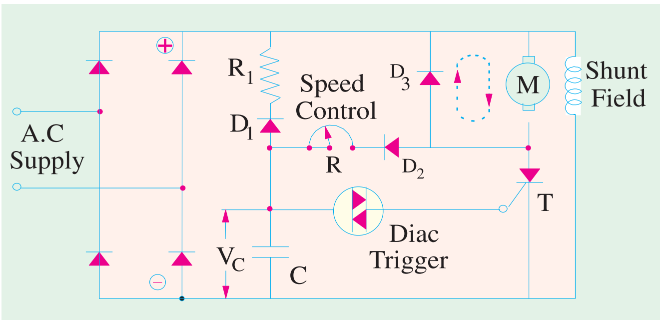


Fig. 30.34

* Like a triac, it has directional switching characteristics.

voltage (and hence, the motor speed) can be changed by turning T_{ON} at different points in each half-cycle with the help of *R*. The thyristor turns OFF only at the end of each half-cycle. Free-wheeling diode D_3 provides a circulating current path (shows dotted) for the energy stored in the armature winding at the time *T* turns OFF. Without D_3. This current would circulate through *T* and the bridge rectifier thereby prohibiting *T* from turning OFF.

At the beginning of each half-cycle, *T* is the OFF state and *C* starts charging up via motor armature, diode D_2 and speed-control variable resistor *R* (it cannot charge through R_1 because of reverse-biased diode D_1). When voltage across *C i.e.*, V_C builds up to the breakover voltage of diac, diac conducts and applies a sudden pulse to *T* thereby turning it ON. Hence, power is supplied to the motor armature for the remainder of that half-cycle. At the end of each half-cycle, *C* is discharged through D_1, R_1 and shunt field winding. The delay angle α depends on the time it takes V_C to become equal to the breakover voltage of the diac. This time, in turn, depends on the time-constant of the *R-C* circuit and the voltage available at point *A*. By changing *R*, V_C can be made to build-up either slowly or quickly and thus change the angle α at will. In this way, the average value of the d.c. voltage across the motor armature can be controlled. It further helps to control the motor speed because it is directly proportional to the armature voltage.

Now, when load is increased, motor tends to slow down. Hence, E_b is reduced. The voltage of point *A* is increased because it is equal to the d.c. output voltage of the bridge rectifier ***minus*** back e.m.f. E_b. Since V_A increases *i.e.*, voltage across the *R-C* charging circuit increases, it builds up V_C more quickly thereby decreasing which leads to early switching ON of *T* in each half-cycle. As a result, power supplied to the armature is increased which increases motor speed thereby compensating for the motor loading.

30.17. Thyristor Speed Control of a Shunt Motor

The speed of a shunt d.c.motor (upto 5 kW) may be regulated over a wide range with the help of the full-wave rectifier using only one main thyristor (or *SCR*) *T* as shown in Fig. 30.35. The firing angle α of *T* is adjusted by R_1 thereby controlling the motor speed. The thyristor and *SUS* (silicon unilateral switch) are reset (*i.e.*, stop conduction) when each half-wave of voltage drops to zero. Before switching on the supply, R_1 is increased by turning it in the counter-clockwise direction. Next, when supply is switched ON, *C* gets charged via motor armature and diode D_1 (being forward biased). It means that it takes much longer for V_C to reach the breakdown voltage of *SUS** due to large time constant of $R_1 – C$ network. Once V_C reaches that value, *SUS* conducts suddenly and triggers *T* into conduction. Since thyristor starts conducting late (*i.e.*, its α is large), it furnishes low voltage to start the motor. As speed selector R_1 is turned clockwise (for less resistance), *C* charges up more rapidly (since time constant is decreased) to the breakover voltage of *SUS* thereby firing *T* into conduction earlier. Hence, average value of the d.c. voltage across the motor armature increases thereby increasing its speed.

While the motor is running at the speed set by R_1, suppose that load on the motor is increased. In that case, motor will tend to slow down thereby decreasing armature back e.m.f. Hence, potential of point 3 will rise which will charge *C* faster to the breakover voltage of *SUS*. Hence, thyristor will be fired earlier thereby applying greater armature voltage which will return the motor speed to its desired value. As seen, the speed is automatically regulated to offset changes in load.

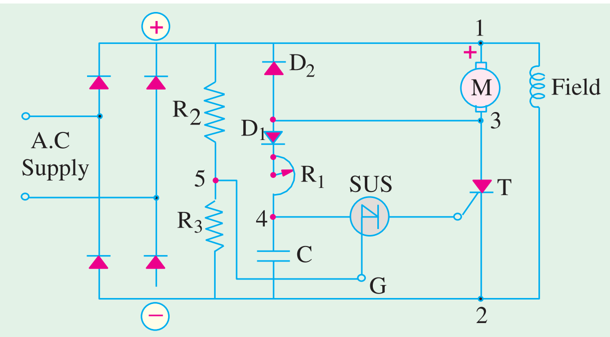


Fig. 30.35

The function of free-wheeling diode D_2 is to allow dissipation of energy stored in motor

* It is a four-layer semiconductor diode with a gate terminal. Unlike diac, it conducts in one direction only.

armature during the time the full-wave rectified voltage drops to zero between half-cycles. If D_2 is not there, then decreasing armature current during those intervals would be forced to flow through T thereby preventing its being reset. In that case, T would not be ready to be fired in the next half-cycle.

Similarly, towards the end of each half cycle as points 1 and 5 decrease towards zero potential, the negative going gate G turns SUS on thereby allowing C to discharge completely through SUS and thyristor gate-cathode circuit so that it can get ready to be charged again in the next half-cycle.

30.18. Thyristor Speed Control of a Series D.C. Motor

Fig. 30.36 shows a simple circuit for regulating the speed of a d.c. motor by changing the average value of the voltage applied across the motor armature by changing the thyristor firing angle α. The trigger circuit $R_1 - R_2$ can give a firing range of almost 180°. As the supply is switched on, full d.c. voltage is applied across $R_1 - R_2$. By changing the variable resistance R_2, drop across it can be made large enough to fire the SCR at any desired angle from 0° – 180°. In this way, output voltage of the bridge rectifier can be changed considerably, thus enabling a wide-range control of the motor speed. The speed control can be made somewhat smoother by joining a capacitor C across R_2 as shown in the figure.

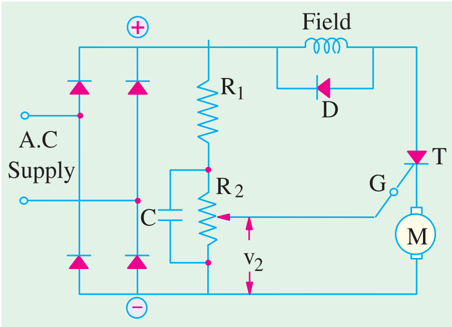


Fig. 30.36

30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = (V - E_b)/R_a$$

where V is the supply voltage, E_b the back e.m.f. and R_a the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resitance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of 0.25 Ω and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of 440/0.25 = 1760 A which is 1760/50 = 35.2 times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

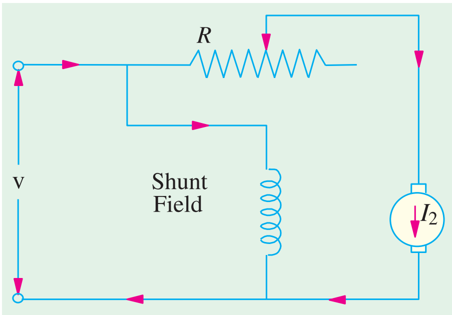


Fig. 30.37

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons :

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance R used for starting a shunt motor is shown. It will be seen that the starting resistance R is in series with the ***armature*** and not with the ***motor*** as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the resistance R. If R wes introduced in the motor circuit, then I_{sh} will be small at the start, hence starting torque T_{st} would be small ($\because\ T_a \propto \Phi I_a$) and there would be experienced some difficulty in starting the motor. Such a simple starter is shown diagramatically in Fig. 30.38.

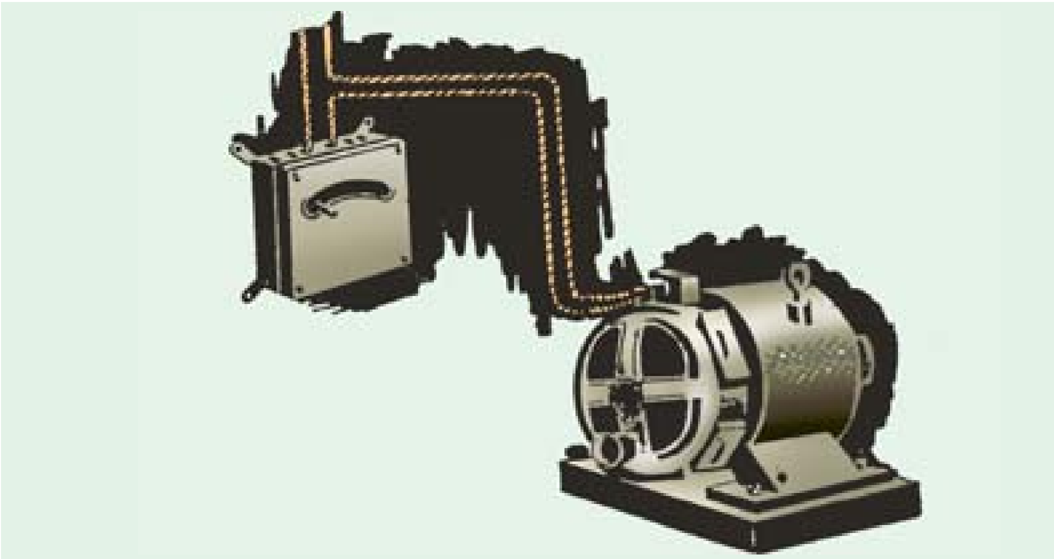


Fig. 30.38

30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked A, B and C. One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point A which is further connected to the starting arm L, through the overcurrent (or overload) release M.

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance R, is placed in series with the armature. The starting current drawn by the armature $= V/(R_a + R_s)$ where R_s is the starting

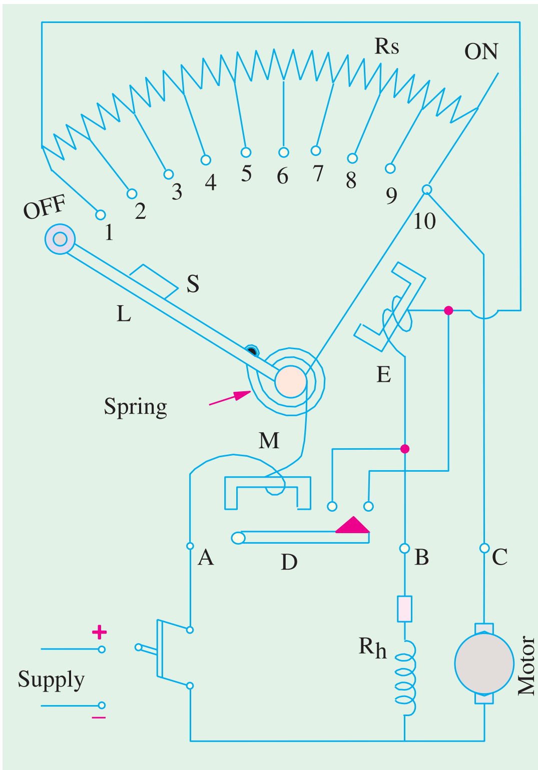


Fig. 30.39

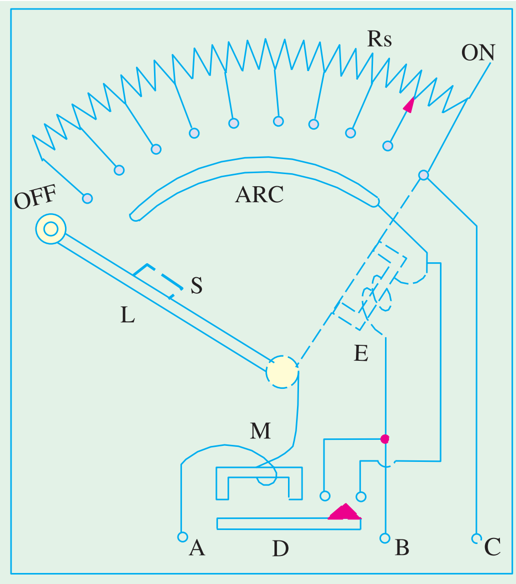


Fig. 30.40

resistance. As the arm is further moved, the starting resistance is gradually cut out till, when the arm reaches the running position, the resistance is all cut out. The arm moves over the various studs against a strong spring which tends to restore it to OFF position. There is a soft iron piece *S* attached to the arm which in the full 'ON' or running position is attracted and held by an electromagnet *E* energised by the shunt current. It is variously known as 'HOLD-ON' coil, LOW VOLTAGE (or NO-VOLTAGE) release.

It will be seen that as the arm is moved from stud NO. 1 to the last stud, the field current has to travel back through that portion of the starting resistance that has been cut out of the armature circuit. This results is slight decrease of shunt current. But as the value of starting resistance is very small as compared to shunt field resistance, this slight decreases in I_{sh} is negligible. This defect can, however, be remedied by using a brass arc which is connected to stud No. 1 (Fig. 30.40). The field circuit is completed through the starting resistance as it did in Fig. 30.39.

Now, we will discuss the action of the two protective devices shown in Fig. 30.39. The normal function of the HOLD-ON coil is to hold on the arm in the full running position when the motor is in normal operation. But, in the case of failure or disconnection of the supply or a break in the field circuit, it is de-energised, thereby releasing the arm which is pulled back by the spring to the OFF position. This prevents the stationary armature from being put across the lines again when the supply is restored after temporary shunt down. This would have happened if the arm were left in the full ON position. One great advantage of connecting the HOLD-ON coil in series with the shunt field is that, should the field circuit become open, the starting arm immediately springs back to the OFF position thereby preventing the motor from running away.

The overcurrent release consists of an electromagnet connected in the supply line. If the motor becomes overloaded beyond a certain predetermined value, then *D* is lifted and short circuits the electromagnet. Hence, the arm is released and returns to OFF position.

The form of overload protection described above is becoming obsolete, because it cannot be made either as accurate or as reliable as a separate well-designed circuit breaker with a suitable time element attachment. Many a times a separated magnetic contactor with an overload relay is also used.

Often the motors are protected by thermal overload relays in which a bimetallic strip is heated by the motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contactor, thereby isolating the motor from the supply.

If it is desired to control the speed of the motor in addition, then a field rheostat is connected in the filed circuit as indicated in Fig.30.39.The motor speed can be increased by weakening the flux (Œ $N \propto I/\Phi$). Obviously, there is a limit to the speed increase obtained in this way, although speed ranges of three to four are possible. The connections of a starter and speed regulator with the motor are shown diagrammatically in Fig. 30.41. But there is one difficulty with such an arrangement for speed control. If too much resistance is 'cut in' by the field rheostat, then field current is reduced very much so that it is unable to create enough electromagnetic pull to overcome the spring tension. Hence, the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-speed motors. This has resulted in widespread application of four-point starter discussed below.

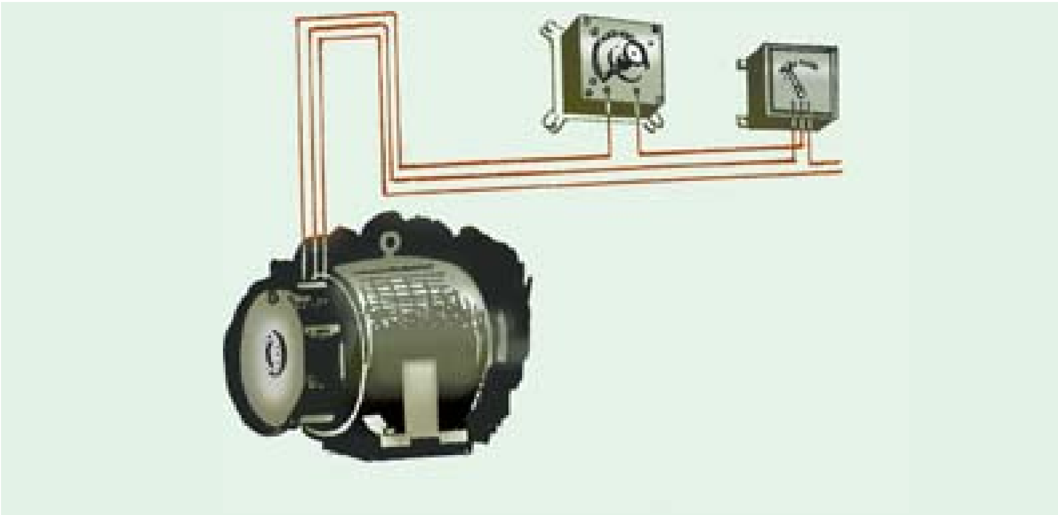
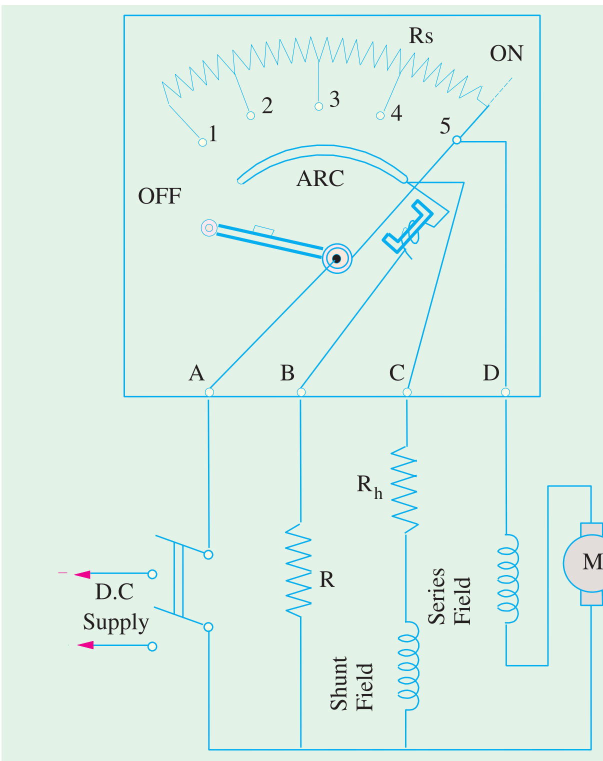


Fig. 30.41

30.22. Four-point Starter



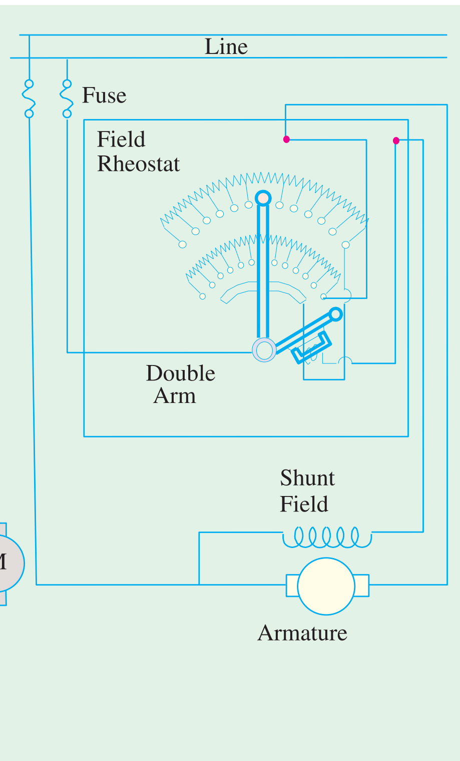


Fig. 30.42 Fig. 30.43

Such a starter with its internal wiring is shown, connected to a long-shunt compound motor in Fig. 30.42. When compared to the three-point starter, it will be noticed that one important change has been made *i.e.*, the HOLD-ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a protecting resistance as shown. When the arm touches stud No. 1, then the line current divides into three parts (*i*) one part passes through starting resistance R_s, series field and motor armature (*ii*) the second part passes through the shunt field and its field rheostat R_h and (*iii*) the third part passes through the HOLD-ON coil and current-protecting resistance R. It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not at all affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means that the electromagnetic pull exerted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting arm to OFF position no matter how the field rheostat or regulator is adjusted.

30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor, whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.

Speed-control Rheostats

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

30.24. Starting and Speed Control of Series Motor

For starting and speed control of series motor either a face-plate type or drum-type controller is used which usually has the reversing feature also. A face-plate type of reversing controller is shown in Fig. 30.44.

Except for a separate overload circuit, no inter-locking or automatic features are required because the operator watches the performance continuously.

As shown, the regulating lever consists of three pieces separated by strips of insulation. The outside parts form the electrical connections and the middle one is insulated from them. By moving the regulating lever, resistance can be cut in and out of the motor circuit. Reversing is obtained by moving the lever in the opposite direction as shown, because in that case, connections to the armature are reversed. Such an arrangement is employed where series motors are used as in the case of cranes, hoists and streetcars etc.

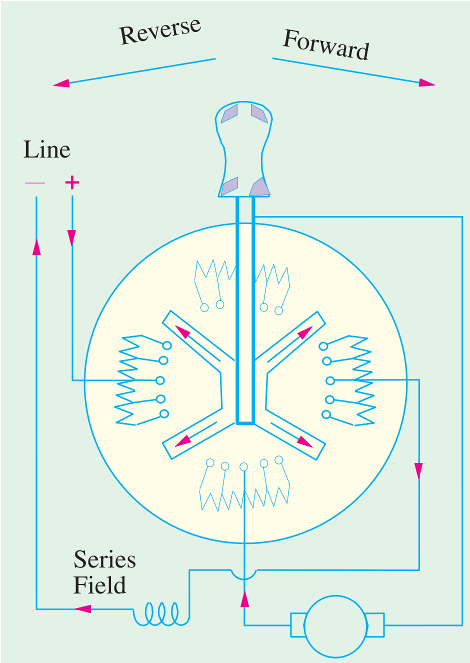


Fig. 30.44

However, for adjustable speed service in connection with the operation of machine tools, a drum controller is preferred. It is called 'controller' because in addition to accelerating the motor to its normal speed, it provides the means for reversing the direction of the motor. Other desirable features such as safety protection against an open field or the temporary failure of power supply and overloads are frequently provided in this type of controller.

The controller consists of armature resistance grids of cross-section sufficient to carry the full-load operating current continuously and are used for adjusting the motor speed to values lower than the base speed obtained with no external resistance in the armature of field circuit. As the operating handle is gradually turned, the resistance is cut out of the armature circuit–there being as yet no resistance in the field circuit at this stage, then when resistance in the armature circuit is completely cut out, further rotation of the handle inserts resistance into the field circuit. Turning of the handle in the opposite direction starts and speeds up the motor in the reverse direction.

30.25. Grading of Starting Resistance for Shunt Motors

T_{st} would be small in designing shunt motor starters, it is usual to allow an overload of 50% for starting and to advance the starter a step when armature current has fallen to definite lower value. Either this lower current limit may be fixed or the number of starter steps may be fixed. In the former case, the number of steps are so chosen as to suit the upper and lower current limits whereas in the latter case, the lower current limit will depend on the number of steps specified. It can be shown that the resistances in the circuit on successive studs from geometrical progression, having a common ratio equal to lower current limit/upper current limit *i.e.*, I_2/I_1.

In Fig. 30.45 the starter connected to a shunt motor is shown. For the sake of simplicity, four live studs have been taken. When arm *A* makes contact with stud No. 1, full shunt field is

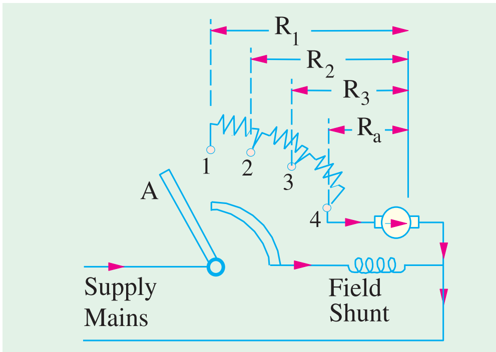


Fig. 30.45

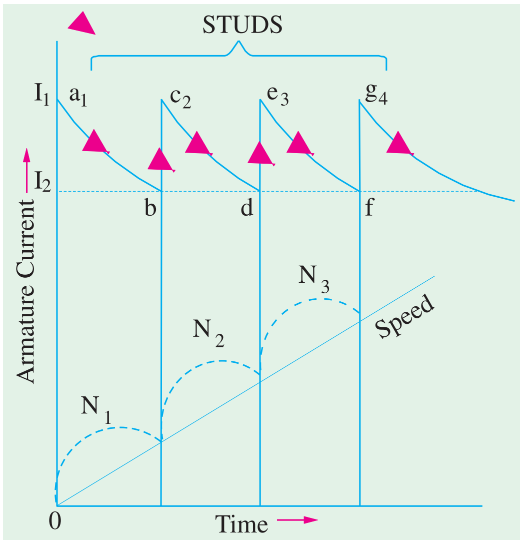


Fig. 30.46

established and at the same time the armature current immediately jumps to a maximum value I_1 given by $I_1 = V/R_1$ where R_1 = armature and starter resistance (Fig. 30.45).

I_1 the maximum permissible armature current at the start (I_{max}) and is, as said above, usually limited to 1.5 times the full-load current of the motor. Hence, the motor develops 1.5 times its full-load torque and accelerates very rapidly. As the motor speeds up, its back e.m.f. grows and hence decreases the armature current as shown by curve *ab* in Fig. 30.46.

When the armature current has fallen to some predetermined value I_2 (also called I_{min}) arm *A* is moved to stud No. 2. Let the value of back e.m.f. be E_{b1} at the time of leaving stud No. 1. Then

$$I_2 = \frac{V - E_{b1}}{R_1} \qquad \text{...(i)}$$

It should be carefully noted that I_1 and I_2 [(I_{max}) and (I_{min})] are respectively the maximum and minimum currents of the motor. When arm *A* touches stud No. 2, then due to diminution of circuit resistance, the current again jumps up to its previous value I_1. Since speed had no time to change, the back e.m.f. remains the same as initially.

$$\therefore \qquad I_1 = \frac{V - E_{b1}}{R_2} \qquad \text{...(ii)}$$

From **(*i*)** and **(*ii*)**, we get
$$\frac{I_1}{I_2} = \frac{R_1}{R_2} \qquad \text{...(iii)}$$

When arm *A* is held on stud No. 2 for some time, then speed and hence the back e.m.f. increases to a value E_{b2}, thereby decreasing the current to previous value I_2, so that

$$I_2 = \frac{V - E_{b3}}{R_2} \qquad \text{...(iv)}$$

Similarly, on first making contact with stud No. 3, the current is

$$I_1 = \frac{V - E_{b2}}{R_3} \qquad \text{...(v)}$$

From **(*iv*)** and **(*v*)**, we again get
$$\frac{I_1}{I_2} = \frac{R_2}{R_3} \qquad \text{...(vi)}$$

When arm *A* is held on stud No. 3 for some time, the speed and hence back e.m.f. increases to a new value E_{b3}, thereby decreasing the armature current to a value I_2 such that

$$I_2 = \frac{V - E_{b3}}{R_3} \qquad \text{...(vii)}$$

On making contact with stud No. 4, current jumps to I_1 given by

$$I_1 = \frac{V - E_{b3}}{R_a} \qquad \text{...(viii)}$$

From **(*vii*)** and **(*viii*)**, we get
$$\frac{I_1}{I_2} = \frac{R_3}{R_a} \qquad \text{...(ix)}$$

From **(*iii*), (*vi*)** and **(*ix*)**, it is seen that

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K \text{ (say)} \qquad \text{...(x)}$$

Obviously,
$$R_3 = KR_a; \; R_2 = KR_3 = K^2R_a$$
$$R_1 = KR_2 = K.K^2R_a = K^3R_a$$

In general, if *n* is the number of live studs and therefore (*n* −1) the number of sections in the starter resistance, then

$$R_1 = K^{n-1}.R_a \text{ or } \frac{R_1}{R_a} = K^{n-1} \text{ or } \left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_a} \qquad \text{...from (x)}$$

Other variations of the above formula are

(*a*) $$K^{n-1} = \frac{R_1}{R_a} = \frac{V}{I_1 R_a} = \frac{V}{I_{max}\, R_a}$$

(*b*) $$K^n = \frac{V}{I_1 R_a} \cdot \frac{I_1}{I_2} = \frac{V}{I_2 R_a} = \frac{V}{I_{min.} R_a} \text{ and}$$

(*c*) $$n = 1 + \frac{\log (V / R_a.I_{max})}{\log K} \text{ from } (a) \text{ above.}$$

Since $R_1 = V/I_1$ and R_a are usually known and K is known from the given values of maximum and minimum currents (determined by the load against which motor has to start), the value of n can be found and hence the value of different starter sections.

When Number of Sections is Specified.

Since I_1 would be given, R_1 can be found from $R_1 = V/I_1$.

Since n is known, K can be found from $R_1/R_a = K^{n-1}$ and the lower current limit I_2 from $I_1/I_2 = K$.

Example 30.49. *A 10 b.h.p. (7.46 kW) 200-V shunt motor has full-load efficiency of 85%. The armature has a resistance of 0.25 Ω. Calculate the value of the starting resistance necessary to limit the starting current to 1.5 times the full-load current at the moment of first switching on. The shunt current may be neglected. Find also the back e.m.f. of the motor, when the current has fallen to its full-load value, assuming that the whole of the starting resistance is still in circuit.*

Solution. Full-load motor current $= 7{,}460/200 \times 0.85 = 43.88$ A

Starting current, $I_1 = 1.5 \times 43.88 = 65.83$ A

$R_1 = V/I_1 = 200/65.83 = 3.038\ \Omega\ ;\ R_a = 0.25\ \Omega$

∴ Starting resistance $= R_1 - R_a = 3.038 - 0.25 = 2.788\ \Omega$

Now, full-load current $I_2 = 43.88$ A

Now, $$I_2 = \frac{V - E_{b1}}{R_1}$$

∴ $E_{b1} = V - I_2 R_1 = 200 - (43.88 \times 3.038) =$ **67 V**

Example 30.50. *A 220-V shunt motor has an armature resistance of 0.5 Ω. The armature current at starting must not exceed 40 A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter.* **(Elect. Machines, AMIE Sec. B, 1992)**

Solution. Since the number of starter sections is specified, we will use the relation.

$R_1/R_a = K^{n-1}$ or $R_1 = R_a K^{n-1}$

Now, $R_1 = 220/40 = 5.5\Omega,\ R_a = 0.4\ \Omega\ ;\ n{-}1 = 6,\ n = 7$

∴ $5.5 = 0.4\,K^6$ or $K^6 = 5.5/0.4 = 13.75$

$6 \log_{10} K = \log_{10} 13.75 = 1.1383\ ;\ K = 1.548$

Now, $R_2 = R_1/K = 5.5/1.548 = 3.553\ \Omega$

$R_3 = R_2/K = 3.553/1.548 = 2.295\ \Omega$

$R_4 = 2.295/1.548 = 1.482\ \Omega$

$R_5 = 1.482/1.548 = 0.958\ \Omega$

$R_6 = 0.958/1.548 = 0.619\ \Omega$

Resistance of Ist section $= R_1 - R_2 = 5.5 - 3.553 =$ **1.947 Ω**

,, 2nd ” $= R_2 - R_3 = 3.553 - 2.295 =$ **1.258 Ω**

,, 3rd ” $= R_3 - R_4 = 2.295 - 1.482 =$ **0.813 Ω**

,, 4th ” $= R_4 - R_5 = 1.482 - 0.958 =$ **0.524 Ω**

,, 5th ” $= R_5 - R_6 = 0.958 - 0.619 =$ **0.339 Ω**

,, 6th ” $= R_6 - R_a = 0.619 - 0.4 =$ **0.219 Ω**

Example 30.51. *Find the value of the step resistance in a 6-stud starter for a 5 h.p. (3.73 kW), 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The total Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is found to be 88%.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Output = 3,730 W

Total loss = 4,238 – 3,730 = 508 W

Armature Cu loss alone = 508/2 = 254 W

Input current = 4,238/200 = 21.19 A

Armature current = 21.19 – 0.6 = 20.59 A

$\therefore \quad 20.59^2 R_a = 254 \quad \therefore \quad R_a = 254/20.59^2 = 0.5989\ \Omega$

Permissible input current = 21.19 × 2 = 42.38 A

Permissible armature current = 42.38 – 0.6 = 41.78 A

$\therefore \quad R_1 = 200/41.78 = 4.787\ \Omega; \quad n = 6\ ; \quad n - 1 =$ **5**

$\therefore \quad 4.787 = K^5 \times 0.5989 \qquad \therefore \quad K^5 = 4.787/0.5989 = 7.993$

or $\quad 5 \log K = \log 7.993 = 0.9027$

$\therefore \quad \log K = 0.1805\ ; \quad K = 1.516$

Now $\quad R_2 = R_1/K = 4.789/1.516 = 3.159\ \Omega$

$R_3 = 2.084\ \Omega\ ; \quad R_4 = 1.376\ \Omega\ ; \quad R_5 = 0.908\ \Omega$

Resistance in 1st step $= R_1 - R_2 = 4.787 - 3.159 =$ **1.628 Ω**

Resistance in 2nd step $= R_2 - R_3 = 3.159 - 2.084 =$ **1.075 Ω**

Resistance in 3rd step $= R_3 - R_4 = 2.084 - 1.376 =$ **0.708 Ω**

Resistance in 4th step $= R_4 - R_5 = 1.376 - 0.908 =$ **0.468 Ω**

Resistance in 5th step $= R_5 - R_a = 0.908 - 0.5989 =$ **0.309 Ω**

The various sections are shown in Fig. 28.47.

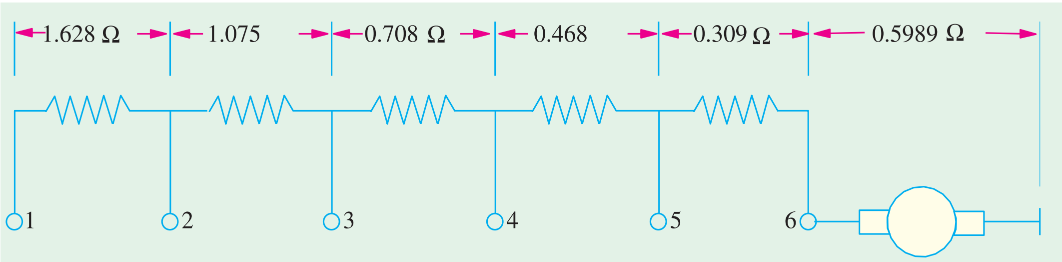


Fig. 30.47

Example 30.52. *Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, d.c. shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.*

(Elec. Machines, Gujarat Univ. 1987)

Solution. Output = 36,775 W ; Input = 36,775/0.92 = 39,980 W

Total Cu loss = 0.05 × 39,980 = 1,999W

Shunt Cu loss $= V^2/R_{sh} = 400^2/200 = 800$ W

Armature Cu loss $= 1{,}999 - 800 = 1199$ W

F.L. input current $= 39{,}980/400 = 99.95$ A

$I_{sh} = 400/200 = 2\text{A}$; $I_a = 99.95 - 2 = 97.95$ A

$\therefore \quad 97.95^2 R_a = 1199$ W or $R_a = 0.125\ \Omega$

Now, minimum armature current equals full-load current *i.e.* $I_a = 97.95$ A. As seen from Art. 30.25 in its formula given in (*b*), we have

$K^n =$ [illegible]

or $\quad K^7 = 400/97.95 \times 0.125 = 32.68$

$\therefore \quad K = 32.68^{1/7} = 1.645$

$I_1 =$ maximum permissible armature current

$= KI_2 = 1.645 \times 97.94 = 161$ A

$\therefore \quad R_1 = V/I_1 = 400/161 = 2.483\ \Omega$

$R_2 = R_1/K = 2.483/1.645 = 1.51\ \Omega$

$R_3 = 1.51/1.645 = 0.917\ \Omega$

$R_4 = 0.917/1.645 = 0.557\ \Omega$

$R_5 = 0.557/1.645 = 0.339\ \Omega$

$R_6 = 0.339/1.645 = 0.206\ \Omega$

$R_7 = 0.206/1.645 = 0.125\ \Omega$

Resistance in 1st step $= R_1 - R_2 =$ **0.973 Ω**

Resistance in 2nd step $= R_2 - R_3 =$ **0.593 Ω**

Resistance in 3rd step $= R_3 - R_4 =$ **0.36 Ω**

Resistance in 4th step $= R_4 - R_5 =$ **0.218 Ω**

Resistance in 5th step $= R_5 - R_6 =$ **0.133 Ω**

Resistance in 6th step $= R_6 - R_a =$ **0.081 Ω**

The various starter sections are shown in Fig 30.48.

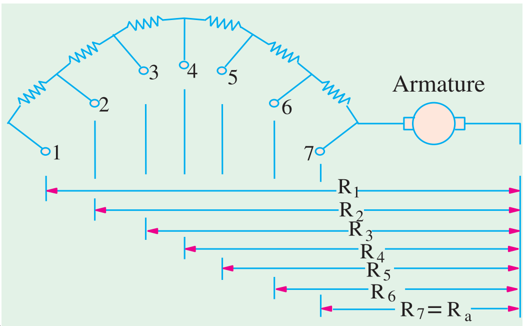


Fig. 30.48

Example 30.53. *Calculate the resistance steps for the starter of a 250-V, d.c. shunt motor having an armature resistance of 0.125 Ω and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A*

. **(Elect.Engineering-I, Bombay Univ. 1989)**

Solution. As the motor is to start against its full-load, the minimum current is its F.L. current *i.e.* 150A. We will use the formula given in Art. 30.25.

$(I_1/I_2)^{n-1} = R_1/R_a$

Here $\quad I_1 = 200$ A ; $I_2 = 150$ A ; $R_1 = 250/200 =$ **1.25 Ω**

$R_a = 0.125\ \Omega$; $n =$ No. of live studs

$\therefore \quad (200/150)^{n-1} = 1.25/0.125 = 10$ or $(4/3)^{n-1} = 10$

$(n - 1) \log 4/3 = \log 10$ or $(n - 1) \times 0.1249 = 1$

$\therefore \quad (n - 1) = 1/0.1249 = 8$

Hence, there are 9 studs and 8 steps.

Now $\quad R_2 = R_1 \times I_2/I_1 = 1.25 \times 3/4 = 0.938\ \Omega$

$R_3 = 0.938 \times 3/4 = 0.703\ \Omega$

$R_4 = 0.703 \times 3/4 = 0.527\ \Omega$

$$R_5 = 0.527 \times 3/4 = 0.395\ \Omega$$
$$R_6 = 0.395 \times 3/4 = 0.296\ \Omega$$
$$R_7 = 0.296 \times 3/4 = 0.222\ \Omega$$
$$R_8 = 0.222 \times 3/4 = 0.167\ \Omega$$
$$R_a = 0.167 \times 3/4 = 0.125\ \Omega$$

∴ Resistance of 1st element = 1.25 – 0.938 = **0.312 Ω**

,, 2nd ,, = 0.938 – 0.703 = **0.235 Ω**

,, 3rd ,, = 0.703 – 0.527 = **0.176 Ω**

,, 4th ,, = 0.527 – 0.395 = **0.132 Ω**

,, 5th ,, = 0.395 – 0.296 = **0.099 Ω**

,, 6th ,, = 0.296 – 0.222 = **0.074 Ω**

,, 7th ,, = 0.222 – 0.167 = **0.055 Ω**

,, 8th ,, = 0.167 – 0.125 = **0.042 Ω**

Example 30.54. *The 4-pole, lap-wound armature winding of a 500-V, d.c. shunt motor is housed in a total number of 60 slots each slot containing 20 conductors. The armature resistance is 1.31 Ω. If during the period of starting, the minimum torque is required to be 218 N-m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.*

(Elect. Machinery-II, Bangalore Univ. 1991)

Solution. From the given minimum torque, we will be able to find the minimum current required during starting. Now

$$T_a = 0.159\ \Phi\ ZI_a\,(P/A)$$

$$\therefore\quad 218 = 0.159 \times 23 \times 10^{-3} \times (60 \times 20)\, I_a \times (4/4) \qquad \therefore\ I_a = 50 \text{ A (approx.)}$$

$$\text{Maximum current} = 50 \times 1.5 = 75 \text{ A}$$

$$\therefore\quad I_1 = 75 \text{ A};\ I_2 = 50 \text{ A} \qquad \therefore\ I_1/I_2 = 75/50 = 1.5$$

$$R_1 = 500/75 = 6.667\ \Omega$$

If n is the number of stater studs, then

$$(I_1/I_2)^{n-1} = R_1/R_a \qquad \text{or} \qquad 1.5^{n-1} = 6.667/1.31 = 5.09$$

$$\therefore\ (n-1)\log_{10} 1.5 = \log_{10} 5.09 \quad \therefore\ (n-1) \times 0.1761 = 0.7067 \quad \therefore\ (n-1) = 4 \text{ or } n = 5$$

Hence, there are five studs and four sections.

$$R_2 = R_1 \times I_2/I_1 = 6.667 \times 2/3 = 4.44\ \Omega$$

$$R_3 = 4.44 \times 2/3 = 2.96\ \Omega;\quad R_4 = 2.96 \times 2/3 = 1.98\ \Omega$$

Resistance of 1st section = $R_1 - R_2$ = 6.67 – 4.44 = **2.23 Ω**

,, 2nd ,, = $R_2 - R_3$ = 4.44 – 2.96 = **1.48 Ω**

,, 3rd ,, = $R_3 - R_4$ = 2.96 – 1.98 = **0.98 Ω**

,, 4th ,, = $R_4 - R_a$ = 1.98 – 1.31 = **0.67 Ω**

30.26. Series Motor Starters

The basic principle employed in the design of a starter for series motor is the same as for a shunt motor *i.e.*, the motor current is not allowed to exceed a certain upper limit as the starter arm moves from one stud to another. However, there is one significant difference. In the case of a series motor, the flux does not remain constant but varies with the current because armature current is also the exciting current. The determination of the number of steps is rather complicated as illustrated in Example 30.55. It may however, be noted that the section resistances form a geometrical progression.

The face-plate type of starter formerly used for d.c. series motor has been almost entirely replaced by automatic starter in which the resistance steps are cut out automatically by means of a contactor operated by electromagnets. Such starters are well-suited for remote control.

However, for winch and crane motors where frequent starting, stopping, reversing and speed variations are necessary, drum type controllers are used. They are called controllers because they can be left in the circuit for any length of time. In addition to serving their normal function of starters, they also used as speed controllers.

Example 30.55. **(*a*)** *Show that, in general, individual resistances between the studs for a rheostat starter for a series d.c. motor with constant ratio of maximum to minimum current at starting, are in geometrical progression, stating any assumptions made.*

(*b*) *Assuming that for a certain d.c. series motor the flux per pole is proportional to the starting current, calculate the resistance of the each rheostat section in the case of a 50 b.h.p. (37.3 kW) 440-V motor with six sections.*

The total armature and field voltage drop at full-load is 2% of the applied voltage, the full-load efficiency is 95% and the maximum starting current is 130% of full-load current.

Solution. **(*a*)** Let I_1 = maximum current, I_2 = minimum current

Φ_1 = flux/pole for I_1 ; Φ_2 = flux/pole for I_2

$$\frac{I_1}{I_2} = K \text{ and } \frac{\Phi_1}{\Phi_2} = \alpha .$$

Let us now consider the conditions when the starter arm is on the nth and $(n + 1)$th stud. When the current is I_2, then $E_b = V - I_2 R_n$.

If, now, the starter is moved up to the $(n + 1)$th stud, then

$$E_b' = \frac{\Phi_1}{\Phi_2} . E_b = \alpha E_b$$

$$\therefore \quad R_{n+1} = \frac{V - E_b'}{I_1} = \frac{V - \alpha E_b}{I_1} = \frac{V - \alpha(V - I_2 R_n)}{I_1} = \frac{V}{I_1}(1 - \alpha) + \alpha \frac{I_2}{I_1} \cdot R_n$$

Now, $V/I_1 = R_1$–the total resistance in the circuit when the starter arm is on the first stud.

$$\therefore \quad R_{n+1} = R_1 (1 - \alpha) + \frac{\alpha}{K} R_n$$

Similarly, by substituting $(n - 1)$ for n, we get $R_n = R_1 (1 - \alpha) + \dfrac{\alpha}{K} R_{n-1}$

Therefore, the resistance between the nth and $(n + 1)$th studs is

$$r_n = R_n - R_{n+1} = \frac{\alpha}{K} R_{n-1} - \frac{\alpha}{K} R_n = \frac{\alpha}{K}(R_{n-1} - R_n) = \frac{\alpha}{K} r_{n-1}$$

$$\therefore \quad \frac{r_n}{r_{n-1}} = \frac{\alpha}{K} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = b \quad \text{– constant}$$

Obviously, the resistance elements form a geometrical progression series.

(*b*) Full-load input current $= \dfrac{37,300}{440 \times 0.95} = 89.2$ A

Max. starting current $I_1 = 1.3 \times 89.2 = 116$ A

Arm. and field voltage drop on full-load = 2% of 440 = 0.02×440 = 8.8 V

Resistance of motor = 8.8/89.2 = 0.0896 Ω

Total circuit resistance on starting, $R_1 = V/I_1$ = 440/116 = 3.79 Ω

Assuming straight line magnetisation, we have $I_1 \propto \Phi_1$ and $I_2 \propto \Phi_2$

$\therefore \quad I_1/I_2 = \Phi_1/\Phi_2 \qquad \therefore \quad \alpha = K$ and $b = \alpha/K = 1 \qquad \therefore \quad r_n = b \times r_{n-1}$

In other words, all sections have the same resistance.

$$\therefore \qquad r = \frac{R_1 - R_{motor}}{\text{No. of sections}} = \frac{3.79 - 0.0896}{6} = \mathbf{0.6176\ \Omega}$$

Example 30.56. *A 75 h.p. (55.95 kW) 650-V, d.c. series tractions motor has a total resistance of 0.51 Ω. The starting current is to be allowed to fluctuate between 140 A and 100 A the flux at 140 A being 20 % greater than at 100 A. Determine the number of steps required in the controller and the resistance of each step.*

Solution. Let R_1 = total resistance on the first stud = 650/140 = 4.65 Ω

When motor speeds up, then back e.m.f. is produced and current falls to I_2.

$$V = E_{b1} + I_2R_1 \qquad ...(i)$$

When the starter moves to the next stud, the speed is still the same, but since current rises to I_1 for which flux is 1.2 times greater than for I_2, hence back e.m.f. becomes 1.2 E_{b1}. Since resistance in the circuit is now R_2.

$$\therefore \qquad V = I_1R_2 + 1.2\,E_{b1} \qquad ...(ii)$$

From (*i*) and (*ii*) we get, $0.2\,V = 1.2\,I_2R_1 - I_1R_2$

$$\therefore \qquad R_2 = (1.2I_2/I_1)\,R_1 - 0.2\,V/I_1 = (1.2I_2/I_1)\,R_1 - 0.2\,R_1 = (1.2I_2/I_1 - 0.2)\,R_1$$

Similarly $\qquad R_3 = (1.2I_2/I_1)\,R_2 - 0.2\,R_1$

In this way, we continue till we reach the value of resistance equal to the armature resistance. Hence, we obtain R_1, R_2 etc. and also the number of steps.

In the present case, $\quad I_1 = 140$ A, $I_2 = 100$A, $V = 650$ V and $I_1/I_2 = 100/140 = 1.14$

$$\therefore \qquad R_1 = 4.65\ \Omega;\ R_2 = (1.2/1.4 - 0.2)\,4.65 = 3.07\ \Omega$$
$$R_3 = (1.2/1.4) \times 3.07 - 0.2 \times 4.65 = 1.70\ \Omega$$
$$R_4 = (1.2/1.4) \times 1.7 - 0.2 \times 4.65 = 0.53\ \Omega$$

We will stop here because R_4 is very near the value of the motor resistance. Hence, there are 4 studs and 3 sections or steps.

$$R_1 - R_2 = 4.65 - 3.07 = 1.58\ \Omega,\quad R_2 - R_3 = 3.07 - 1.7 = 1.37\ \Omega$$
$$R_3 - R_4 = 1.70 - 0.53 = 1.17\ \Omega$$

Note. It will be seen that

$$R_2 - R_3 = (1.2\,I_2/I_1)\,(R_1 - R_2) \text{ and } R_3 - R_4 = (1.2\,I_2/I_1)\,(R_2 - R_3) \text{ and so on.}$$

or
$$\frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} = 1.2\frac{I_2}{I_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = \frac{\alpha}{K} = b$$

It is seen that individual resistances of various sections decrease in the ratio of $\alpha/K = b$.

30.27. Thyristor Controller Starters

The moving parts and metal contacts etc., of the resistance starters discussed in Art. 30.21 can be eliminated by using thyristors which can short circuit the resistance sections one after another. A thyristor can be switched on to the conducting state by applying a suitable signal to its gate terminal. While conducting, it offers zero resistance in the forward (*i.e.*, anode-to-cathode) direction and thus acts as a short-circuit for the starter resistance section across which it is connected. It can be switched off (*i.e.*, brought back to the non-conducting state) by reversing the polarity of its anode-cathode voltage. A typical thyristor-controlled starter for d.c. motors is shown in Fig. 30.49.

After switching on the main supply, when switch S_1 is pressed, positive signal is applied to gate G of thyristor T_1 which is, therefore, turned ON. At the same time, shunt field gets established since it is directly connected across the d.c. supply. Consequently, motor armature current I_a flows *via* T_1,

R_2, R_3 and R_4 because T_2, T_3 and T_4 are, as yet in the non-conducting state. From now onwards, the starting procedure is automatic as detailed below :

1. As S_1 is closed, capacitor C starts charging up with the polarity as shown when I_a starts flowing.
2. The armature current and field flux together produce torque which accelerates the motor and load.

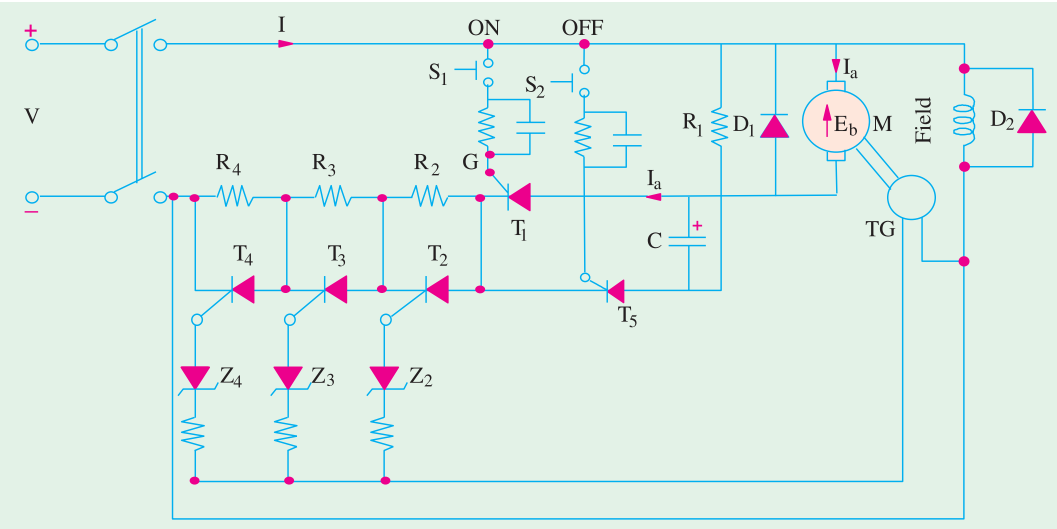


Fig. 30.49

3. As motor speeds up, voltage provided by tachogenerator (*TG*) is proportionately increased because it is coupled to the motor.
4. At some motor speed, the voltage provided by *TG* becomes large enough to breakdown Zener diode Z_2 and hence trigger T_2 into conduction. Consequently, R_2 is shorted out and now I_a flows via motor armature, T_1, T_2, R_3 and R_4 and back to the negative supply terminal.
5. As R_2 is cut out, I_a increases, armature torque increases, motor speed increases which further increases the voltage output of the tachogenerator. At some speed, Z_3 breaks down, thereby triggering T_3 into conduction which cuts out R_3.
6. After sometime, R_4 is cut out as Z_4 breaks down and triggers T_4 into conduction. In fact, Zener diodes Z_2, Z_3 and Z_4 can be rated for 1/3, 1/2 and 3/4 full speed respectively.

For stopping the motor, switch S_2 is closed which triggers T_5 into conduction, thereby establishing current flow via R_1. Consequently, capacitor C starts discharging thereby reverse-biasing T_1 which stops conducting. Hence I_a ceases and, at the same time, T_2, T_3 and T_4 also revert back to their non-conducting state.

Incidentally, it may be noted that the function of C is to switch T_1, ON and OFF. Hence, it is usually called ***commutating capacitor.***

The function of the diodes D_1 and D_2 is to allow the decay of inductive energy stored in the motor armature and field when supply is disconnected. Supply failure will cause the thyristors to block because of this current decay, thereby providing protection usually given by no-voltage release coil.

Recently, thyristor starting circuits have been introduced which use no starting resistance at all, thereby making the entire system quite efficient and optimized as regards starting time. These are based on the principle of 'voltage chopping' (Art. 30.12). By varying the chopping frequency, the ratio of the time the voltage is ON to the time it is OFF can be varied. By varying this ratio, the average voltage applied to the motor can be changed. A low average voltage is needed to limit the armature current while the motor is being started and gradually the ratio is increased to reach the maximum at the rated speed of the motor.

Tutorial Problems 32.2

1. A shunt-wound motor runs at 600 r.p.m. from a 230-V supply when taking a line current of 50 A. Its armature and field resistances are 0.4 Ω and 104.5 Ω respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (*a*) the no-load speed if the no-load line current is 5A (*b*) the resistance to be placed in armature circuit in order to reduce the speed to 500 r.p.m. when motor is taking a line current of 50 A (*c*) the percentage reduction in the flux per pole in order that the speed may be 750 r.p.m. when the armature current is 30 A with no added resistance in the armature circuit. **[(*a*) 652 r.p.m. (*b*) 0.73 Ω (*c*) 1.73 %]**

2. The resistance of the armature of a 250-V shunt motor is 0.3 Ω and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run ? Neglect armature reaction. **[0.94 Ω; 932 r.p.m.]**

3. A 230-V d.c. shunt motor takes an armature current of 20 A on a certain load. Resistance of the armature is 0.5 Ω. Find the resistance required in series with the armature to half the speed if (*a*) the load torque is constant (*b*) the load torque is proportional to the square of the speed.
[(*a*) 5.5 Ω (*b*) 23.5 Ω]

4. A 230-V series motor runs at 1200 r.p.m. at a quarter full-load torque, taking a current of 16 A. Calculate its speed at half and full-load torques. The resistance of the armature brushes, and field coils is 0.25 Ω. Assume the flux per pole to be proportional to the current. Plot torque/speed graph between full and quarter-load. **[842 r.p.m. ; 589 r.p.m.]**

5. A d.c. series motor drives a load the torque of which is proportional to the square of the speed. The motor current is 20 A when speed is 500 r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistance of the same value as the field winding. Neglect all motor losses and assume that the magnetic field is unsaturated. **[595 r.p.m. ; 33.64 A]**
(Electrical Machines-I, Aligarh Muslim Univ. 1979)

6. A d.c. series motor, with unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load takes 50 A at 500 V. If the load torque varies as the cube of the speed, find the resistance which should be connected in series with machine to reduce the speed by 25 per cent. **[7.89 Ω]**
(Electrical Engg-I, M.S. Univ. Baroda 1980)

7. A series motor runs at 500 r.p.m. on a certain load. Calculate the resistance of a divertor required to raise the speed to 650 r.p.m. with the same load current, given that the series field resistance is 0.05 Ω and the field is unsaturated. Assume the ohmic drop in the field and armature to be negligible.
[0.1665 Ω]

8. A 230-V d.c. series motor has armature and field resistances of 0.5 Ω and 0.3 Ω respectively. The motor draws a line current of 40 A while running at 400 r.p.m. If a divertor of resistance 0.15 W is used, find the new speed of the motor for the same armature current.

It may be assumed that flux per pole is directly proportional to the field current. **[1204 r.p.m.]**
(Electrical Engineering Grad. I.E.T.E. June 1986)

9. A 250-V, d.c. shunt motor runs at 700 r.p.m. on no-load with no extra resistance in the field and armature circuit. Determine :

(*i*) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 A.

(*ii*) the resistance to be placed in series with the field to produce a speed of 1,000 r.p.m. when taking an armature current of 18 A.

Assume that the useful flux is proportional to the field. Armature resistance = 0.35 Ω, field resistance = 125 Ω. **[(*i*) 7.3 Ω (*ii*) 113 Ω] (Elect. Engg. Grad. I.E.T.E., June 1984)**

10. A d.c. series motor is operating from a 220-V supply. It takes 50 A and runs at 1000 r.p.m. The resistance of the motor is 0.1 Ω. If a resistance of 2 Ω is placed in series with the motor, calculate the resultant speed if the load torque is constant. **[534 r.p.m.]**

11. A d.c. shunt motor takes 25 A when running at 1000 r.p.m. from a 220-V supply.

Calculate the current taken form the supply and the speed if the load torque is halved, a resistance of 5 Ω is placed in the armature circuit and a resistance of 50 Ω is placed in the field circuit.

Armature resistance = 0.1 Ω ; field resistance = 100 Ω

Assume that the field flux per pole is directly proportional to the field current. **[17.1 A; 915 r.p.m.]**

(Elect. Technology, Gwalior Univ. Nov. 1977)

12. A 440-V shunt motor takes an armature current of 50 A and has a flux/pole of 50 mWb. If the flux is suddenly decreased to 45 mWb, calculate (*a*) instantaneous increase in armature current (*b*) percentage increase in the motor torque due to increase in current (*c*) value of steady current which motor will take eventually (*d*) the final percentage increase in motor speed. Neglect brush contact drop and armature reaction and assume an armature resistance of 0.6 Ω.

[(*a*) 118 A (*b*) 112 % (*c*) 5.55 A (*d*) 10%]

13. A 440-V shunt motor while running at 1500 r.p.m. takes an armature current of 30 A and delivers a mechanical output of 15 h.p. (11.19 kW). The load torque varies as the square of the speed. Calculate the value of resistance to be connected in series with the armature for reducing the motor speed to 1300 r.p.m. and the armature current at that speed. **[2.97 Ω, 22.5 A]**

14. A 460-V series motor has a resistance of 0.4 Ω and takes a current of 25 A when there is no additional controller resistance in the armature circuit. Its speed is 1000 r.p.m. The control resistance is so adjusted as to reduce the field flux by 5%. Calculate the new current drawn by the motor and its speed. Assume that the load torque varies as the square of the speed and the same motor efficiency under the two conditions of operation.

[22.6 A; 926 r.p.m.] (Elect. Machines, South Gujarat Univ. Oct. 1977)

15. A 460-V, series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change and torque if the load is reduced so that the motor is taking 30 A. Total resistance of armature and field circuit is 0.8 Ω. Assume flux proportional to the field current.

[680 r.p.m. 43.75%]

16. A 440-V, 25 h.p (18.65 kW) motor has an armature resistance of 1.2 Ω and full-load efficiency of 85%. Calculate the number and value of resistance elements of a starter for the motor if maximum permissible current is 1.5 times the full-load current. **[1.92 Ω, 1.30 Ω, 0.86 Ω; 0.59 Ω]**

(Similar example in JNTU, Hyderabad, 2000)

17. A 230-V, d.c. shunt motor has an armature resistance of 0.3 Ω. Calculate (*a*) the resistance to be connected in series with the armature to limit the armature current to 75 A at starting and (*b*) value of the generated e.m.f. when the armature current has fallen to 50 A with this value of resistance still in circuit. **[(*a*) 2.767 Ω (*b*) 76.7 A]**

18. A 200-V, d.c. shunt motor takes full-load current of 12 A. The armature circuit resistance is 0.3 Ω and the field circuit resistance is 100 Ω. Calculate the value of 5 steps in the 6-stud starter for the motor. The maximum starting current is not to exceed 1.5 times the full-load current.

[6.57 Ω, 3.12 Ω, 1.48 Ω, 0.7 Ω, 0.33 Ω]

19. The resistance of a starter for a 200-V, shunt motor is such that maximum starting current is 30 A. When the current has decreased to 24 A, the starter arm is moved from the first to the second stud.

Calculate the resistance between these two studs if the maximum current in the second stud is 34 A. The armature resistance of the motor is 0.4 Ω. **[1.334 Ω]**

20. A totally-enclosed motor has thermal time constant of 2 hr. and final temperature rise at no-load and 40° on full load.

Determine the limits between which the temperature fluctuates when the motor operates on a load cycle consisting of alternate period of 1 hr. on full-load and 1 hr. on no-load, steady state conditions having been established. **[28.7° C, 21.3°C]**

21. A motor with a thermal time constant of 45 min. has a final temperature rise of 75°C on continuous rating (*a*) What is the temperature rise after one hour at this load ? (*b*) If the temperature rise on one-hour rating is 75°C, find the maximum steady temperature at this rating (*c*) When working at its one-hour rating, how long does it take the temperature to increase from 60°C to 75°C ? **[(*a*) 55 °C (*b*) 102 °C (*c*) 20 min]**

(Electrical Technology, M.S. Univ. Baroda. 1976)

OBJECTIVE TESTS – 30

1. The speed of a d.c. motor can be controlled by varying
(*a*) its flux per pole
(*b*) resistance of armature circuit
(*c*) applied voltage
(*d*) all of the above

2. The most efficient method of increasing the speed of a 3.75 kW d.c. shunt motor would be the ....method.
(*a*) armature control
(*b*) flux control
(*c*) Ward-Leonard
(*d*) tapped-field control

3. Regarding Ward-Leonard system of speed control which statement is **false ?**
(*a*) It is usually used where wide and very sensitive speed control is required.
(*b*) It is used for motors having ratings from 750 kW to 4000 kW
(*c*) Capital outlay involved in the system is right since it uses two extra machines.
(*d*) It gives a speed range of 10 : 1 but in one direction only.
(*e*) It has low overall efficiency especially at light loads.

4. In the rheostatic method of speed control for a d.c. shunt motor, use of armature divertor makes the method
(*a*) less wasteful
(*b*) less expensive
(*c*) unsuitable for changing loads
(*d*) suitable for rapidly changing loads

5. The chief advantage of Ward-Leonard system of d.c. motor speed control is that it
(*a*) can be used even for small motors
(*b*) has high overall efficiency at all speeds
(*c*) gives smooth, sensitive and wide speed control
(*d*) uses a flywheel to reduce fluctuations in power demand

6. The flux control method using paralleling of field coils when applied to a 4-pole series d.c. motor can give .... speeds.
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 6

7. The series-parallel system of speed control of series motors widely used in traction work gives a speed range of about
(*a*) 1 : 2 (*b*) 1 : 3
(*c*) 1 : 4 (*d*) 1 : 6

8. In practice, regenerative braking is used when
(*a*) quick motor reversal is desired
(*b*) load has overhauling characteristics
(*c*) controlling elevators, rolling mills and printing presses etc.
(*d*) other methods can not be used.

9. **Statement 1.** A direct-on-line (**DOL**) starter is used to start a small d.c. motor because
Statement 2. it limits initial current drawn by the armature circuit.
(*a*) both statement 1 and 2 are incorrect
(*b*) both statement 1 and 2 are correct
(*c*) statement 1 is correct but 2 is wrong
(*d*) statement 2 is correct but 1 is wrong

10. Ward-Leonard system of speed control is **NOT** recommended for
(*a*) wide speed range
(*b*) constant-speed operation
(*c*) frequent motor reversals
(*d*) very low speeds

11. Thyristor chopper circuits are employed for
(*a*) lowering the level of a d.c. voltage
(*b*) rectifying the a.c. voltage
(*c*) frequency conversion
(*d*) providing commutation circuitry

12. An invertor circuit is employed to convert
    (*a*) a.c. voltage into d.c. voltage
    (*b*) d.c.voltage into a.c. voltage
    (*c*) high frequency into low frequency
    (*d*) low frequency into high frequency
13. The phase-control rectifiers used for speed of d.c. motors convert fixed a.c. supply voltage into
    (*a*) variable d.c. supply voltage
    (*b*) variable a.c. supply voltage
    (*c*) full-rectified a.c. voltage
    (*d*) half-rectified a.c. voltage
14. If some of the switching devices in a convertor are controlled devices and some are diodes, the convertor is called
    (*a*) full convertor (*b*) semiconvertor
    (*c*) solid-state chopper
    (*d*) d.c. convertor
15. A solid-state chopper converts a fixed-voltage d.c. supply into a
    (*a*) variable-voltage a.c. supply
    (*b*) variable-voltage d.c. supply
    (*c*) higher-voltage d.c. supply
    (*d*) lower-voltage a.c. supply
16. The d.c. motor terminal voltage supplied by a solid-state chopper for speed control purposes varies...........with the duty ratio of the chopper
    (*a*) inversely (*b*) indirectly
    (*c*) linearly (*d*) parabolically

ANSWERS

1. *d*	**2.** *b*	**3.** *d*	**4.** *d*	**5.** *c*	**6.** *b*	**7.** *c*	**8.** *b*	**9.** *c*	**10.** *b*	**11.** *a*
12. *b*	**13.** *a*	**14.** *b*	**15.** *b*	**16.** *c*						

GO To FIRST