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## C H A P T E R

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## TESTING <br> OF D.C. <br> MACHINES



Testing is performed on D.C. machines to determine efficiency and power losses

### 31.1 Brake Test

It is a direct method and consists of applying a brake to a water-cooled pulley mounted on the motor shaft as shown in Fig. 31.1. The brake band is fixed with the help of wooden blocks gripping the pulley. One end of the band is fixed to earth via a spring balance $S$ and the other is connected to a suspended weight $W_{1}$. The motor is running and the load on the motor is adjusted till it carries its full load current.

Let $\quad W_{1}=$ suspended weight in kg

$$
W_{2}=\text { reading on spring balance in } \mathrm{kg} \text {-wt }
$$

The net pull on the band due to friction at the pulley is $\left(W_{1}-W_{2}\right) \mathrm{kg}$. wt. or $9.81\left(W_{1}-W_{2}\right)$ newton.

If $\quad R=$ radius of the pulley in metre
and $\quad N=$ motor or pulley speed in r.p.s.


Then, shaft torque $T_{s h}$ developed by the motor

$$
=\left(W_{1}-W_{2}\right) R \mathrm{~kg}-\mathrm{m}=9.81\left(W_{1}-W_{2}\right) R \mathrm{~N}-\mathrm{m}
$$

Motor output power $=T_{s h} \times 2 \pi N$ watt

$$
\begin{aligned}
& =2 \pi \times 9.81 N\left(W_{1}-W_{2}\right) R \text { watt } \\
& =61.68 N\left(W_{1}-W_{2}\right) R \text { watt }
\end{aligned}
$$

Let $V=$ supply voltage ; $I=$ full-load current taken by the motor.
Then, input power $=V I$ watt

$$
\therefore \quad \eta=\frac{\text { Output }}{\text { Input }}=\frac{61.68 N\left(W_{1}-W_{2}\right) R}{V I}
$$

The simple brake test described above can be used for small motors only, because in the case of large motors, it is difficult to dissipate the large amount of heat generated at the brake.

Another simple method of measuring motor output is by the use of poney brake one form of which is shown in Fig. 31.2 (a). A rope is wound round the pulley and its two ends are attached to two spring balances $S_{1}$ and $S_{2}$. The tension of the rope can be adjusted with the help of swivels.


Fig. 31.1 Obviously, the force acting tangentially on the pulley is equal to the difference between the readings of the two spring balances. If $R$ is the pulley radius, the torque at the pulley is $T_{\text {sh }}$ $=\left(S_{1}-S_{2}\right) R$. If $\omega(=2 \pi N)$ is the angular velocity of the pulley, then
motor output $=T_{s h} \times \omega=2 \pi N\left(S_{1}-S_{2}\right) R \mathrm{~m}-\mathrm{kg} . \mathrm{wt} .=9.81 \times 2 \pi N\left(S_{1}-S_{2}\right) R$ watt.
The motor input may be measured as shown in Fig. 31.2 (b). Efficiency may, as usual, be found by using the relation $\eta=$ output/input.

Example 31.1. In a brake test the effective load on the branch pulley was 38.1 kg , the effective diameter of the pulley 63.5 cm and speed 12 r.p.s. The motor took 49 A at 220 V . Calculate the output power and the efficiency at this load.

Solution. Effective load $\left(W_{1}-W_{2}\right)=38.1 \mathrm{~kg}$. wt ; radius $=0.635 / 2=0.3175 \mathrm{~m}$
Shaft torque $=38.1 \times 0.3175 \mathrm{~kg}-\mathrm{m}=9.81 \times 38.1 \times 0.3175=118.6 \mathrm{~N}-\mathrm{m}$
Power output $=$ torque $\times$ angular velocity in rad $/ \mathrm{s}=118.6 \times 2 \pi \times 12=8,945 \mathrm{~W}$
Now, motor input $=49 \times 220 \mathrm{~W} \quad \therefore \quad$ Motor $\eta=\frac{8,945}{49 \times 220}=\mathbf{0 . 8 3}$ or $\mathbf{8 3} \%$


Fig. 31.2 (a)


Fig. 31.2 (b)

Example 31.2(a). The following readings are obtained when doing a load test on a d.c. shunt motor using a brake drum :

Spring balance reading
Speed of the motor
Line current

10 kg and 35 kg
950 r.p.m.
30 A

Diameter of the drum 40 cm
Applied voltage 200 V

Calculate the output power and the efficiency. (Electrical Engineering, Madras Univ. 1986)
Solution. Force on the drum surface $F=(35-10)=25 \mathrm{~kg} \mathrm{wt}=25 \times 9.8 \mathrm{~N}$
Drum radius $\quad R=20 \mathrm{~cm}=0.2 \mathrm{~m}$; Torque $T_{s h}=F \times R=25 \times 9.8 \times 0.2=49 \mathrm{~N}$

$$
N=950 / 60=95 / 6 \text { r.p.s. } ; \omega=2 \pi(95 / 6)=99.5 \mathrm{rad} / \mathrm{s}
$$

Motor output $=T_{\text {sh }} \times \omega$ watt $=49 \times 99.5=4,876 \mathrm{~W}$
Motor input $=200 \times 30=6000 \mathrm{~W} ; \eta=4876 / 6000=0.813$ or $81.3 \%$
Example 31.2(b). In a brake-test, on a d.c. shunt motor, the tensions on the two sides of the brake were 2.9 kg and 0.17 kg . Radius of the pulley was 7 cm . Input current was 2 amp at 230 volts. The motor speed was 1500 rpm. Find the torque, power-output and efficiency.
(Bharathiar Univ. April 1998)


Fig. 31.3. D.C. Shunt Motor Brake Test
Solution. Net force on pulley $=2.90-0.17=2.73 \mathrm{~kg}$

$$
=2.73 \times 9.81=26.78 \mathrm{Nw}
$$

$$
\begin{aligned}
\text { Net torque }=\text { Force } \times \text { Radius } & =26.78 \times 7 / 100 \\
& =1.8746 \mathrm{Nw}-\mathrm{m} \\
\text { Power output } & =\text { Torque } \times \text { Radians } / \mathrm{sec} . \\
& =1.8746 \times 2 \pi 1500 / 60 \\
& =294 \text { watts }
\end{aligned}
$$

$$
\text { Efficiency }=294 /(230 \times 2)=0.639
$$

$$
\% \text { efficiency }=63.9 \%
$$

### 31.2. Swinbume's* Test (or No-load Test or Losses Method)

It is a simple method in which losses are measured separately and from their knowledge, efficiency at any desired load can be predetermined in advance. The only running test needed is no-load test. However, this test is applicable to those machines in which flux is practically constant i.e. shunt and compound-wound machines.

The machine is running as a motor on no-load at its rated voltage i.e. voltage stamped on the nameplate. The speed is adjusted to the rated speed with the help of shunt regulator as shown in Fig. 31.4.

The no-load current $I_{0}$ is measured by the
 ammeter $A_{1}$ whereas shunt field current $I_{s h}$ is given by ammeter $A_{2}$. The no-load armature current is $\left(I_{0}-I_{s h}\right)$ or $I_{a 0}$.

Let, supply voltage $=V$ no-load input $=V I_{0}$ watt
$\therefore$ Power input to armature $=V\left(I_{0}-I_{s h}\right) ;$ Power input to shunt $=V I_{s h}$
No-load power input to armature supplies the following :
(i) Iron losses in core
(ii) friction loss
(iii) windage loss and
(iv) armature Cu loss, $\left(I_{0}-I_{\text {sh }}\right)^{2} R_{a}$ or $I_{a 0}^{2} R_{a}$

In calculating armature Cu loss, 'hot' resistance of armature should be used. A stationary measurement of armature circuit resistance at the room-temperature of, say, $15^{\circ} \mathrm{C}$ is made by passing current through the armature from a low voltage d.c. supply [Fig. 31.5 (a)].


Fig. 31.5

[^0]Then, the 'hot' resistance, allowing a temperature rise of $50^{\circ} \mathrm{C}$ is found thus :
$R_{15}=R_{0}\left(1+15 \alpha_{0}\right) ; R_{65}=\left(1+65 \alpha_{0}\right), R_{65}=R_{15} \times \frac{1+65 \alpha_{0}}{1+15 \alpha_{0}}$
Taking $\alpha_{0}=1 / 234.5$, we have $\quad R_{65}=R_{15} \times \frac{234.5+65}{234.5+15}=1.2 R_{15}$ (approx.*)
If we subtract from the total input the no-load armature Cu loss, then we get constant losses.
$\therefore$ Constant losses

$$
W_{c}=V I_{0}-\left(I_{0}-I_{s h}\right)^{2} R_{a}
$$

Knowing the constant losses of the machine, its efficiency at any other load can be determined as given below. Let $I=$ load current at which efficiency is required.

$$
\text { Then, armature current is } \quad \begin{aligned}
I_{a} & =I-I_{s h} \\
& =I+I_{s h}
\end{aligned} \quad \text {...if machine is motoring }
$$

## Efficiency when running as a motor

$$
\begin{aligned}
& \text { Input }=V I, \quad \text { Armature Cu loss }=I_{a}^{2} R_{a}=\left(I-I_{s h}\right)^{2} R_{a} \\
& \text { Constant losses }=W_{c} \\
\therefore \quad & \text { Total losses }=\left(I-I_{s h}\right)^{2} R_{a}+W_{c} ; \eta_{m}=\frac{\text { input }- \text { losses }}{\text { input }}=\frac{V I-\left(I-I_{s h}\right)^{2} R_{a}-W_{c}}{V I}
\end{aligned}
$$

## Efficiency when running as a generator

Output $=V I$; Armature, Cu loss $=\left(I+I_{s h}\right)^{2} R_{a} ;$ Constant loss $=W_{c} \quad$...found above
$\therefore \quad$ Total losses $=\left(I+I_{s h}\right)^{2} R+W_{c} ; \eta_{g}=\frac{\text { output }}{\text { output }+ \text { losses }}=\frac{V I}{V I+\left(I+I_{s h}\right)^{2} R_{a}+W_{c}}$

### 31.3. Advantages of Swinbume's Test

1. It is convenient and economical because power required to test a large machine is small i.e. only no-load input power.
2. The efficiency can be predetermined at any load because constant-losses are known.

### 31.4. Main Disadvantages

1. No account is taken of the change in iron losses from no-load to full-load. At full-load, due to armature reaction, flux is distorted which increases the iron losses in some cases by as much as $50 \%$.
2. As the test is on no-load, it is impossible to know whether commutation would be satisfactory at full-load and whether the temperature rise would be within the specified limits.

Example 31.3. A 220 V, d.c. shunt motor at no load takes a current of 2.5 A. The resistances of the armature and shunt field are $0.8 \Omega$ and $200 \Omega$ respectively. Estimate the efficiency of the motor when the input current is 20 A. State precisely the assumptions made.
(Electrical Technology, Kerala Univ. 1986)
Solution. No-load input $=220 \times 2.5=550 \mathrm{~W}$
This input meets all kinds of no-load losses i.e. armature Cu loss and constant losses.
$I_{s h}=220 / 200=1.1 \mathrm{~A}$. No-load arm current, $I_{a 0}=2.5-1.1=1.4 \mathrm{~A}$

* The armature resistance is found to decrease slightly with increasing armature current as shown in Fig. 31.5
(b). This is due to the fact that brush contact resistance is inversely proportional to the armature current.

No-load armature Cu loss $=I_{a 0}^{2} R_{a}=1.4^{2} \times 0.8=1.6 \mathrm{~W}$
Constant losses $=550-1.6=548.4 \mathrm{~W}$
When input current is 20 A
$I_{a}=32-1.1=30.9 \mathrm{~A}$; Armature Cu loss $=30.9^{2} \times 0.8=764 \mathrm{~W}$
Total loss $=764+548.4=1312 \mathrm{~W}$ (approx.) ; Input $=220 \times 20=4,400 \mathrm{~W}$
Output $=4,400-1,312=3,088 \mathrm{~W} ;$ Efficiency $=(3088 / 4400) \times 100=70.2 \%$
In the above calculations, it has been assumed that :

1. mechanical losses remain constant even through motor speed changes from no-load to the given load.
2. effect of armature reaction on main pole flux with a consequent change in iron losses has been neglected.
3. decrease in flux due to increase in shunt resistance by heating has been neglected.

Example 31.4. When running on no-load, a 400-V shunt motor takes 5 A. Armature resistance is $0.5 \Omega$ and field resistance $200 \Omega$. Find the output of the motor and efficiency when running on fullload and taking a current of 50 A. Also, find the percentage change in speed from no-load to fullload.
(Electro Mechanics, Allahabad Univ. 1991)
Solution. No-load input $=400 \times 5=2,000 \mathrm{~W}$
This input goes to meet all kinds of no-load losses i.e. armature Cu loss and constant losses.

$$
I_{s h}=400 / 200=2 \mathrm{~A} ; \text { No-load } I_{a}=5-2=3 \mathrm{~A}
$$

No-load arm. Cu loss $=3^{2} \times 0.5=4.5 \mathrm{~W}$; Constant losses $=2,000-4.5=1,995.5 \mathrm{~W}$
When line current is 50 A

$$
I_{a}=50-2=48 \mathrm{~A} ; \text { Arm. Cu loss }=48^{2} \times 0.5=1,152 \mathrm{~W}
$$

Total loss on F.L. $=1,152+1,995.5=3,147.5 \mathrm{~W}$; Input $=50 \times 400=20,000 \mathrm{~W}$
Output $=20,000-3,147.5=\mathbf{1 6 , 8 5 2 . 5} \mathbf{W}=\mathbf{1 6 . 8} \mathbf{k W}$
F.L. efficiency $=16,852.5 / 20,000=0.8426$ or $\mathbf{8 4 . 2 6 \%}$

Now,

$$
\begin{aligned}
& E_{b 1}=400-(3 \times 0.5)=398.5 \mathrm{~V} ; E_{b 2}=400-(48 \times 0.5)=376 \mathrm{~V} \\
& \frac{N_{1}}{N_{2}}=\frac{E_{b 1}}{E_{b 2}}=\frac{398.5}{376} \quad \therefore \quad \frac{N_{1}-N_{2}}{N_{2}}=\frac{22.5}{376}=0.0598
\end{aligned}
$$

$\therefore \quad$ percentage change in speed $=5.98$
Example 31.5. The no-load test of a $44.76 \mathrm{~kW}, 220-\mathrm{V}$, d.c. shunt motor gave the following figures:

Input current $=13.25$ A ; field current $=2.55 \mathrm{~A}$; resistance of armature at $75^{\circ} \mathrm{C}=0.032 \Omega$ and brush drop $=2 \mathrm{~V}$. Estimate the full-load current and efficiency.
(Electrical Engineering, Madras Univ. 1987)

## Solution. No-load Condition

No-load input $=220 \times 13.25=2915 \mathrm{~W}$; Armature current $=13.25-2.55=10.7 \mathrm{~A}$
Armature Cu loss $=10.7^{2} \times 0.032=3.6 \mathrm{~W}$
Loss due to brush drop $=2 \times 10.7=21.4 \mathrm{~W}$
Variable loss $=21.4+3.6=25 W_{e}$, Constant losses $W_{c}=2915-25=2890 \mathrm{~W}$
Full-load Condition
If $I_{a}$ is the full-load armature current, then full-load motor input current is $\left(I_{a}+2.55\right) \mathrm{A}$.
F.L. motor power input $=220\left(I_{a}+2.55\right) \mathrm{W}$

This input must be equal to the sum of
(i) output $=44.76 \mathrm{~kW}=44,760 \mathrm{~W}$
(ii) $W_{c}=2,890 \mathrm{~W}$
(iii) brush loss $=2 I_{a}$ watt
(iv) Arm. Cu loss $=0.032 I_{a}{ }^{2}$

```
\(\therefore \quad 220\left(I_{a}+2.55\right)=44,750+2,890+2 I_{a}+0.032 I_{a}^{2}\)
```

or $0.032 I_{a}^{2}-218 I_{a}+47,090=0$
or

$$
I_{a}=\frac{218 \pm \sqrt{218^{2}-4 \times 0.032 \times 47,090}}{2 \times 0.032}=223.5 \mathrm{~A}
$$

Line input current $I=I_{a}+I_{s h}=223.5+2.55=226 \mathbf{A}$
F.L. power input $=226 \times 220=49,720 \mathrm{~W}$
$\therefore \quad$ F.L.efficiency $=44,760 / 49,720=0.9$ or $90 \%$.
Example 31.6. A 200-V, shunt motor develops an output of 17.158 kW when taking 20.2 kW . The field resistance is $50 \Omega$ and armature resistance $0.06 \Omega$. What is the efficiency and power input when the output is 7.46 kW ?
(Elect. Machines-I, Aligarh Muslim Univ. 1989)
Solution. In the first case :

$$
\begin{aligned}
\text { Output } & =17,158 \mathrm{~W} \quad \text { Input }=20,200 \mathrm{~W} \\
\text { Total losses } & =20,200-17,158=3,042 \mathrm{~W} ; \text { Input current }=20,200 / 200=101 \mathrm{~A} \\
I_{\text {sh }} & =200 / 50=4 \mathrm{~A} ; I_{a}=101-4=97 \mathrm{~A}
\end{aligned}
$$

$\therefore \quad$ Armature Cu loss $=97^{2} \times 0.06=564.5 \mathrm{~W}$
$\therefore \quad$ Constant losses $=3,042-564.5=2,477.5=2478 \mathrm{~W}$ (approx.)
In the second case :

$$
\begin{array}{lrl}
\text { Let, } & I_{a} & =\text { armature current } \quad \text { Input current }=\left(I_{a}+4\right) \mathrm{A} \\
\text { Now, } & \text { input power } & =\text { output }+I_{a}^{2} R_{a}+\text { constant losses } \\
\therefore & 200\left(I_{a}+4\right) & =7,460+0.06 I_{a}^{2}+2,478 \\
\text { or } & 0.06 I_{a}^{2}-200 I_{a}+9,138=0 \\
\therefore & & I_{a}=\frac{200 \pm \sqrt{200^{2}-4 \times 0.06 \times 9,138}}{2 \times 0.06}=\frac{200 \pm 194}{0.12}=3,283.3 \mathrm{~A} \text { or } 46 \mathrm{~A}
\end{array}
$$

We will reject the larger value because it corresponds to unstable operation of the motor. Hence, take $I_{a}=46 \mathrm{~A}$.
$\therefore$ Input current $I=I_{a}+I_{s h}=46+4=50 \mathrm{~A}$

$$
\text { Power input }=\frac{50 \times 200}{1000}=10 \mathrm{~kW} \quad \therefore \quad \eta=\frac{7,460 \times 100}{10,000}=74.6 \%
$$

Example 31.7. A 200-V, 14.92 kW dc shunt motor when tested by the Swinburne method gave the following results :

Running light : armature current was 6.5 A and field current 2.2 A . With the armature locked, the current was 70 A when a potential difference of 3 V was applied to the brushes. Estimate the efficiency of the motor when working under full-load conditions.
(Electrical Engg.-I, Bombay Univ. 1985)
Solution. No-load input current $=6.5+2.2=8.7 \mathrm{~A}$
No-load power input $=200 \times 8.7=1,740 \mathrm{~W}$
No-load input equals Cu losses and stray losses.

$$
\begin{aligned}
\text { Field Cu loss } & =200 \times 2.2=440 \mathrm{~W} & \\
\text { Armature Cu loss } & =6.5^{2} \times 0.04286=1.8 \mathrm{~W} & \left(\because R_{a}=3 / 70=0.04286 \Omega\right) \\
\therefore \quad \text { Constant losses } & =1,740-1.8=1738 \mathrm{~W} &
\end{aligned}
$$

We will assume that constant losses are the same at full-load also.
Let, $\quad I_{a}=$ full-load armature current

```
    F.L. armature Cu loss \(=0.04286 I_{a}^{2} \mathrm{~W}\); Constant losses \(=1,738 \mathrm{~W}\)
            F.L. total loss \(=1,738+0.04286 I_{a}^{2}\)
                F.L. output \(=14,920 \mathrm{~W}\); F.L. input \(=200\left(I_{a}+2.2\right) \mathrm{W}\)
We know,
                    input \(=\) output + losses
or
    \(200 I_{a}+440=14,920+1,738+0.04286 I_{a}^{2}\)
or \(\quad 0.04286 I_{a}^{2}-200 I_{a}+16,218=0 \quad \therefore \quad I_{a}=82.5 \mathrm{~A}\)
\(\therefore \quad\) Inputcurrent \(=82.5+2.2=84.7 \mathrm{~A}\)
        F.L. power input \(=200 \times 84.7 \mathrm{~A}=\mathbf{1 6 , 9 4 0} \mathrm{W}\)
\(\therefore \quad \eta=14,920 \times 100 / 16,940=\mathbf{8 8 \%}\)
```

Example 31.8. In a test on a d.c. shunt generator whose full-load output is 200 kW at 250 V , the following figures were obtained :
(a) When running light as a motor at full speed, the line current was 36 A, the field current 12 A , and the supply voltage 250.
(b) With the machine at rest, a p.d. of $6 V$ produced a current of 400 A through the armature circuit. Explain how these results may be utilised to obtain the efficiency of generator at full-load and half-load. Neglect brush voltage drop.

## Solution. At no-load :

$$
I_{a}=36-12=24 \mathrm{~A} ; R_{a}=6 / 400=0.015 \Omega
$$

$\therefore$ Armature Cu loss $=24^{2} \times 0.015=8.64$ watt
No-load input $=$ total losses in machine $=250 \times 36=9,000 \mathrm{~W}$
Constant losses $=9,000-8.64=8,991.4 \mathrm{~W}$
At full-load :

$$
\text { Output }=200,000 \mathrm{~W} ; \text { Output current }=200,000 / 250=800 \mathrm{~A} ; I_{s h}=12 \mathrm{~A}
$$

$\therefore \quad$ F.L. armature current $=800+12=812 \mathrm{~A}$
$\therefore \quad$ F.L. armature Cu losses $=812^{2} \times 0.015=9890 \mathrm{~W}$
$\therefore \quad$ F.L. total losses $=9890+8,991.4=18881 \mathrm{~W} \quad \therefore \quad \eta=\frac{200,000 \times 100}{200,000+18,881}=91.4 \%$
Athalf-load:

$$
\begin{aligned}
\text { Output } & =100,000 \mathrm{~W} ; \text { Output current }=100,000 / 250=400 \mathrm{~A} \\
I_{a} & =400+12=412 \mathrm{~A} \quad \therefore \quad I_{a}^{2} R_{a}=412^{2} \times 0.015=2,546 \mathrm{~W} \\
\text { Total losses } & =8,991.4+2,546=11,537 \mathrm{~W} \quad \therefore \quad \eta=\frac{100,000 \times 100}{111,537}=89.6 \%
\end{aligned}
$$

Example 31.9. A 250-V, 14.92 kW shunt motor has a maximum efficiency of $88 \%$ and a speed of 700 r.p.m. when delivering $80 \%$ of its rated output. The resistance of its shunt field is $100 \Omega$. Determine the efficiency and speed when the motor draws a current of 78 A from the mains.

Solution. Full-load output $=14,920 \mathrm{~W}$

$$
\begin{aligned}
80 \% \text { of F.L. output } & =0.8 \times 14.920=11,936 \mathrm{~W} ; \eta=0.88 \\
\text { Input } & =11,936 / 0.88=13,564 \mathrm{~W} \\
\text { Total losses } & =13,564-11,936=1,628 \mathrm{~W}
\end{aligned}
$$

As efficiency is maximum at this load, the variable loss is equal to constant losses.
$\therefore$

$$
W_{c}=I_{a}^{2} R_{a}=1,628 / 2 \quad \therefore \quad I_{a}^{2} R_{a}=814 \mathrm{~W}
$$

Now, $\quad$ input current $=13,564 / 250=54.25 \mathrm{~A}$
$\therefore \quad I_{\text {sh }}=250 / 100=2.5 \mathrm{~A} \quad \therefore \quad I_{a}=54.25-2.5=51.75 \mathrm{~A}$

$$
\therefore \quad 51.75^{2} R_{a}=814 \quad \therefore \quad R_{a}=814 / 51.75^{2}=0.3045 \Omega
$$

When input current is 78 A

$$
I_{a}=78-2.5=75.5 \mathrm{~A} \quad \therefore \quad I_{a}^{2} R_{a}=75.5^{2} \times 0.3045=1,736 \mathrm{~W}
$$

$$
\text { Total losses }=1,736+814=2,550 \mathrm{~W} ; \text { Input }=250 \times 78=19,500 \mathrm{~W}
$$

$$
\eta=\frac{19,500-2,550}{19,550} \times 100=86.9 \%
$$

Speed :

$$
\frac{N_{2}}{N_{1}}=\frac{E_{b 2}}{E_{b 1}} \quad \text { or } \quad \frac{N_{2}}{700}=\frac{250-(75.5 \times 0.3045)}{250-(51.75 \times 0.3045)}=\frac{227}{234.25} ; N_{2}=680 \text { r.p.m. }
$$

### 31.5. Regenerative or Hopkinson's Test (Back-to-Back Test)

By this method, full-load test can be carried out on two shunt machines, preferably identical ones, without wasting their outputs. The two machines are mechanically coupled and are so adjusted electrically that one of them runs as a motor and the other as a generator. The mechanical output of the motor drives the generator and the electrical output of generator is used in supplying the greater part of input to the motor. If there were no losses in the machines, they would have run without any external power supply. But due to these losses, generator output is not sufficient to drive the motor and vice-versa. The losses are supplied either by an extra motor which is belt-connected to the motor-generator set or as suggested by Kapp, electrically from the supply mains.

Essential connections for the test are shown in Fig. 31.6. The two shunt machines are connected in parallel. They are, to begin with, started as unloaded motors. Then, the field of one is weakened and that of the other is strengthened so that the former runs as a motor and the latter as a generator. The usual method of procedure is as follows :

Machine $M$ is started up from the supply mains with the help of a starter (not shown) whereas main switch $S$ of the other machine is kept open. Its


Fig. 31.6 speed is adjusted to normal value by means of its shield regulator. Machine $M$ drives machine $G$ as a
 generator and its voltage is read on voltmeter $V_{1}$. The voltage of $G$ is adjusted by its field regulator until voltmeter $V_{1}$ reads zero, thereby showing that its voltage is the same, both in polarity and magnitude as that of the main supply. Thereafter, $S$ is closed to parallel the machines. By adjusting the respective field regulators, any load can now be thrown on to the machines. Generator current $I_{1}$ can be adjusted to any desired value by increasing the excitation of $G$ or by reducing the excitation of $M$ and the corresponding values of different ammeters are read.

The electrical output of the generator plus the small power taken from the supply, is taken by the motor and is given out as a mechanical power after supplying the motor losses.

If supply voltage is $V$, then
Motor input $=V\left(I_{1}+I_{2}\right)$, where $I_{2}$ is the current taken from the supply.
Generator output $=V I_{1}$
Assuming that both machines have the same efficiency $\boldsymbol{\eta}$,
Output of motor $=\eta \times$ input $=\eta V\left(I_{1}+I_{2}\right)=$ generator input
Output of generator $=\eta \times$ input $=\eta \times \eta V\left(I_{1}+I_{2}\right)=\eta^{2} V\left(I_{1}+I_{2}\right)$
Hence, from (i) and (ii), we get

$$
\eta^{2} V\left(I_{1}+I_{2}\right)=V I_{1} \quad \text { or } \quad \eta=\sqrt{\frac{I_{1}}{I_{1}+I_{2}}}
$$

However, it is not quite correct to assume equal efficiencies for two machines because their armature currents as well as excitations are different. We will not find the efficiencies separately.

Let

$$
\begin{aligned}
R_{a} & =\text { armature resistance of each machine } \\
I_{3} & =\text { exciting current of the generator } \\
I_{4} & =\text { exciting current of the motor }
\end{aligned}
$$

Armature Cu loss in generator $=\left(I_{1}+I_{3}\right)^{2} R_{a}$; Armature Cu loss in motor $=\left(I_{1}+I_{2}-I_{4}\right)^{2} R_{a}$ Shunt Cu loss in generator $=V I_{3} ;$ Shunt Cu loss in motor $=V I_{4}$
But total motor and generator losses are equal to the power supplied by the mains.
Power drawn from supply $=V I_{2}$
If we subtract the armature and shunt Cu losses from this, we get the stray losses of both machines.
$\therefore$ Total stray losses for the set

$$
=V I_{2}-\left[\left(I_{1}+I_{3}\right)^{2} R_{a}+\left(I_{1}+I_{2}-I_{4}\right)^{2} R_{a}+V I_{3}+V I_{4}\right]=\mathrm{W} \text { (say) }
$$

Making one assumption that stray losses are equally divided between the two machines, we have
Stray loss per machine $=\mathrm{W} / 2$

## For Generator

Total losses $\quad=\left(I_{1}+I_{3}\right)^{2} R_{a}+V I_{3}+\mathrm{W} / 2=W_{g}$ (say)
Output

$$
=V I_{1} \quad \therefore \quad \eta_{g}=\frac{V I_{1}}{V I_{1}+W_{g}}
$$

Total losses

$$
=\left(I_{1}+I_{2}-I_{4}\right)^{2} R_{a}+V I_{d}+\mathrm{W} / 2=W_{m}(\text { say })
$$

Input

$$
=V\left(I_{1}+I_{2}\right) \quad \therefore \quad \eta_{m}=\frac{V\left(I_{1}+I_{2}\right)-W_{m}}{V\left(I_{1}+I_{2}\right)}
$$

### 31.6. Altemative Connections for Hopkinson's Test

In Fig. 31.7 is shown in slightly different method of connecting the two machines to the supply. Here, the main difference is that the shunt windings are directly connected across the lines. Hence, the line input current is $I_{1}$ excluding the field currents. The efficiencies can be calculated as detailed below :

Motor armature Cu loss $=\left(I_{1}+I_{2}\right)^{2} R_{a}$; Generator armature Cu loss $=I_{2}^{2} R_{a}$
Power drawn from the supply $=V I_{1}$
$\therefore$ Total stray losses i.e. iron, friction and windage losses for the two machines are

$$
=V I_{1}-\left[\left(I_{1}+I_{2}\right)^{2} R_{a}-I_{2}^{2} R_{a}\right]=\mathrm{W}(\text { say })
$$

$\therefore \quad$ stray loss for each machine $=\mathrm{W} / 2$

## Motor Efficiency

Motor input $=$ armature input + shunt field input $=V\left(I_{1}+I_{2}\right)+V I_{3}+W_{\text {input }}$
Motor losses $=$ armature Cu loss + shunt Cu loss + stray losses

$$
=\left(I_{1}+I_{2}\right)^{2} R_{a}+V I_{3}+W / 2=W_{m}(\text { say })
$$

Motor

$$
\eta=\frac{W_{\text {input }}-W_{m}}{W_{\text {input }}} \times 100
$$

## Generator Efficiency

Generator output $=V I_{2} ;$ Generator losses $=I_{2}^{2} R_{a}+V I_{4}+W / 2=W_{g}$ (say)
$\therefore$ Generator $\eta=\frac{V I_{2}}{V I_{2}+W_{g}}$

### 31.7. Merits of Hopkinson's Test

1. Power required for the test is small as compared to the full-load powers of the two machines.
2. As machines are being tested under full-load conditions, the temperature rise and the commutation qualities of the machines can be observed.


Fig. 31.7
3. Because of full-load conditions, any change in iron loss due to flux distortion at full-load, is being taken into account.

The only disadvantage is with regard to the availability of two identical machines.
Example 31.10 (a). In a Hopkinson's test on two $220-\mathrm{V}, 100-\mathrm{kW}$ generators, the circulating current is equal to the full-load current and, in addition, 90 A are taken from the supply. Obtain the efficiency of each machine.

Solution. Output current of the generator

$$
I_{1}=\frac{100,000}{220}=\frac{5,000}{11}=454.4 \mathrm{~A}, I_{2}=90 \mathrm{~A}
$$

Assuming equal efficiencies, from Art. 29.5, we have

$$
\eta=\sqrt{\frac{I_{1}}{I_{1}+I_{2}}}=\sqrt{\frac{454.5}{454.5+90}}=0.914 \text { or } 91.4 \%
$$

Example 31.10 (b). In the Hopkinson's test on two d.c. machines, machine $A$ has a field current of 1.4 A and machine B has a field current of 1.3 A . Which machine acts as a generator ?
(Bharathithasan University April 1997)

Solution. In Hopkinson's test (on two identical d.c. shunt machines), since the two machines are coupled, the speed is common and is decided by the field current of the motor. The field windings of both the machines are in parallel with a separate D.C. source.

Since the machines are identical and are running at the same speed, their e.m.fs are in proportion to their field currents.

$$
\frac{\text { E.M.F. induced in the armature of machine } \mathrm{A}}{\text { E.M.F. induced in the armature of machine } \mathrm{B}}=\frac{1.4}{1.3}
$$

$$
E_{A}=(1.4 / 1.3) \times E_{B}=1.077 E_{B}
$$

Since $E_{A}$ is larger than $E_{B}$, Machine A supplies power to Machine B . It means, A is working as a generator, and B is motoring.

Example 31.11. Two shunt machines loaded for the Hopkinson's test take 15 A at 200 V from the supply. The motor current is 100 A and the shunt currents are 3 A and 2.5 A . If the armature resistance of each machine is 0.05 ohm , calculate the efficiency of each machine for this particular load-condition.
(Bharathithasan Univ. April 1997)
Solution. Line current into armature circuits $=15 \mathrm{~A}$, Motor armature copper-loss $=500 \mathrm{~W}$
Motor-armature-current $=100 \mathrm{~A}$, Generator armature copper loss $=361 \mathrm{~W}$
Hence generator-armature-current $=85 \mathrm{~A}$
For each machine, No load Mechanical losses + Core-loss + Stray losses

$$
\begin{aligned}
& =1 / 2\left(V I_{a}-I_{a m}^{2} r_{a m}-I_{a g}^{2} r_{a g}\right) \\
& =1 / 2\left(200 \times 15-100^{2} \times 0.05-85^{2} \times 0.05\right) \\
& =1 / 2(3000-500-361)=1069.5 \mathrm{~W}
\end{aligned}
$$

Motor field copper-loss $=200 \times 3=600 \mathrm{~W} \cong 1.07 \mathrm{~kW}$
Generator field copper-loss $=200 \times 2.5=500 \mathrm{~W}$

$$
\text { Total Losses in motor }=600+1069.5+500=2169.5 \mathrm{~W}
$$

Total Losses in Generator $=500+1069.5+361=1931 \mathrm{~W}$
Efficiency of motor $=\frac{\text { Motor output }}{\text { Motor input }} \times 100 \%$
Motor Input: (a) $200 \times 100=20 \mathrm{~kW}$ to armature
(b) 0.6 kW to field winding

Total Input to motor $=20.6 \mathrm{~kW}$
From armature side, losses to be catered are :
(i) Stray losses + No Load Mech. Losses + Core Losses $=1.07 \mathrm{~kW}$
(ii) Armature copper-loss $=0.5 \mathrm{~kW}$

Motor Output from armature $=20-0.5-1.07=18.43 \mathrm{~kW}$
Motor efficiency $=\frac{18.43}{20.6} \times 100 \%=89.47 \%$
Generator armature output $=200 \times 85 \times 10^{-3}=17 \mathrm{~kW}$
Generator losses: (a) Field wdg : 0.5 kW
(b) Total no-load-losses : 1.07 kW
(c) armature copper-loss $=0.36 \mathrm{~kW}$

Total losses in Generator $=1.93 \mathrm{~kW}$
Generator efficiency $=\frac{17}{17+1.93} \times 100 \%=\mathbf{8 9 . 8 0 \%}$
Special Note: 15 A current for d.c. supply is related here to armature-input for two machines which are under back-to-back regenerative tests. There are different variations in handling and giving the test data. It is always desirable to draw the circuit diagram according to which the calculations are being related.

Example 31.12. The Hopkinson's test on two similar shunt machines gave the following fullload data :

| Line voltage | $=110 \mathrm{~V}$ | Field currents are 3 A and 3.5 A |
| :--- | :--- | :--- |
| Line current | $=48 \mathrm{~A}$ | Arm. resistance of each is $0.035 \Omega$ |
| Motor arm. current | $=230 \mathrm{~A}$ |  |

Motor arm. current $=230 \mathrm{~A}$
Calculate the efficiency of each machine assuming a brush contact drop of 1 volt per brush.
(Electrical Machines, Nagpur Univ. 1992)
Solution. The motor-generator set is shown in Fig. 31.8. It should also be noted that the machine with lesser excitation is motoring. We will find the total armature Cu losses and brush contact loss for both machines.
Motor

$$
\begin{aligned}
\text { Arm. Cu loss } & =230^{2} \times 0.035=1,851.5 \mathrm{~W} \\
\text { Brush contact loss } & =230 \times 2=460 \mathrm{~W} \\
\text { Total arm. Cu loss } & =1851.5+460 \quad=2,312 \mathrm{~W} \\
\text { Shunt } \mathrm{Cu} \text { loss } & =110 \times 3=330 \mathrm{~W} \\
\text { Total } \mathrm{Cu} \text { loss } & =2,312+330=2,642 \mathrm{~W}
\end{aligned}
$$

## Generator

Generator arm. current $=233-48+3.5$

$$
=188.5 \mathrm{~W}
$$

Arm. Cu loss $=188.5^{2} \times 0.035=1,244 \mathrm{~W}$
Brush contact Cu loss $=188.5 \times 2=377 \mathrm{~W}$
Total arm. Cu loss $=1,244+377=1,621 \mathrm{~W}$
Shunt Cu loss $=110 \times 3.5=385 \mathrm{~W}$;
Total Cu loss $=1,621+385=2,006 \mathrm{~W}$

## For the Set

Total arm. and shunt Cu loss for the set


$$
=2,642+2,006=4,648 \mathrm{~W}
$$

Total input $=110 \times 48=5,280 \mathrm{~W}$; Stray losses for the set $=5,280-4,648=632 \mathrm{~W}$
Stray losses per machine $=632 / 2=316 \mathrm{~W}$

## Motor Efficiency

$$
\begin{array}{rlr}
\text { Arm. } \mathrm{Cu}+\text { brush drop loss }=2,312 \mathrm{~W} & \text { Shunt Cu loss }=330 \mathrm{~W} \\
\text { Stray losses } & =316 \mathrm{~W} & \text { Total loss }=2,312+330+316=2,958 \mathrm{~W} \\
\text { Motor input } & =110 \times 233=25,630 \mathrm{~W} ; \text { Motor output }=25,630-2,958=22,672 \\
\therefore \quad \eta & \quad \eta & =22,672 \times 100 / 25,630=\mathbf{8 8 . 8 \%}
\end{array}
$$

## Generator Efficiency

Total losses $=2,006+316=2,322 \mathrm{~W}$; Output $=110 \times 185=20,350 \mathrm{~W}$
Generator input $=20,350+2,322=22,672 \mathrm{~W}=$ motor input

$$
\eta=20,350 / 22,672=0.894 \text { or } \mathbf{8 9 . 4 \%}
$$

Example 31.13. In a Hopkinson's test on a pair of 500-V, 100-kW shunt generators, the following data was obtained :

Auxiliary supply, 30 A at 500 V : Generator output current, 200 A
Field currents, 3.5 A and 1.8 A
Armature circuit resistances, $0.075 \Omega$ each machine. Voltage drop at brushes, 2 V(each machine). Calculate the efficiency of the machine acting as a generator.
(Elect. Technology-1, Gwalior Univ. 1986)

Solution. Motor arm. current $=200+30=230$ A, as shown in Fig. 31.9.
Motor arm. Culoss $=230^{2} \times 0.075+230 \times 2=4,428 \mathrm{~W}$
Motor field Cu loss $=500 \times 1.8=900 \mathrm{~W}$
Generator arm. Cu loss $=200^{2} \times 0.075+200 \times 2=3,400 \mathrm{~W}$
Geneator field Cu loss $=500 \times 3.5=1,750 \mathrm{~W}$
Total Cu loss for two machines

$$
\begin{aligned}
& =4,428+900+3400+1750 \\
& =10,478 \mathrm{~W}
\end{aligned}
$$

Power taken from auxiliary supply

$$
=500 \times 30=15,000 \mathrm{~W}
$$

Stray losses for the two machines

$$
=15,000-10,478=4,522 \mathrm{~W}
$$

Stray loss per machine $=4,522 / 2=2,261 \mathrm{~W}$


Fig. 31.9

Total losses in generator $=3400+1750+2261=7,411 \mathrm{~W}$

$$
\text { Generator output }=500 \times 200=100,000 \mathrm{~W}
$$

$$
\therefore \quad \eta_{g}=\frac{\text { output }}{\text { output }+ \text { losses }}=\frac{100,000}{107,411} \times 100=93.09 \%
$$

Example 31.14. Explain the Hopkinson's test on a pair of shunt motors.
In such a test on 250-V machines, the line current was 50 A and the motor current 400 A not including the field currents of 6 A and 5 A. The armature resistance of each machine was $0.015 \Omega$. Calculate the efficiency of each machine.
(Adv. Elect. Machines, A.M.I.E. Sec. B, 1991)
Solution. The connections are shown in Fig. 31.10.
Motor armature Cu loss

$$
=400^{2} \times 0.015=2,400 \mathrm{~W}
$$

Generator armature Cu loss

$$
=350^{2} \times 0.015=1,838 \mathrm{~W}
$$

Power drawn from supply

$$
=250 \times 50=12,500 \mathrm{~W}
$$

$\therefore$ Iron, friction and windage losses for the two machines

$$
\begin{aligned}
& =12,500-(2,400+1,838) \\
& =8,262 \mathrm{~W}
\end{aligned}
$$

Iron, friction and windage loss per machine

$$
=8.262 / 2=4,130 \mathrm{~W}^{*} \text { (approx.) }
$$



Fig. 31.10

## Motor Losses and Efficiency

Motor arm. Cu loss $=2,400 \mathrm{~W}$; Motor field Cu loss $=250 \times 5=1,250 \mathrm{~W}$
Iron, friction and windage losses $=4,130 \mathrm{~W}$
Total motor losses $=2,400+1,250+4,130=7,780 \mathrm{~W}$
Motor input $=250 \times 400+250 \times 5=101,250 \mathrm{~W}$
$\therefore \quad$ Motor efficiency $=(101,250-7,780) / 101,250=0.923$ or $92.3 \%$

* $\overline{\text { We could }}$ also get this value as follows:

Total supply input $=250 \times 61=15,250 \mathrm{~W}$; Gen. and motor field Cu loss $=250 \times 6+250 \times 5=2,750 \mathrm{~W}$
Iron, friction and windage losses for both machines

$$
=15,250-(2,400+1,838+2,750)=\mathbf{8 , 2 6 2} \mathbf{W} \quad-\text { as before }
$$

## Generator Losses and Efficiency

Generator arm. Cu loss $=1,838 \mathrm{~W}$; Generator field Cu loss $=250 \times 6=1,500 \mathrm{~W}$
Iron, friction and windage loss $=4,130 \mathrm{~W}$
Total losses $=1,838+1,500+4,130=7.468 \mathrm{~W}$
Generator output $=250 \times 350=87,500 \mathrm{~W}$
$\therefore$ Generator efficiency $=(87,500-7.468) / 87,500=0.915$ or $91.5 \%$
Example 31.15. The Hopkinson's test on two shunt machines gave the following results for fullload :

Line voltage $=250 \mathrm{~V}$; current taken from supply system excluding field currents $=50 \mathrm{~A}$; motor armature current $=380 \mathrm{~A}$; field currents 5 A and 4.2 A . Calculate the efficiency of the machine working as a generator. Armature resistance of each machine is $0.2 \Omega$.
(Electrical Machinery-I Mysore Univ. 1988)
Solution. The connections are shown in Fig. 31.11.
Motor arm. Cu loss $=380^{2} \times 0.02=2,888 \mathrm{~W}$
Generator arm. Cu loss $=330^{2} \times 0.02=2,178 \mathrm{~W}$
Power drawn from supply $=250 \times 50=12,500 \mathrm{~W}$
Stray losses for the two machines

$$
=12,500-(2,888+2,178)=7.434 \mathrm{~W}
$$

Stray losses per machine $=7,434 / 2=3,717 \mathrm{~W}$
Motor Efficiency
Arm. Culoss $=2,888 \mathrm{~W}$
Field Culoss $=250 \times 4.2=1050 \mathrm{~W}$
Stray losses $=3,717 \mathrm{~W}$
Total loss $=2,888+1050+3,717$
$=7,655 \mathrm{~W}$
Motor input $=250 \times 380+250 \times 4.2$

$$
=96,050 \mathrm{~W}
$$

Motor output $=96,050-7,655$


Fig. 31.11

$$
\therefore \quad \eta=88,395 / 96,050=0.9203 \text { or } 92.03 \%
$$

Generator Efficiency

$$
\begin{aligned}
\text { Arm. Cu loss } & =2,178 \mathrm{~W} \text {; Field Cu loss }=250 \times 5=1250 \mathrm{~W} \\
\text { Stray losses } & =3,717 \mathrm{~W} ; \text { Total losses }=7,145 \mathrm{~W} \\
\text { Generator output } & =250 \times 330=82,500 \mathrm{~W} \\
\text { Generator input } & =82,500+7,145=89,645 \mathrm{~W} \\
\eta & =82,500 / 89,645=0.9202 \text { or } \mathbf{9 2 . 0 2 \%}
\end{aligned}
$$

### 31.8. Retardation or Running Down Test

This method is applicable to shunt motors and generators and is used for finding stray losses. Then, knowing the armature and shunt Cu losses at a given load current, efficiency can be calculated.

The machine under test is speeded up slightly beyond its normal speed and then supply is cut off from the armature while keeping the field excited. Consequently, the armature slows down and its kinetic energy is used to meet the rotational losses i.e. friction, windage and iron losses.*

Kinetic energy of the armature is K.E. $=\frac{1}{2} I \omega^{2}$
where $\quad I=$ moment of inertia of the armature and $\omega=$ angular velocity
$\therefore \quad$ Rotational losses, $W=$ Rate of loss of $K . E$.

[^1]$\therefore \quad W=\frac{d}{d t}\left(\frac{1}{2} I \omega^{2}\right)=I \omega \cdot \frac{d \omega}{d t}$
Two quantities need be known (i) moment of inertia (I) of the armature and (ii) $\frac{d \omega}{d t}$ or $\frac{d N}{d t}$
because $\omega \propto N$. These are found as follows:
(a) Finding $\frac{d \omega}{d t}$


Fig. 31.12
Fig. 31.13
As shown in Fig. 31.12, a voltmeter $V$ is connected across the armature. This voltmeter is used as a speed indicator by suitably graduating it, because $E \propto N$. When supply is cut off, the armature speed and hence voltmeter reading falls. By noting different amounts of voltage fall in different amounts of time, a curve is drawn between time and the speed (obtained from voltage values) as shown in Fig. 31.13.

From any point $P$ which corresponds to normal speed, a tangent $A B$ is drawn.

$$
\begin{aligned}
& \text { Then } \quad \frac{d N}{d t}=\frac{O B \text { (in r.p.m.) }}{O A \text { (in seconds) }} \\
& \text { From }(i), \text { above } \quad W=I \omega \frac{d \omega}{d t}
\end{aligned}
$$



Now

$$
\begin{align*}
\omega & =\frac{2 \pi N}{60} \\
W & =I\left(\frac{2 \pi N}{60}\right) \times \frac{d}{d t}\left(\frac{2 \pi N}{60}\right) ; W=\left(\frac{2 \pi}{60}\right)^{2} I . N . \frac{d N}{d t}=0.011 I . N . \frac{d N}{d t} \tag{ii}
\end{align*}
$$

(ii) Finding Moment of Inertia (I)
(a) First Method-where I is calculated.

First, slowing down curve is drawn with armature alone. Next, a fly-wheel of known moment of inertia $I_{1}$ is keyed onto the shaft and slowing down curve is drawn again. Obviously, slowing down time will be longer due to combined increased moment of inertia of the two. For any given speed, $\left(d N / d t_{1}\right)$ and $\left(d N / d t_{2}\right)$ are determined as before. It should be noted that the losses in both cases would be almost the same, because addition of a fly-wheel will not make much difference to the losses.

Hence, from equation (ii) above
In the first case, $\quad W=\left(\frac{2 \pi}{60}\right)^{2} I N\left(\frac{d N}{d t_{1}}\right)$

In the second case, $W=\left(\frac{2 \pi}{60}\right)^{2}\left(I+I_{1}\right) N \times\left(\frac{d N}{d t_{2}}\right)$

$$
\begin{aligned}
\therefore & \left(I+I_{1}\right)\left(\frac{d N}{d t_{2}}\right) & =I\left(\frac{d N}{d t_{1}}\right) \text { or }\left(\frac{I+I_{1}}{I}\right)=\left(\frac{d N}{d t_{1}}\right) /\left(\frac{d N}{d t_{2}}\right) \\
\therefore & I & =I_{1} \times \frac{\left(d N / d t_{2}\right)}{\left(d N / d t_{1}\right)-\left(d N / d t_{2}\right)}=I_{1} \frac{d t_{1}}{d t_{2}-d t_{1}} ; I=I_{1} \frac{t_{1}}{t_{2}-t_{1}}
\end{aligned}
$$

(b) Second Method-where I is eliminated.

In this method, first, time taken to slow down, say by $5 \%$, is noted with armature alone. Next, a retarding torque-mechanical or preferably electrical, is applied to the armature and again time is noted. The method using electrical torque is shown in Fig. 31.12. The double-throw switch $S$ while cutting off the armature from supply, automatically joins it to a non-inductive resistance $R$ as shown. The power drawn by this resistance acts as a retarding torque on the armature, thereby making it slow down comparatively quickly. The additional loss is $I_{a}^{2}\left(R_{a}+R\right)$ or $V I_{a}$,
where $\quad I_{a}=$ average current through $R ; V=$ average voltage across $R$.
Let $W^{\prime}$ be this power. Then from $(i)$ above

$$
\left.\begin{array}{rl}
W & =\left(\frac{2 \pi}{60}\right)^{2} I . N . \frac{d N}{d t_{1}} \\
W+W^{\prime} & =\left(\frac{2 \pi}{60}\right)^{2} I . N . \frac{d N}{d t_{2}}
\end{array}\right\} \text { If } d N \text { is the same } 1 \text { or } \quad \therefore \quad \begin{aligned}
& W=W^{\prime} \times \frac{d t_{2}}{d t_{1}-d t_{2}} \quad \text { or } W=W^{\prime} \times \frac{t_{2}}{t_{1}-t_{2}}
\end{aligned}
$$

where $\quad \frac{d N}{d t_{1}}=$ rate of change of speed without extra load

$$
\frac{d N}{d t_{2}}=\text { rate of change of speed with extra electrical load. }
$$

Example 31.16. In a retardation test on a separately-excited motor, the induced e.m.f. in the armature falls from 220 V to 190 V in 30 seconds on disconnecting the armature from the supply. The same fall takes place in 20 seconds if, immediately after disconnection, armature is connected to a resistance which takes 10 A (average) during this fall. Find stray losses of the motor.
(Adv. Elect. Machines, A.M.I.E. Sec. B, 1992)
Solution. Let $\quad W=$ stray losses (mechanical and magnetic losses)
Average voltage across resistance $=(200+190) / 2=195$ V, Average current $=10 \mathrm{~A}$
$\therefore \quad$ Power absorbed $W^{\prime}=1950 \mathrm{~W}$
Using the relation $\quad \frac{W}{W^{\prime}}=\frac{t_{2}}{t_{1}-t_{2}}$; we get $W=1950 \times \frac{20}{30-20}=3,900$ watt
Example 31.17. In a retardation test on a d.c. motor, with its field normally excited, the speed fell from 1525 to 1475 r.p.m. in 25 seconds. With an average load of 1.0 kW supplied by the armature, the same speed drop occurred in 20 seconds. Find out the moment of inertia of the rotating parts in kg.m ${ }^{2}$.
(Electrical Machines-III, Gujarat Univ. 1984)
Solution. As seen from Art. 31.8 (ii) (b).

$$
W^{\prime}=\left(\frac{2 \pi}{60}\right)^{2} I . N . \frac{d N}{d t} . \text { Also } W=W^{\prime} \times \frac{t_{2}}{t_{1}-t_{2}}
$$

Here,

$$
W^{\prime}=1 \mathrm{~kW}=1000 \mathrm{~W}, t_{1}=25 \text { second, } t_{2}=20 \text { second }
$$

$$
\begin{array}{lrlrl}
\therefore & W & =1000 \times 20 /(25-20)=4000 \mathrm{~W} \\
& \text { Now, } & N & =1500 \text { r.p.m. (average speed) } ; d N=1525-1475=50 \text { r.p.m. } ; d t=25 \\
\therefore & 4000 & =(2 \pi / 60)^{2} I .1500 \times 50 / 25 & \therefore \quad I=\mathbf{1 2 1 . 8} \mathbf{k g . \mathrm { m } ^ { 2 } .}
\end{array}
$$

Example 31.18. A retardation test is made on a separately-excited d.c. machine as a motor. The induced voltage falls from 240 V to 225 V in 25 seconds on opening the armature circuit and 6 seconds on suddenly changing the armature connection from supply to a load resistance taking 10 A (average). Find the efficiency of the machines when running as a motor and taking a current of 25 A on a supply of 250 V . The resistance of its armature is $0.4 \Omega$ and that of its field winding is $250 \Omega$.
(Elect. Technology, Allahabad Univ. 1991)
Solution. Average voltage across load

$$
=(240+225) / 2=232.5 \mathrm{~V} ; I_{a v}=10 \mathrm{~A}
$$

$\therefore \quad$ Power absorbed $W^{\prime}=232.5 \times 10=2,325 \mathrm{~W}$
and $\quad t_{1}=30$ second, $t_{2}=6$ second ; $W=$ stray loss
Using $\quad \frac{W}{W^{\prime}}=\frac{t_{2}}{t_{1}-t_{2}}=734.1 \mathrm{~W}$, we get
Stray losses $\quad W=2325 \times \frac{6}{25-6}=734.1 \mathrm{~W}$ Input current $=25 \mathrm{~A} ; I_{s h}=250 / 250=1 \mathrm{~A} ; I_{a}=25-1=24 \mathrm{~A}$ Armature Cu loss $=24^{2} \times 0.4=230.4 \mathrm{~W}$; Shunt Cu loss $=250 \times 1=250 \mathrm{~W}$
$\therefore \quad$ Total losses $=734.1+230.4+250=1,215 \mathrm{~W}$ (approx.)

$$
\text { Input }=250 \times 25=6,250 \mathrm{~W} ; \text { Output }=6,250-1,215=5,035 \mathrm{~W}
$$

$\therefore \quad \eta=5,035 / 6,250=0.806$ or $\mathbf{8 0 . 6 \%}$
Example 31.19. A retardation test is carried out on a 1000 r.p.m. d.c. machine. The time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m. is :
(a) 36 seconds with no excitation
(b) 15 seconds with full excitation and
(c) 9 seconds with full excitation and the armature supplying an extra load of 10 A at 219 V .

Calculate (i) the moment of inertia of the armature in $\mathrm{kg} . \mathrm{m}^{2}$ (ii) iron losses and (iii) the mechanical losses at the mean speed of 1000 r.p.m.

Solution. It should be noted that
(i) when armature slows down with no excitation, its kinetic energy is used to overcome mechanical losses only ; because due to the absence of flux, there is no iron loss.
(ii) with excitation, kinetic energy is used to supply mechanical and iron losses collectively known as stray losses.
(iii) If $I$ is taken in kg-m ${ }^{2}$ unit, then rate of loss of energy is in watts.

Mechanical loss

$$
W_{m}=\left(\frac{2 \pi}{60}\right)^{2} I \cdot N \cdot \frac{d N}{d t}
$$

Here

$$
d N=1030-970=60 \text { r.p.m., } d t=36 \text { seconds, } N=1000 \text { r.p.m. }
$$

$$
\begin{equation*}
W_{m}=\left(\frac{2 \pi}{60}\right)^{2} \text { I.N. } \frac{60}{36} \tag{i}
\end{equation*}
$$

Similarly

$$
\begin{equation*}
W_{s}=\left(\frac{2 \pi}{60}\right)^{2} \text { I.N. } \frac{60}{15} \tag{ii}
\end{equation*}
$$

Also $W_{s}$

$$
=W^{\prime} \frac{t_{2}}{t_{1}-t_{2}}=219 \times 10 \times \frac{9}{15-9}=3,285 \mathrm{~W}
$$

Using equation (ii), we get

$$
\begin{aligned}
3,285 & =\left(\frac{2 \pi}{60}\right)^{2} \times I \times 1000 \times \frac{60}{15} \\
\text { (i) } \therefore \quad I & =75 \mathrm{~kg} . \mathrm{m}^{2}
\end{aligned}
$$

Dividing (i) by (ii), we get $\frac{W_{m}}{W_{s}}=\frac{15}{36}$
(ii) $\therefore \quad W_{m}=3,285 \times 15 / 36=1,369 \mathbf{W}$
(iii) $\therefore \quad$ Iron losses $=W_{s}-W_{m}=3,285-1,369=1,916 \mathbf{W}$

### 31.9. Field's Test for Series Motor

This test is applicable to two similar series motors. Series motors which are mainly used for traction work are easily available in pairs. The two machines are coupled mechanically.

One machine runs normally as a motor and drives generator whose output is wasted in a variable load $R$ (Fig. 31.14). Iron and friction losses of two machines are made equal $(i)$ by joining the series field winding of the generator in the motor armature circuit so that both


Fig. 31.14

machines are equally excited and (ii) by running them at equal speed. Load resistance $R$ is varied till the motor current reaches its full-load value indicated by ammeter $A_{1}$. After this adjustment for full-load current, different ammeter and voltmeter readings are noted.
Let $V=$ supply voltage ; $I_{1}=$ motor current ; $V_{2}=$ terminal p.d. of generator ; $I_{2}=$ load current.
$\therefore$ Intake of the whole set $=V I_{1} ;$ output $=V_{2} I_{2}$.
Total losses in the set, $W_{t}=V I_{1}-V_{2} I_{2}$
Armature and field Cu losses $W_{c u}=\left(R_{a}+2 R_{s e}\right) I_{1}^{2}+I_{2}^{2} R_{a}$ where $R_{a}=$ hot armature resistance of each machine $R_{s e}=$ hot series field resistance of each machine
$\therefore \quad$ Stray losses for the set $=W_{t}-W_{c u}$
Stray losses per machine $\quad W_{s}=\frac{W_{t}-W_{c u}}{2}$
Stray losses are equally divided between the machines because of their equal excitation and speed.

## Motor Efficiency

$$
\begin{aligned}
\text { Motor input } & =V_{1} I_{1} \\
\text { Motor losses } & =\text { armature }+ \text { field Cu losses }+ \text { stray losses } \\
& =\left(R_{a}+R_{s e}\right) I_{1}^{2}+W_{s}=W_{m} \text { (say) } \\
\eta_{m} & =\frac{V_{1} I_{1}-W_{m}}{V_{1} I_{1}}
\end{aligned}
$$

## Generator Efficiency

The generator efficiency will be of little use because it is running under abnormal conditions of separate excitation. However, the efficiency under these unusual conditions can be found if desired.

$$
\begin{array}{ll}
\text { Generator output } & =V_{2} I_{2} \\
\text { Field Cu loss } & =I_{1}^{2} R_{s e} \\
\text { Armature Cu loss } & =I_{2}{ }^{2} R_{a} ; \text { Stray losses }=W_{s} \quad(\because \text { Motor current is passing through it. }) \\
\text { Total losses } & =I_{1}{ }^{2} R_{s e}+I_{2}^{2} R_{a}+W_{s}=W_{g}(\text { say }) \\
& \eta_{g} \\
=\frac{V_{2} I_{2}}{V_{2} I_{2}+W_{g}}
\end{array}
$$

It should be noted that although the two machines are mechanically coupled yet it is not a regenerative method, because the generator output is wasted instead of being fed back into the motor as in Hopkinson's (back-to-back) test.

Example 31.20. A test on two coupled similar tramway motors, with their fields connected in series, gave the following results when one machine acted as a motor and the other as a generator. Motor: $\quad$ Armature current $=56$ A ; Armature voltage $=590 \mathrm{~V}$

$$
\text { Voltage drop across field winding }=40 \mathrm{~V}
$$

Generator :

$$
\begin{aligned}
\text { Armature current } & =44 \mathrm{~A} ; \text { Armature voltage }=400 \mathrm{~V} \\
\text { Field voltage drop } & =40 \mathrm{~V} ; \text { Resistance of each armature }=0.3 \Omega
\end{aligned}
$$

Calculate the efficiency of the motor and gearing at this load.
(Elect. Machinery-II, Nagpur Univ. 1992 \& JNTU, Hyderabad, 2000)
Solution. The connection for the two machines are shown in Fig. 31.15.

$$
\begin{aligned}
\text { Total input } & =630 \times 56=35,280 \mathrm{~W} \\
\text { Output } & =400 \times 44=17,600 \mathrm{~W}
\end{aligned}
$$

Total losses in the two machines are

$$
=35,280-17,600=17,680 \mathrm{~W}
$$

Series field resistance

$$
\begin{aligned}
R_{\text {se }} & =40 / 56 \\
& =0.714 \Omega
\end{aligned}
$$

Total Cu loss $=(0.3+2 \times 0.714) \times 56^{2}+44^{2}$ $\times 0.3=5,425+581=6,006 \mathrm{~W}$

Stray losses of the set=17,680-6,006

$$
=11,674 \mathrm{~W}
$$

$\therefore$ Stray losses $/$ machine $=11,674 / 2=5,837 \mathrm{~W}$

## Motor Efficiency

Motor armature input $=$ arm. voltage $\times$ motor


Fig. 31.15 current $=590 \times 56=33,040 \mathrm{~W}$

Armature circuit Cu loss

$$
=(0.3+0.714) \times 56^{2}=3,180 \mathrm{~W}
$$

Stray loss $=5,837 \mathrm{~W}-$ found above

$$
\text { Total losses }=3,180+5,837=9,017 \mathrm{~W}, \text { Output }=33,040-9,017=24,023 \mathrm{~W}
$$

$\therefore \quad \eta_{m}=24,023 / 33,040=0.727$ or $72.7 \%$

## Generator Efficiency

$$
\begin{aligned}
\text { Armature Cu loss } & =44^{2} \times 0.3=581 \mathrm{~W}, \text { Series field Cu loss }=40 \times 56=2,240 \mathrm{~W} \\
\text { Stray losses } & =5,837 \mathrm{~W} ; \text { Total losses }=581+2,240+5,837=8,658 \mathrm{~W} \\
\text { Output } & =400 \times 44=17,600 \mathrm{~W} \\
\therefore \quad \eta_{g} & =17,600 /(17,600+8,658)=0.67 \text { or } 67 \%
\end{aligned}
$$

## Tutorial Problem No. 31.1

1. A $500-\mathrm{V}$, shunt motor takes a total current of 5 A when running unloaded. The resistance of armature circuit is $0.25 \Omega$ and the field resistance is $125 \Omega$. Calculate the efficiency and output when the motor is loaded and taking a current of 100 A .
[ $90.4 \%$; 45.2 kW ]
2. A d.c. shunt motor rated at 12.5 kW output runs at no-load at 1000 r.p.m. from a $250-\mathrm{V}$ supply consuming an input current of 4 A . The armature resistance is $0.5 \Omega$ and shunt field resistance is $250 \Omega$. Calculate the efficiency of the machine when delivering full-load output of 12.5 kW while operating at 250 V.
[81.57\%] (Elect. Technology-I Madras Univ. 1979)
3. The following results were obtained during Hopkinson's test on two similar $230-\mathrm{V}$ machines; armature currents 37 A and 30 A ; field currents 0.85 A and 0.8 A . Calculate the efficiencies of machines if each has an armature resistance of $0.33 \Omega$.
[Generator 87.9\%, Motor 87.7\%]
4. In a Field's test on two $230-\mathrm{V}, 1.492 \mathrm{~kW}$ mechanically-coupled similar series motors, the following figures were obtained. Each has armature and compole winding resistance of $2.4 \Omega$, series field resistance of $1.45 \Omega$ and total brush drop of 2 V . The p.d. across armature and field was 230 V with a motor current of 10.1 A . The generator supplied a current of 8.9 A at a terminal p.d. of 161 V . Calculate the efficiency and output of the motor for this load.
[76.45\%, 1.775 kW ]
5. Describe the Hopkinson's test for obtaining the efficiency of two similar shunt motors. The readings obtained in such a test were as follows; line voltage 100 V ; motor current 30 A ; generator current 25 A ; armature resistance of each machine $0.25 \Omega$. Calculate the efficiency of each machine from these results, ignoring the field currents and assuming that their iron and mechanical losses are the same.
[Motor 90.05\%, Generator 92.5\%]
6. The Hopkinson's test on two similar d.c. shunt machines gave the following results :

Line voltage $=220 \mathrm{~V}$; line current excluding field currents $=40 \mathrm{~A}$; the armature current of motoring machine $=200 \mathrm{~A}$; field currents 6 A and 7 A . Calculate the efficiency of each of the machines at the given load conditions. The armature resistance of each of the machines is $0.05 \Omega$.

$$
\left[\eta_{m}=86.58 \% ; \eta_{g}=86.3 \%\right] \text { (Electrical Engg-I, M.S. Univ. Baroda 1980) }
$$

## OBJECTIVE TEST - 31

1. One of the main advantages of Swinburne's test is that it
(a) is applicable both to shunt and compound motors
(b) needs one running test
(c) is very economical and convenient
(d) ignores any charge in iron loss
2. The main disadvantage of Hopkinson's test for finding efficiency of shunt d.c. motors is that it
(a) requires full-load power
(b) ignores any change in iron loss
(c) needs one motor and one generator
(d) requires two identical shunt machines
3. The most economical method of finding no-load losses of a large d.c. shunt motor is-test.
(a) Hopkinson's
(b) Swinburne's
(c) retardation
(d) Field's
4. Retardation test on a d.c. shunt motor is used for finding-losses.
(a) stray
(b) copper
(c) friction
(d) iron
5. The main thing common between Hopkinson's test and Field's test is that both
(a) require two electrically-coupled series motors
(b) need two similar mechanically-coupled motors
(c) use negligible power
(d) are regenerative tests
6. The usual test for determining the efficiency of a traction motor is the $\qquad$ test.
(a) Field's
(b) retardation
(c) Hopkinson's
(d) Swinburne's

## ANSWERS

## $\begin{array}{llllll}\text { 1. } c & \text { 2. } d & 3 . b & \text { 4. } a & \text { 5. } b & \text { 6. } a\end{array}$

## QUESTIONS AND ANSWERS ON D.C. MOTORS

Q. 1. How may the direction of rotation of a d.c. motor be reversed ?

Ans. By reversing either the field current or current through the armature. Usually, reversal of current through the armature is adopted.
Q. 2. What will happen if both currents are reversed ?

Ans. The motor will run in the original direction.
Q. 3. What will happen if the field of a d.c. shunt motor is opened ?

Ans. The motor will achieve dangerously high speed and may destroy itself.
Q. 4. What happens if the direction of current at the terminals of a series motor is reversed ?

Ans. It does not reverse the direction of rotation of motor because current flows through the armature in the same direction as through the field.
Q. 5. Explain what happens when a d.c. motor is connected across an a.c. supply ?

Ans. 1. Since on a.c. supply, reactance will come into the picture, the a.c. supply will be offered impedance (not resistance) by the armature winding. Consequently, with a.c. supply, current will be much less. The motor will run but it would not carry the same load as it would on d.c. supply.
2. There would be more sparking at the brushes.
3. Though motor armature is laminated as a rule, the field poles are not. Consequently, eddy currents will cause the motor to heat up and eventually burn on a.c. supply.
Q. 6. What will happen if a shunt motor is directly connected to the supply line ?

Ans. Small motors up to 1 kW rating may be line-started without any adverse results being produced. High rating motors must be started through a suitable starter in order to avoid the huge starting current which will
(i) damage the motor itself and (ii) badly affect the voltage regulation of the supply line.
Q. 7. What is the function of interpoles and how are interpole windings connected ?

Ans. Interpoles are small poles placed in between the main poles. Their function is to assist commutation by producing the auxiliary or commutating flux. Consequently, brush sparking is practically eliminated. Interpole windings are connected in series with the armature windings.
Q. 8. In rewinding the armature of a d.c. motor, progressive connections are changed to retrogressive ones. Will it affect the operation in any way?
Ans. Yes. Now, the armature will rotate in the opposite direction.
Q. 9. A d.c. motor fails to start when switched on. What could be the possible reasons and remedies?
Ans. Any one of the following reasons could be responsible :

1. Open-circuit in controller-should be checked for open starting resistance or open switch or open fuse.
2. Low terminal voltage-should be adjusted to name-plate value.
3. Overload-should be reduced if possible otherwise larger motor should be installed.
4. Excessive friction-bearing lubrication should be checked.
Q. 10. A d.c. motor is found to stop running after a short period of time. What do you think could be the reasons? How would you remedy each?
Ans. Possible causes are as under :
5. Motor not getting enough power-check voltage at motor terminals as well as fuses, clups and overload relay.
6. Weak or no field-in the case of adjustable-speed motors, check if rheostat is correctly set. Also, check field winding for any 'open'. Additionally, look for any loose winding or broken connection.
7. Motor torque insufficient for driving the given load-check line voltage with name-plate voltage. If necessary, use larger motor to match the load.
Q. 11. What are the likely causes if a d.c. motor is found to run too slow under load? And the remedy?
Ans. 1. Supply line voltage too low-remove any excessive resistance in supply line, connections or controller.
8. Brushes ahead of neutral-set them on neutral.
9. Overload-reduce it to allowable value or use larger motor.
Q. 12. Why does a d.c. motor sometime run too fast when under load? Give different possible causes and their remedies.
Ans. Different possible causes are as under :
10. Weak field-remove any extra resistance in shunt field circuit. Also, check for 'grounds'.
11. Line voltage too high-reduce it to name-plate value.
12. Brushes back of neutral-set them on neutral.
Q. 13. Under what conditions is sparking produced at the brushes of a d.c. motor ? How would you remedy it ?
Ans. 1. Commutator in bad condition-clean and reset brushes.
13. commutator either eccentric or rough-grind and true the commutator. Also, undercut mica.
14. Excessive vibration-balance armature. Make sure that brushes ride freely in holders.
15. Brush-holding spring broken or sluggish-replace spring and adjust pressure to recommended value.
16. Motor overloaded-reduce load or install motor of proper rating.
17. Short-circuit in armature circuit-remove any metallic particles between commutator segments and check for short between adjacent commutator risers. Locate and repair internal armature short if any.
Q. 14. Sometimes a hissing noise (or brush chatter) is heard to emanate from the commutator end of a running d.c. motor. What could it be due to and how could it be removed?

Ans. Any one of the following causes could produce brush chatter :

1. Excessive clearance of brush holders-adjust properly
2. Incorrect angle of brushes-adjust to correct value
3. Unsuitable brushes-replace them
4. High mica-undercut it
5. Wrong brush spring pressure-adjust to correct value.
Q. 15. What are the possible causes of excessive sparking at brushes in a d.c. motor ?

Ans. 1. Poor brush fit on commutator-sand-in the brushes and polish commutator.
2. Brushes binding in the brush holders-clean holders and brushes and remove any irregularities on surfaces of brush holders or rough spots on brushes.
3. Excessive or insufficient pressure on brushes-adjust pressure.
4. Brushes off neutral-set them on neutral.
Q. 16. Why does a d.c. motor sometime spark on light load ?

Ans. Due to the presence of paint spray, chemical, oil or grease etc. on commutator.
Q. 17. When is the armature of a d.c. motor likely to get over-heated?

Ans. 1. When motor is over-loaded.
2. When it is installed at a place having restricted ventilation.
3. When armature winding is shorted.
Q. 18. What causes are responsible for over-heating of commutator in a d.c. motor?

Ans. It could be due either to the brushes being off neutral or being under excessive spring pressure. Accordingly, brushes should be adjusted properly and the spring pressure should be reduced but not to the point where sparking is introduced.


[^0]:     electric machines and synthetic resins.

[^1]:    * If armature slows down with no excitation, then energy of the armature is used to overcome mechanical losses only, there being no iron losses (see Ex. 31.19).

