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CHAPTER 35

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COMPUTATIONS AND CIRCLE DIAGRAMS

This chapter explains you how to derive performance characteristics of induction motors using circular diagrams

35.1. General

In this chapter, it will be shown that the performance characteristics of an induction motor are derivable from a circular locus. The data necessary to draw the circle diagram may be found from no-load and blocked-rotor tests, corresponding to the open-circuit and short-circuit tests of a transformer. The stator and rotor Cu losses can be separated by drawing a torque line. The parameters of the motor, in the equivalent circuit, can be found from the above tests, as shown below.

35.2. Circle Diagram for a Series Circuit

It will be shown that the end of the current vector for a series circuit with constant reactance and voltage, but with a variable resistance is a circle. With reference to Fig. 35.1, it is clear that

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R^2 + X^2)}}$$

$$= \frac{V}{X} \times \frac{X}{\sqrt{(R^2 + X^2)}} = \frac{V}{X}\sin\phi$$

$$\because \sin\phi = \frac{X}{\sqrt{(R^2 + X^2)}} \qquad \text{– Fig. 35.2}$$

$$\therefore \quad I = (V/X)\sin\phi$$

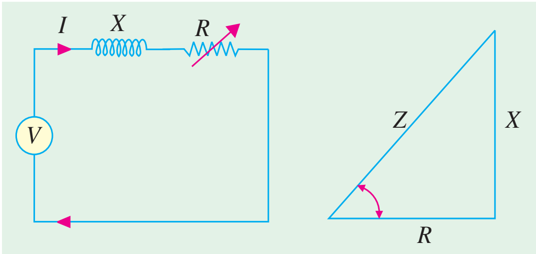


Fig. 35.1 Fig. 35.2

It is the equation of a circle in polar co-ordinates, with diameter equal to V/X. Such a circle is drawn in Fig. 35.3, using the magnitude of the current and power factor angle ϕ as polar co-ordinates of the point A. In other words, as resistance R is varied (which means, in fact, ϕ is changed), the end of the current vector lies on a circle with diameter equal to V/X. For a lagging current, it is usual to orientate the circle of Fig. 35.3 (*a*) such that its diameter is horizontal and the voltage vector takes a vertical position, as shown in Fig. 35.3 (*b*). There is no difference between the two so far as the magnitude and phase relationships are concerned.

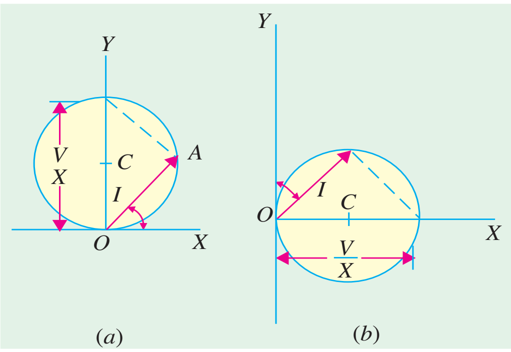


Fig. 35.3

35.3. Circle Diagram for the Approximate Equivalent Circuit

The approximate equivalent diagram is redrawn in Fig. 35.4. It is clear that the circuit to the right of points *ab* is similar to a series circuit, having a constant voltage V_1 and reactance X_{01} but variable resistance (corresponding to different values of slip s).

Hence, the end of current vector for I_2' will lie on a circle with a diameter of V/X_{01}. In Fig. 35.5, I_2' is the rotor current referred to stator, I_0 is no-load current (or exciting current) and I_1 is the total stator current and is the vector sum of the first two. When I_2' is lagging and $\phi_2 = 90°$, then the position of vector for I_2' will be along OC *i.e.* at right angles to the voltage vector OE. For any other value of ϕ_2, point A will move along the circle shown dotted. The exciting current I_0 is drawn lagging V by an angle ϕ_0. If conductance G_0 and susceptance

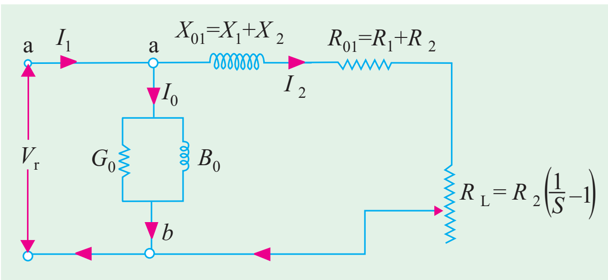


Fig. 35.4

B_0 of the exciting circuit are assumed constant, then I_0 and ϕ_0 are also constant. The end of current vector for I_1 is also seen to lie on another circle which is displaced from the dotted circle by an amount I_0. Its diameter is still V/X_{01} and is parallel to the horizontal axis *OC*. Hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.

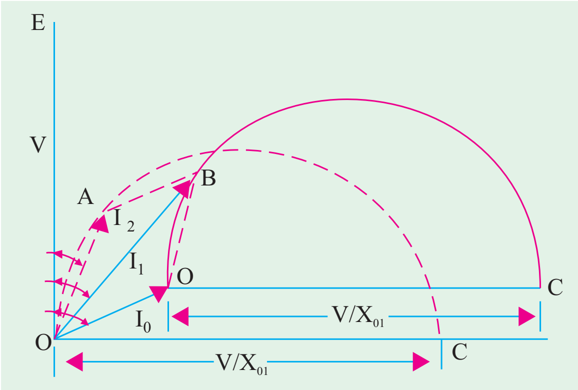


Fig. 35.5

35.4. Determination of G_0 and B_0

If the total leakage reactance X_{01} of the motor, exciting conductance G_0 and exciting susceptance B_0 are found, then the position of the circle *O′BC′* is determined uniquely. One method of finding G_0 and B_0 consists in running the motor synchronously so that slip s = 0. In practice, it is impossible for an induction motor to run at synchronous speed, due to the inevitable presence of friction and windage losses. However, the induction motor may be run at synchronous speed by another machine which supplies the friction and windage losses. In that case, the circuit to the right of points *ab* behaves like an open circuit, because with $s = 0$, $R_L = \infty$ (Fig. 35.6). Hence, the current drawn by the motor is I_0 only. Let

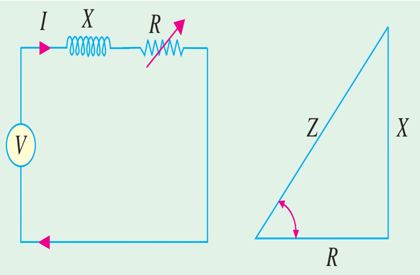


Fig. 35.6

V = applied voltage/phase; I_0 = motor current / phase

W = wattmeter reading *i.e.* input in watt ; Y_0 = exciting admittance of the motor. Then, for a 3-phase induction motor

$$W = 3G_0V^2 \quad \text{or} \quad G_0 = \frac{W}{3V^2} \quad \text{Also, } I_0 = V\,Y_0 \quad \text{or} \quad Y_0 = I_0/V$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{[(I_0/V)^2 - G_0^2]}$$

Hence, G_0 and B_0 can be found.

35.5. No-load Test

In practice, it is neither necessary nor feasible to run the induction motor synchronously for getting G_0 and B_0. Instead, the motor is run without any external mechanical load on it. The speed of the rotor would not be synchronous, but very much near to it ; so that, for all practical purposes, the speed may be assumed synchronous. The no load test is carried out with different values of applied voltage, below and above the value of normal voltage. The power input is measured by two wattmeters,

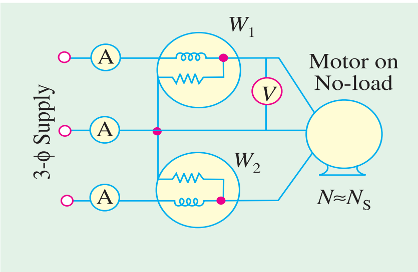


Fig. 35.7

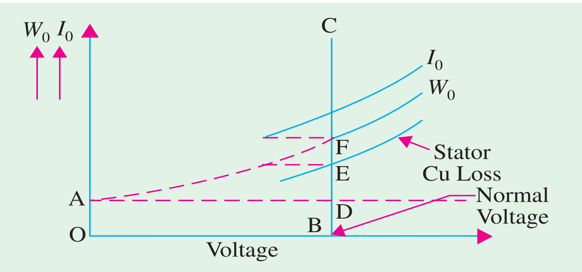


Fig. 35.8

I_0 by an ammeter and V by a voltmeter, which are included in the circuit of Fig. 35.7. As motor is running on light load, the p.f. would be low *i.e*. less than 0.5, hence total power input will be the difference of the two wattmeter readings W_1 and W_2. The readings of the total power input W_0, I_0 and voltage V are plotted as in Fig. 35.8. If we extend the curve for W_0, it cuts the vertical axis at point A. OA represents losses due to friction and windage. If we subtract loss corresponding to OA from W_0, then we get the no-load electrical and magnetic losses in the machine, because the no-load input W_0 to the motor consists of

(*i*) small stator Cu loss $3\, I_0^2\, R_1$ (*ii*) stator core loss $W_{CL} = 3G_0\, V^2$

(*iii*) loss due to friction and windage.

The losses (*ii*) and (*iii*) are collectively known as fixed losses, because they are independent of load. OB represents normal voltage. Hence, losses at normal voltage can be found by drawing a vertical line from B.

BD = loss due to friction and windage DE = stator Cu loss EF = core loss

Hence, knowing the core loss W_{CL}, G_0 and B_0 can be found, as discussed in Art. 35.4.

Additionally, ϕ_0 can also be found from the relation $W_0 = \sqrt{3}\, V_L\, I_0 \cos\phi_0$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{\sqrt{3}\, V_L\, I_0}$$ where V_L = line voltage and W_0 is no-load stator input.

Example 35.1. *In a no-load test, an induction motor took 10 A and 450 watts with a line voltage of 110 V. If stator resistance/phase is 0.05 Ω and friction and windage losses amount to 135 watts, calculate the exciting conductance and susceptance/phase.*

Solution. stator Cu loss $= 3\, I_0^2\, R_1 = 3 \times 10^2 \times 0.05 = 15$ W

∴ stator core loss $= 450 - 135 - 15 = 300$ W

Voltage/phase $V = 110/\sqrt{3}$ V ; Core loss $= 3\, G_0\, V^2$

$$300 = 3\, G_0 \times (110/\sqrt{3})^2 \;;\; G_0 = \frac{300}{3 \times (110/\sqrt{3})^2}$$

$= \mathbf{0.025\ siemens/phase}$

$$Y_0 = I_0 / V = (10 \times \sqrt{3})/110 = 0.158 \text{ siemens/phase}$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{(0.158^2 - 0.025^2)}$$

$= \mathbf{0.156\ siemens/phase.}$

35.6. Blocked Rotor Test

It is also known as locked-rotor or short-circuit test. This test is used to find–

1. short-circuit current with ***normal*** voltage applied to stator
2. power factor on short-circuit

Both the values are used in the construction of circle diagram

3. total leakage reactance X_{01} of the motor as referred to primary (*i.e.* stator)
4. total resistance of the motor R_{01} as referred to primary.

This vertical test stand is capable of absorbing up to 10,000 N-m of torque at continuous load rating (max 150.0 hp at 1800 rpm). It helps to develop speed torque curves and performs locked rotor testing

In this test, the rotor is locked (or allowed very slow rotation) and the rotor windings are short-circuited at slip-rings, if the motor has a wound rotor. Just as in the case of a short-circuit test on a transformer, a reduced voltage (up to 15 or 20 per cent of normal value) is applied to the stator terminals and is so adjusted that full-load current flows in the stator. As in this case $s = 1$, the equivalent circuit of the motor is exactly like a transformer, having a short-circuited secondary. The values of current, voltage and power input on short-circuit are measured by the ammeter, voltmeter and wattmeter connected in the circuits as before. Curves connecting the above quantities may also be drawn by taking two or three additional sets of readings at progressively reduced voltages of the stator.

(*a*) It is found that relation between the short-circuit current and voltage is approximately a straight line. Hence, if V is normal stator voltage, V_s the short-circuit voltage (a fraction of V), then short-circuit or standstill rotor current, if normal voltage were applied to stator, is found from the relation

$$I_{SN} = I_s \times V/V_s$$

where I_{SN} = short-circuit current obtainable with normal voltage

I_s = short-circuit current with voltage V_S

(*b*) Power factor on short-circuit is found from

$$W_S = \sqrt{3}\, V_{SL} I_{SL} \cos\phi_S \; ; \qquad \therefore \quad \cos\phi_S = W_S / (\sqrt{3}\, V_{SL} I_{SL})$$

where W_S = total power input on short-circuit

V_{SL} = line voltage on short-circuit

I_{SL} = line current on short-circuit.

(*c*) Now, the motor input on short-circuit consists of

(*i*) mainly stator and rotor Cu losses

(*ii*) core-loss, which is small due to the fact that applied voltage is only a small percentage of the normal voltage. This core-loss (if found appreciable) can be calculated from the curves of Fig. 35.8.

$$\therefore \quad \text{Total Cu loss} = W_S - W_{CL}$$

$$3 I_s^2 R_{01} = W_s - W_{CL} : \quad R_{01} = (W_s - W_{CL}) / 3 I_s^2$$

(*d*) With reference to the approximate equivalent circuit of an induction motor (Fig. 35.4), motor leakage reactance per phase X_{01} as referred to the stator may be calculated as follows :

$$Z_{01} = V_S / I_S \qquad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$$

Usually, X_1 is assumed equal to X_2' where X_1 and X_2 are stator and rotor reactances per phase respectively as referred to stator. $X_1 = X_2' = X_{01} / 2$

If the motor has a wound rotor, then stator and rotor resistances are separated by dividing R_{01} in the ratio of the d.c. resistances of stator and rotor windings.

In the case of squirrel-cage rotor, R_1 is determined as usual and after allowing for 'skin effect' is subtracted from R_{01} to give R_2' – the effective rotor resistance as referred to stator.

$$\therefore \quad R_2' = R_{01} - R_1$$

Example 35.2. *A 110-V, 3-ϕ, star-connected induction motor takes 25 A at a line voltage of 30 V with rotor locked. With this line voltage, power input to motor is 440 W and core loss is 40 W. The d.c. resistance between a pair of stator terminals is 0.1 Ω. If the ratio of a.c. to d.c. resistance is 1.6, find the equivalent leakage reactance/phase of the motor and the stator and rotor resistance per phase.* **(Electrical Technology, Madras Univ. 1987)**

Solution. S.C. voltage/phase, $V_s = 30 / \sqrt{3} = 17.3 \text{ V} : I_s = 25$ A per phase

$Z_{01} = 17.3/25 = 0.7\ \Omega$ (approx.) per phase

$$\text{Stator and rotor Cu losses} = \text{input} - \text{core loss} = 440 - 40 = 400 \text{ W}$$

$$\therefore \quad 3 \times 25^2 \times R_{01} = 400 \qquad \therefore R_{01} = 400/3 \times 625 = \mathbf{0.21\ \Omega}$$

where R_{01} is equivalent resistance/phase of motor as referred to stator.

$$\text{Leakage reactance/phase} \quad X_{01} = \sqrt{(0.7^2 - 0.21^2)} = 0.668\ \Omega$$

$$\text{d.c. resistance/phase of stator} = 0.1/2 = 0.05\ \Omega$$

$$\text{a.c. resistance/phase} \quad R_1 = 0.05 \times 1.6 = 0.08\ \Omega$$

Hence, effective resistance/phase of rotor as referred to stator

$$R_2' = 0.21 - 0.08 = \mathbf{0.13\ \Omega}$$

35.7. Construction of the Circle Diagram

Windings inside a motor

Circle diagram of an induction motor can be drawn by using the data obtained from **(1) no-load (2) short-circuit test** and **(3) stator resistance test**, as shown below.

Step No. 1

From no-load test, I_0 and ϕ_0 can be calculated. Hence, as shown in Fig. 35.9, vector for I_0 can be laid off lagging ϕ_0 behind the applied voltage V.

Step No. 2

Next, from blocked rotor test or short-circuit test, short-circuit current I_{SN} ***corresponding to normal voltage*** and ϕ_S are found. The vector ***OA*** represents $I_{SN} = (I_S V/V_S)$ in magnitude and phase. Vector $O'A$ represents rotor current I_2' as referred to stator.

Clearly, the two points O' and A lie on the required circle. For finding the centre C of this circle, chord $O'A$ is bisected at right angles–its bisector giving point C. The diameter $O'D$ is drawn perpendicular to the voltage vector.

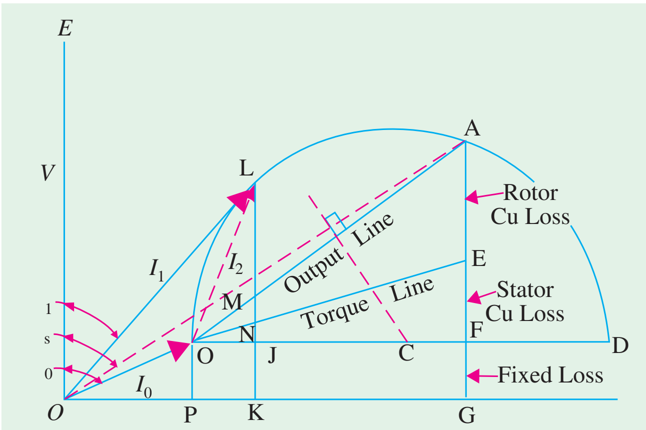


Fig. 35.9

As a matter of practical contingency, it is recommended that the scale of current vectors should be so chosen that the diameter is more than 25 cm, in order that the performance data of the motor may be read with reasonable accuracy from the circle diagram. With centre C and radius = CO', the circle can be drawn. The line $O'A$ is known as **out-put line.**

It should be noted that as the voltage vector is drawn vertically, all vertical distances represent the active or power or energy components of the currents. For example, the vertical component $O'P$ of no-load current OO' represents the no-load input, which supplies core loss, friction and windage loss and a negligibly small amount of stator I^2R loss. Similarly, the vertical component AG of short-circuit current OA is proportional to the motor input on short-circuit or if measured to a proper scale, may be said to equal power input.

Step No. 3

Torque line. ***This is the line which separates the stator and the rotor copper losses***. When the rotor is locked, then all the power supplied to the motor goes to meet core losses and Cu losses in the stator and rotor windings. The power input is proportional to AG. Out of this, FG (= $O'P$) represents fixed losses *i.e.* stator core loss and friction and windage losses. AF is proportional to the sum of the stator and rotor Cu losses. The point E is such that

$$\frac{AE}{EF} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

As said earlier, line $O'E$ is known as torque line.

How to locate point E ?

(i) Squirrel-cage Rotor. Stator resistance/phase *i.e.* R_1 is found from stator-resistance test. Now, the short-circuit motor input W_s is approximately equal to motor Cu losses (neglecting iron losses).

$$\text{Stator Cu loss} = 3I_S^2 R_1 \qquad \therefore \text{ rotor Cu loss} = W_S - 3\,I_S^2 R_1 \qquad \therefore \quad \frac{AE}{EF} = \frac{W_S - 3I_S^2 R_1}{3\,I_S^2 R_1}$$

(ii) Wound Rotor. In this case, rotor and stator resistances per phase r_2 and r_1 can be easily computed. For any values of stator and rotor currents I_1 and I_2 respectively, we can write

$$\frac{AE}{EF} = \frac{I_2^2 r_2}{I_1^2 r_1} = \frac{r_2}{r_1}\left(\frac{I_2}{I_1}\right)^2; \qquad \text{Now,} \quad \frac{I_1}{I_2} = K = \text{transformation ratio}$$

$$\frac{AE}{EF} = \frac{r_2}{r_1} \times \frac{1}{K^2} = \frac{r_2/K^2}{r_1} = \frac{r_2'}{r_1} = \frac{\text{equivalent rotor resistance per phase}}{\text{stator resistance per phase}}$$

Value of K may be found from short-circuit test itself by using two ammeters, both in stator and rotor circuits.

Let us assume that the motor is running and taking a current OL (Fig. 35.9). Then, the perpendicular JK represents fixed losses, JN is stator Cu loss, NL is the rotor input, NM is rotor Cu loss, ML is rotor output and LK is the total motor input.

From our knowledge of the relations between the above-given various quantities, we can write :

$$\sqrt{3}\,.\,V_L\,.\,LK = \text{motor input} \qquad \sqrt{3}\,.\,V_L\,.\,JK = \text{fixed losses}$$

$$\sqrt{3}\,.\,V_L\,.\,JN = \text{stator copper loss} \qquad \sqrt{3}\,.\,V_L\,.\,MN = \text{rotor copper loss}$$

$$\sqrt{3}\,.\,V_L\,.\,MK = \text{total loss} \qquad \sqrt{3}\,.\,V_L\,.\,ML = \text{mechanical output}$$

$$\sqrt{3}\,.\,V_L\,.\,NL = \text{rotor input} \propto \text{torque}$$

1. ML/LK = output/input = efficiency

2. MN/NL = (rotor Cu loss)/(rotor input) = slip, s.

3. $$\frac{ML}{NL} = \frac{\text{rotor output}}{\text{rotor input}} = 1 - s = \frac{N}{N_S} = \frac{\text{actual speed}}{\text{synchronous speed}}$$

4. $$\frac{LK}{OL} = \text{power factor}$$

Hence, it is seen that, at least, theoretically, it is possible to obtain all the characteristics of an induction motor from its circle diagram. As said earlier, for drawing the circle diagram, we need **(*a*)** stator-resistance test for separating stator and rotor Cu losses and **(*b*)** the data obtained from **(*i*)** no-load test and **(*ii*)** short-circuit test.

35.8. Maximum Quantities

It will now be shown from the circle diagram (Fig. 35.10) that the maximum values occur at the positions stated below :

(*i*) Maximum Output

It occurs at point M where the tangent is parallel to output line $O'A$. Point M may be located by drawing a line *CM* from point *C* such that it is perpendicular to the output line *O′A*. Maximum output is represented by the vertical MP.

(*ii*) Maximum Torque or Rotor Input

It occurs at point *N* where the tangent is parallel to torque line *O′E*. Again, point *N* may be found by drawing *CN* perpendicular to the torque line. Its value is represented by *NQ* . Maximum torque is also known as stalling or pull-out torque.

(*iii*) Maximum Input Power

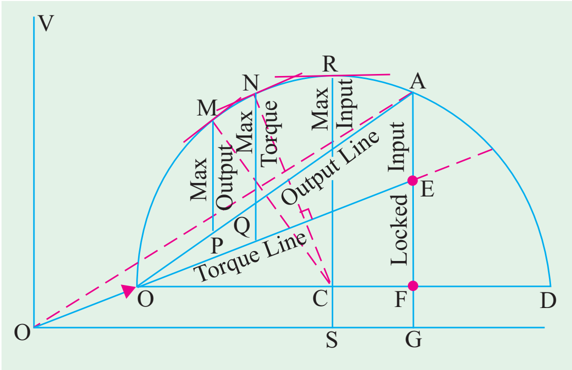


Fig. 35.10

It occurs at the highest point of the circle *i.e*. at point R where the tangent to the circle is horizontal. It is proportional to RS. As the point R is beyond the point of maximum torque, the induction motor will be unstable here. However, the maximum input is a measure of the size of the circle and is an indication of the ability of the motor to carry short-time over-loads. Generally, RS is twice or thrice the motor input at rated load.

Example 35.3. *A 3-ph, 400-V induction motor gave the following test readings;*
No-load : 400 V, 1250 W, 9 A, Short-circuit : 150 V, 4 kW, 38 A
Draw the circle diagram.
If the normal rating is 14.9 kW, find from the circle diagram, the full-load value of current, p.f. and slip. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. $\cos\phi_0 = \dfrac{1250}{\sqrt{3}\times 400\times 9} = 0.2004\ ;\quad \phi_0 = 78.5°$

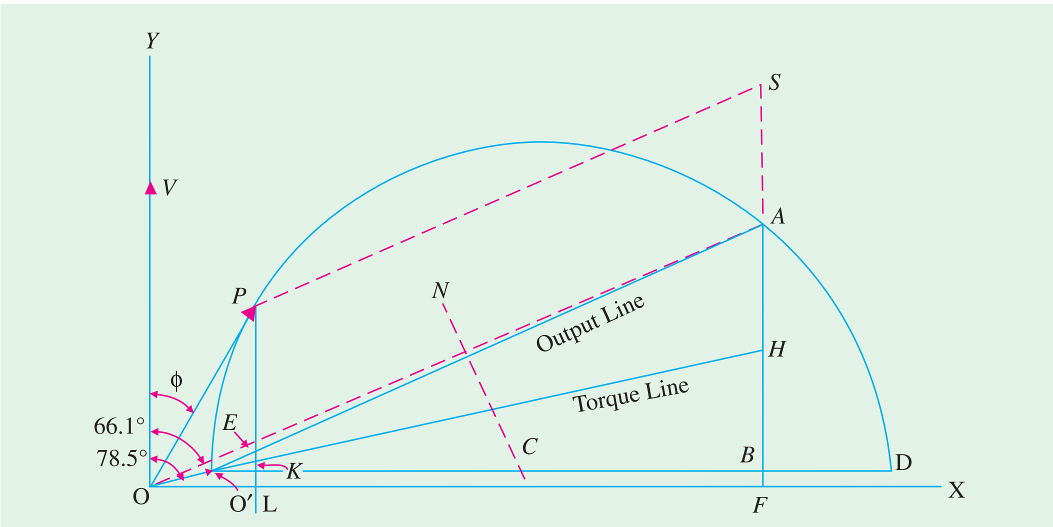


Fig. 35.11

$$\cos\phi_S = \frac{4000}{\sqrt{3}\times 150\times 38} = 0.405\ ;\quad \phi_S = 66.1°$$

Short-circuit current with normal voltage is I_{SN} = 38 (400/150) = 101.3 A. Power taken would be = 4000 $(400/150)^2$ = 28,440 W. In Fig. 33.11, *OO′* represents I_0 of 9 A. If current scale is 1 cm = 5 A, then vector OO' = 9/5 = 1.8 cm* and is drawn at an angle of ϕ_0 = 78.5° with the vertical OV (which represents voltage). Similarly, OA represents I_{SN} (S.C. current with normal voltage applied) equal to 101.3 A. It measures 101.3/5 = 20.26* cm and is drawn at an angle of 66.1°, with the vertical OV.

Line $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$. The semi-circle $O'AD$ is drawn with C as the centre. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit. Now, AF represents 28,440 W and measures 8.1 cm. Hence, power scale becomes : 1 cm = 28,440/8.1 = 3,510 W. Now, full-load motor output = 14.9×10^3 = 14,900 W. According to the above calculated power scale, the intercept between the semi-circle and output line $O'A$ should measure = 14,900/3510 = 4.25 cm. For locating full-load point P, BA is extended. AS is made equal to 4.25 cm and SP is drawn parallel to output line $O'A$. PL is perpendicular to OX.

$$\text{Line current} = OP = 6 \text{ cm} = 6 \times 5 = \textbf{30 A}; \ \phi = 30° \text{ (by measurement)}$$

$$\text{p.f.} = \cos 30° = \textbf{0.886} \text{ (or } \cos\phi = PL/OP = 5.2/6 = 0.865)$$

Now, $$\text{slip} = \frac{\text{rotor Cu loss}}{\text{rotor input}}$$

In Fig. 35.11, EK represents rotor Cu loss and PK represents rotor input.

$\therefore$ $$\text{slip} = \frac{EK}{PK} = \frac{0.3}{4.5} = 0.067 \text{ or } \textbf{6.7\%}$$

Example 35.4. *Draw the circle diagram for a 3.73 kW, 200-V, 50-Hz, 4-pole, 3-ϕ star-connected induction motor from the following test data :*

No-load : Line voltage 200 V, line current 5 A; total input 350 W

Blocked rotor : Line voltage 100 V, line current 26 A; total input 1700 W

Estimate from the diagram for full-load condition, the line current, power factor and also the maximum torque in terms of the full-load torque. The rotor Cu loss at standstill is half the total Cu loss. **(Electrical Engineering, Bombay Univ. 1987)**

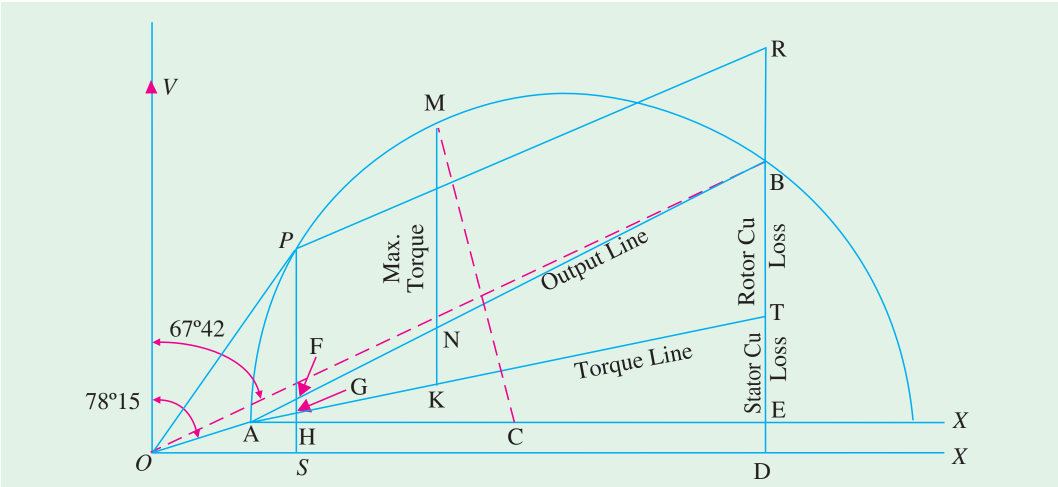


Fig. 35.12

Solution. No-load test

$$I_0 = 5 \text{ A}, \cos\phi_0 = \frac{350}{\sqrt{3} \times 200 \times 5} = 0.202 \ ; \ \phi_0 = 78°15'$$

* The actual lengths are different from these values, due to reduction in block making.

Blocked-rotor test :

$$\cos \phi_s = \frac{1700}{\sqrt{3} \times 100 \times 26} = 0.378 \; ; \; \phi_s = 67°42'$$

Short-circuit current with normal voltage, $I_{SN} = 26 \times 200/100 = 52$ A

Short-circuit/blocked rotor input with normal voltage = $1700(52/26)^2$ = 6,800 W

In the circle diagram of Fig. 35.12, voltage is represented along OV which is drawn perpendicular to OX. Current scale is 1 cm = 2 A

Line OA is drawn at an angle of $\phi_0 = 78°15'$ with OV and 2.5 cm in length. Line AX' is drawn parallel to OX. Line OB represents short-circuit current with normal voltage *i.e.* 52 A and measures 52/2 = 26 cm. AB represents output line. Perpendicular bisector of AB is drawn to locate the centre C of the circle. With C as centre and radius = CA, a circle is drawn which passes through points A and B. From point B, a perpendicular is drawn to the base. BD represents total input of 6,800 W for blocked rotor test. Out of this, ED represents no-load loss of 350 W and BE represents 6,800 – 350 = 6,450 W. Now BD = 9.8 cm and represents 6,800 W

∴ power scale = 6,800/9.8 = 700 watt/cm or 1 cm = **700 W**

BE which represents total copper loss in rotor and stator, is bisected at point T to separate the two losses. AT represents torque line.

Now, motor output = 3,730 watt. It will be represented by a line = 3,730/700 = 5.33 cm

The output point P on the circle is located thus :

DB is extended and BR is cut = 5.33 cm. Line RP is drawn parallel to output line AB and cuts the circle at point P. Perpendicular PS is drawn and P is joined to origin O.

Point M corresponding to maximum torque is obtained thus :

From centre C, a line CM is drawn such that it is perpendicular to torque line AT. It cuts the circle at M which is the required point. Point M could also have been located by drawing a line parallel to the torque line. MK is drawn vertical and it represents maximum torque.

Now, in the circle diagram, OP = line current on full-load = 7.6 cm. Hence, OP represents 7.6 × 2 = 15.2 A

$$\text{Power factor on full-load} = \frac{SP}{OP} = \frac{6.45}{7.6} = 0.86$$

$$\frac{\text{Max. torque}}{\text{F.L. torque}} = \frac{MK}{PG} = \frac{10}{5.6} = 1.8$$

∴ Max. torque = 180% of full-load torque.

Example. 35.5. *Draw the circle diagram from no-load and short-circuit test of a 3-phase. 14.92 k W, 400-V, 6-pole induction motor from the following test results (line values).*

No-load	: 400-V,	11 A,	p.f. = 0.2
Short-circuit	: 100-V,	25 A,	p.f. = 0.4

Rotor Cu loss at standstill is half the total Cu loss.

From the diagram, find (a) line current, slip, efficiency and p.f. at full-load (b) the maximum torque. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. No-load p.f. = 0.2; $\phi_0 = \cos^{-1}(0.2) = 78.5°$

Short-circuit p.f. = 0.4: $\phi_s = \cos^{-1}(0.4) = 66.4°$

S.C. current I_{SN} if normal voltage were applied = 25 (400/100) = 100 A

S.C. power input with this current = $\sqrt{3} \times 400 \times 100 \times 0.4$ = 27,710 W

Assume a current scale of 1 cm = 5 A.* The circle diagram of Fig. 35.13 is constructed as follows :

(i) No-load current vector OO' represents 11 A. Hence, it measures 11/5 = 2.2 cm and is drawn at an angle of 78.5º with OY.

(ii) Vector OA represents 100 A and measures 100/5 = 20 cm. It is drawn at an angle of 66.4º with OY.

(iii) $O'D$ is drawn parallel to OX. NC is the right angle bisector of $O'A$.

(iv) With C as the centre and CO' as radius, a semicircle is drawn as shown.

(v) AF represents power input on short-circuit with normal voltage applied. It measures 8 cm and (as calculated above) represents 27,710 W. Hence, power scale becomes

$$1 \text{ cm} = 27{,}710/8 = 3{,}465 \text{ W}$$

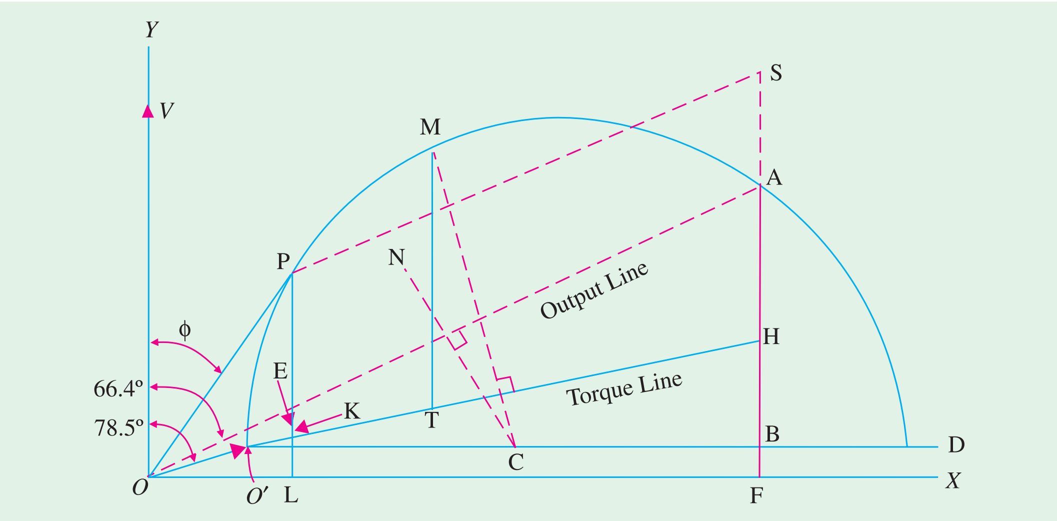


Fig. 35.13

(a) F.L motor output = 14,920 W. According to the above power scale, the intercept between the semicircle and the output line $O'A$ should measure = 14,920/3,465 = 4.31 cm. Hence, vertical line PL is found which measures 4.31 cm. Point P represents the full-load operating point.**

(a) Line current = OP = 6.5 cm which means that full-load line current = 6.5 × 5 = **32.5 A.** ϕ = 32.9º (by measurement)

$\therefore$ cos 32.9º = **0.84** (or $\cos\phi = PL/OP = 5.4/6.5 = 0.84$)

$$\text{slip} = \frac{EK}{PK} = \frac{0.3}{5.35} = 0.056 \quad \text{or} \quad \mathbf{5.6\%} \; ; \quad \eta = \frac{PE}{PL} = \frac{4.3}{5.4} = 0.8 \quad \text{or} \quad \mathbf{80\%}$$

(b) For finding maximum torque, line CM is drawn ⊥ to torque line $O'H$. MT is the vertical intercept between the semicircle and the torque line and represents the maximum torque of the motor in synchronous watts

Now, MT = 7.8 cm (by measurement) $\therefore$ T_{max} = 7.8 × 3465 = **27,030 synch. watt**

Example 35.6. *A 415-V, 29.84 kW, 50-Hz, delta-connected motor gave the following test data :*

No-load test	*: 415 V,*	*21 A,*	*1,250 W*
Locked rotor test	*: 100 V,*	*45 A,*	*2,730 W*

Construct the circle diagram and determine

* The actual scale of the book diagram is different because it has been reduced during block making.

** The operating point may also be found by making AS = 4.31 cm and drawing SP parallel to $O'A$.

(*a*) *the line current and power factor for rated output* (*b*) *the maximum torque.*
Assume stator and rotor Cu losses equal at standstill. **(A.C. Machines-I, Jadavpur Univ. 1990)**

Solution. Power factor on no-load is $= \dfrac{1250}{\sqrt{3} \times 415 \times 21} = 0.0918$

$\therefore \quad \phi_0 = \cos^{-1}(0.0918) = 84°44'$

Power factor with locked rotor is $= \dfrac{2,730}{\sqrt{3} \times 100 \times 45} = 0.3503$

$\therefore \quad \phi_S = \cos^{-1}(0.3503) = 69°30'$

The input current I_{SN} on short-circuit if normal voltage were applied = 45 (415/100) = 186.75 A and power taken would be = 2,730 $(415/100)^2$ = 47,000 W.

Let the current scale be 1 cm = 10 A. The circle diagram of Fig. 35.14 is constructed as follows :

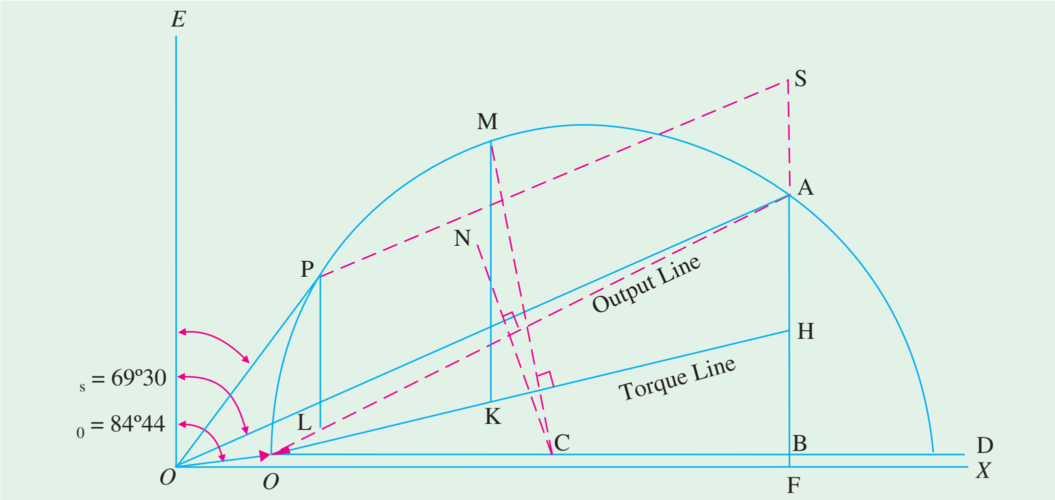


Fig. 35.14

(*i*) Vector OO' represents 21 A so that it measures 2.1 cm and is laid at an angle of 84°44′ with OE (which is vertical *i.e.* along Y-axis).

(*ii*) Vector OA measures 186.75/10 = 18.675 cm and is drawn at an angle of 69°30′ with OE.

(*iii*) $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$

(*iv*) With C as the centre and CO' as radius, a semi-circle is drawn as shown. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit.

(*v*) The vertical AF represents power input on short-circuit with normal voltage applied. AF measures 6.6 cm and (as calculated above) represents 47,000 W. Hence, power scale becomes, 1 cm = 47,000/6.6 = 7,120 W

(*a*) Full-load output = 29,840 W. According to the above power scale, the intercept between the semicircle and output line $O'A$ should measure 29,840/7,120 = 4.19 cm. Hence, line PL is found which measures 4.19 cm. Point P represents the full-load operating point.*

Phase current $= OP = 6$ cm $= 6 \times 10 = 60$ A; Line current $= \sqrt{3} \times 60$ = **104 A**

Power factor $= \cos \angle POE = \cos 35° =$ **0.819**

(*b*) For finding the maximum torque, line CM is drawn ⊥ to the torque line $O'H$. Point H is such that

$$\frac{AH}{BH} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

* The operating point may also by found be making AS = 4.19 cm and drawing SP parallel to $O'A$.

Since the two Cu losses are equal, point *H* is the mid-point of *A B*.

Line *MK* represents the maximum torque of the motor in synchronous watts

$$MK = 7.3 \text{ cm (by measurement)} = 7.3 \times 7{,}120 = \mathbf{51{,}980 \text{ synch. watt.}}$$

Example 35.7. *Draw the circle diagram for a 5.6 kW, 400-V, 3-φ, 4-pole, 50-Hz, slip-ring induction motor from the following data :*

No-load readings : 400 V, 6 A, $\cos \phi_0 = 0.087$ *: Short-circuit test : 100 V, 12 A, 720 W.*

The ratio of primary to secondary turns = 2.62, stator resistance per phase is 0.67 Ω and of the rotor is 0.185 Ω. Calculate

(i) *full-load current* **(ii)** *full-load slip*

(iii) *full-load power factor*

(iv) $\dfrac{\text{maximum torque}}{\text{full-load torque}}$ **(v)** *maximum power.*

Solution. No-load condition

$$\phi_0 = \cos^{-1}(0.087) = 85°$$

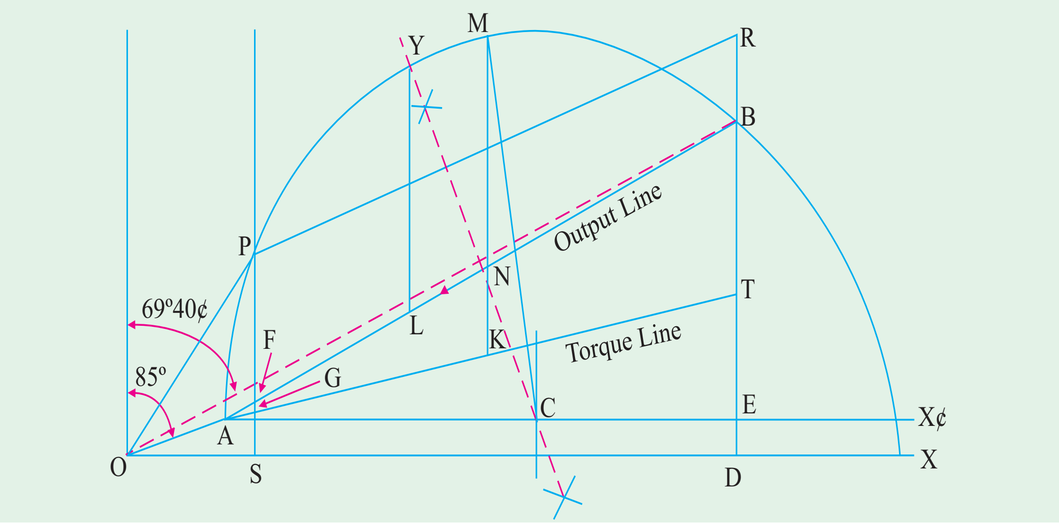


Fig. 35.15

Short-circuit condition

Short-circuit current with normal voltage = 12 × 400/100 = 48 A

$$\text{Total input} = 720 \times (48/12)^2 = 11.52 \text{ kW}$$

$$\cos \phi_s = \frac{720}{\sqrt{3} \times 100 \times 12} = 0.347 \quad \text{or} \quad \phi_s = 69° \, 40'$$

Current scale is, 1 cm = 2 A

In the circle diagram of Fig. 35.15, *OA* = 3 cm and inclined at 85° with *OV*. Line *OB* represents short-circuit current with normal voltage. It measures 48/2 = 24 cm and represent 48 A. *BD* is perpendicular to *OX*.

For Drawing Torque Line

$K = 2.62$ $\quad R_1 = 0.67\ \Omega$ $\quad R_2 = 0.185\ \Omega$

(in practice, an allowance of 10% is made for skin effect)

$$\therefore \quad \frac{\text{rotor Cu loss}}{\text{stator Cu loss}} = 2.62^2 \times \frac{0.185}{0.67} = 1.9 \qquad \therefore \quad \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{1.9}{2.9} = 0.655$$

Now $\quad BD = 8.25$ cm and represents 11.52 kW

$$\text{power scale} = 11.52/8.25 = 1.4 \text{ kW/cm}$$

$\therefore \quad 1 \text{ cm} = 1.4 \text{ kW}$

BE represents total Cu loss and is divided at point *T* in the ratio 1.9 : 1.

$$BT = BE \times 1.9/2.9 = 0.655 \times 8 = 5.24 \text{ cm}$$

AT is the torque line

$$\text{Full-load output} = 5.6 \text{ kW}$$

It is represented by a line = 5.6/1.4 =4 cm

DB is produced to *R* such that *BR* = 4 cm. Line *RP* is parallel to output line and cuts the circle at *P*. OP represents full-load current.

PS is drawn vertically. Points *M* and *Y* represent points of maximum torque and maximum output respectively.

(*i*) $\text{F.L. current} = OP = 5.75 \text{ cm} = 5.75 \times 2 =$ **11.5 A**

(*ii*) $\text{F.L. slip} = \dfrac{FG}{PG} = \dfrac{0.2}{4.25} = 0.047$ or **4.7%**

(*iii*) $\text{p.f.} = \dfrac{SP}{OP} = \dfrac{4.6}{5.75} =$ **0.8**

(*iv*) $\dfrac{\text{max. torque}}{\text{full-load torque}} = \dfrac{MK}{PG} = \dfrac{10.05}{4.25} =$ **2.37**

(*v*) Maximum output is represented by *Y L* = 7.75 cm.

$\therefore \quad \text{Max. output} = 7.75 \times 1.4 =$ **10.8 kW**

Example 35.8. *A 440-V, 3-ϕ, 4-pole, 50-Hz slip-ring motor gave the following test results :*

No-load reading : 440 V, 9 A, p.f. = 0.2

Blocked rotor test : 110 V, 22 A, p.f. = 0.3

The ratio of stator to rotor turns per phase is 3.5/1. The stator and rotor Cu losses are divided equally in the blocked rotor test. The full-load current is 20 A. Draw the circle diagram and obtain the following :

(*a*) *power factor, output power, efficiency and slip at full-load*

(*b*) *standstill torque or starting torque.*

(*c*) *resistance to be inserted in the rotor circuit for giving a starting torque 200 % of the full-load torque. Also, find the current and power factor under these conditions.*

Solution. No-load p.f. = 0.2 $\quad \therefore \quad \phi_0 = \cos^{-1}(0.2) = 78.5°$

Short-circuit p.f. = 0.3 $\quad \therefore \quad \phi_S = 72.5°$

Short-circuit current at normal voltage = 22 × 440/110 = 88 A

$$\text{S.C. input} = \sqrt{3} \times 440 \times 88 \times 0.3 = 20{,}120 \text{ W} = 20.12 \text{ kW}$$

Take a current scale of 1 cm = 4 A

In the circle diagram of Fig. 35.16, *OA* = 2.25 cm drawn at an angle of 78.5º behind *OV*. Similarly, *OB* = 88/4 = 22 cm and is drawn at an angle of 72.50 behind OV. The semi-circle is drawn as usual. Point *T* is such that *BT* = *TD*. Hence, torque line *AT* can be drawn. *BC* represents 20.12 kW. By measurement *BC* = 6.6 cm.

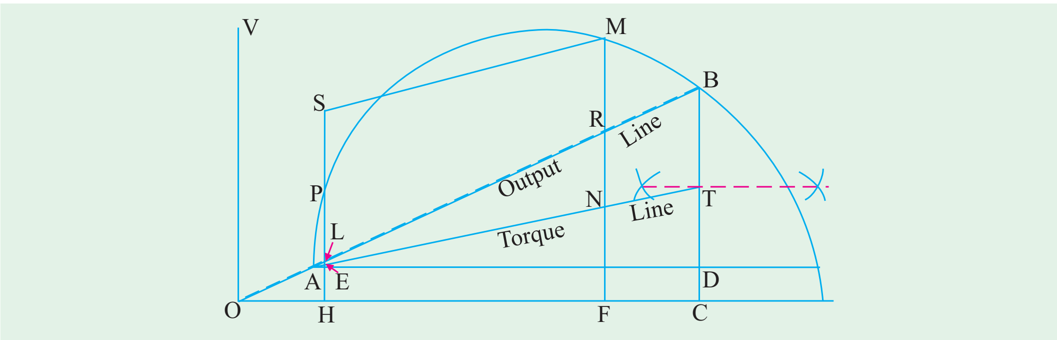


Fig. 35.16

$\therefore$ power scale $= 20.12/6.6 = 3.05$

$\therefore$ 1 cm $= 3.05$ kW

Full-load current = 20 A. Hence, it is represented by a length of 20/4 = 5 cm. With *O* as centre and 5 cm as radius, an arc is drawn which cuts the semi-circle at point *P*. This point represents full-load condition. *PH* is drawn perpendicular to the base *OC*.

(*a*) **(*i*)** p.f. = $\cos\phi = PH/OP = 4.05/5 =$ **0.81**

(*ii*) Torque can be found by measuring the input.

$$\text{Rotor input} = PE = 3.5 \text{ cm} = 3.5 \times 3.05 = 10.67 \text{ kW}$$

Now $N_s = 120 \times 50/4 = 1500$ r.p.m.

$\therefore$ $T_g = 9.55\, P_2/N_s = 9.55 \times 10{,}670/1500 =$ **61 N-m**

(*iii*) output $= PL = 3.35 \times 3.05 =$ **10.21 kW**

(*iv*) $$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{PL}{PH} = \frac{3.35}{4.05} = 0.83 \text{ or } \mathbf{83\%}$$

(*v*) $$\text{slip } s = \frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{LE}{PE} = \frac{0.1025}{3.5} = 0.03 \text{ or } \mathbf{3\%}$$

(*b*) Standstill torque is represented by *BT*.

$$BT = 3.1 \text{ cm} = 3.1 \times 3.05 = 9.45 \text{ kW} \qquad \therefore \quad T_{st} = 9.55 \times \frac{9.45 \times 10^3}{1500} = \mathbf{60.25\ N\text{-}m}$$

(*c*) We will now locate point *M* on the semi-circle which corresponds to a starting torque twice the full-load torque *i.e.* 200% of F.L. torque.

Full-load torque = *PE*. Produce *EP* to point *S* such that *PS* = *PE*. From point *S* draw a line parallel to torque line *AT* cutting the semi-circle at *M*. Draw *MN* perpendicular to the base.

At starting when rotor is stationary, *MN* represents total rotor copper losses.

NR = Cu loss in rotor itself as before ; *RM* = Cu loss in external resistance

RM = 4.5 cm = 4.5×3.05 = 13.716 kW = 13,716 watt.

Cu loss/phase = 13,716/3 =4,572 watt

Rotor current *AM* = 17.5 cm = 17.5×4 = 70 A

Let r_2' be the additional external resistance in the rotor circuit (as referred to stator) then

$$r_2' \times 70^2 = 4{,}572 \quad \text{or} \quad r_2' = 4{,}572/4{,}900 = 0.93\ \Omega$$

Now $K = 1/3.5$

$\therefore$ rotor resistance/phase, $r_2 = r_2' \times K^2 = 0.93/3.5^2 =$ **0.076 Ω**

Stator current = $OM = 19.6 \times 4 =$ **78.4 A** ; power factor $= \dfrac{MF}{OM} = \dfrac{9.75}{18.7} =$ **0.498**

Example 35.9. *Draw the circle diagram of a 7.46 kW, 200-V, 50-Hz, 3-phase slip-ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of 0.38 ohm/phase and a rotor resistance of 0.24 ohm/phase. The following are the test readings ;*

No-load : 200 V, 7.7 A, $\cos\phi_0 = 0.195$

Short-circuit : 100 V, 47.6 A, $\cos\phi_s = 0.454$

Find **(*a*)** *starting torque and* **(*b*)** *maximum torque, both in synchronous watts*

(*c*) *the maximum power factor* **(*d*)** *the slip for maximum torque*

(*e*) *the maximum output* **(Elect. Tech.-II, Madras Univ. 1989)**

Solution. $\phi_0 = \cos^{-1}(0.195) = 78°45'$; $\phi_S = \cos^{-1}(0.454) = 63°$

The short-circuit I_{SN} with normal voltage applied is $= 47.6 \times (200/100) = 95.2$ A

The circle diagram is drawn as usual and is shown in fig. 35.17.

With a current scale of 1 cm = 5 A, vector *OO′* measures 7.7/5 =1.54 cm and represents the no-load current of 7.7 A.

Similarly, vector OA represents I_{SN} *i.e.* short-circuit current with normal voltage and measures 95.2/5 = 19.04 cm

Both vectors are drawn at their respective angles with OE.

The vertical line AF measures the power input on short-circuit with normal voltage and is $= \sqrt{3} \times 200 \times 95.2 \times 0.454 = 14,970$ W.

Since AF measures 8.6 cm, the power scale is 1 cm = 14,970/8.6 = 1740 W

The point H is such that

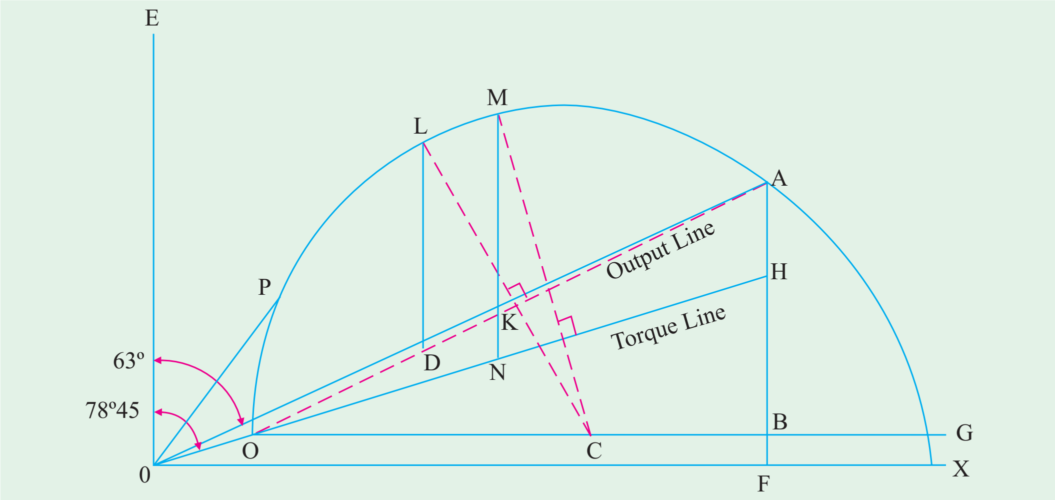


Fig. 35.17

$$\frac{AH}{AB} = \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{\text{rotor resistance}^*}{\text{rotor + stator resistance}} = \frac{0.24}{0.62}$$

Now AB = 8.2 cm (by measurement) $\therefore$ $AH = 8.2 \times 0.24/0.62 = 3.2$ cm

(a) Starting torque = AH = 3.2 cm = 3.2 × 1740 = **5,570 synch. watt.**

(b) Line CM is drawn perpendicular to the torque line $O'H$. The intercept MN represents the maximum torque in synchronous watts.

Maximum torque = MN = 7.15 cm = 7.15 × 1740 = **12,440 synch. watts.**

(c) For finding the maximum power, line OP is drawn tangential to the semi-circle.

$\angle POE$ = 28.5º

$\therefore$ maximum p.f. = cos 28.5º = **0.879**

(d) The slip for maximum torque is = KN/MN= 1.4/7.15 = **0.195**

(e) Line CL is drawn perpendicular to the output line $O'A$. From L is drawn the vertical line LD. It measures 5.9 cm and represents the maximum output.

$\therefore$ maximum output = 5.9 × 1740 = **10,270 W**

Tutorial Problems 35.1

1. A 300 h.p. (223.8 kW), 3000-V, 3-ϕ, induction motor has a magnetising current of 20 A at 0.10 p.f. and a short-circuit (or locked) current of 240 A at 0.25 p.f. Draw the circuit diagram, determine the p.f. at full-load and the maximum horse-power. **[0.85 p.f. 621 h.p. (463.27 kW)] (*I.E.E. London*)**

2. The following are test results for a 18.65 kW, 3-ϕ, 440.V slip-ring induction motor :

Light load : 440-V, 7.5 A, 1350 W (including 650 W friction loss).

* Because $K = 1$, otherwise it should be $R_2' = R_2/K^2$.

S.C. test : 100 V, 32 A, 1800 W

Draw the locus diagram of the stator current and hence obtain the current, p.f. and slip on full-load. On short-circuit, the rotor and stator copper losses are equal. **[30 A, 0.915, 0.035] (*London Univ*)**

3. Draw the circle diagram for 20 h.p. (14.92 kW), 440-V, 50-Hz, 3-ϕ induction motor from the following test figures (line values) :

No-load : 440 V, 10A, p.f. 0.2 Short-circuit : 200 V, 50 A, p.f. 0.4

From the diagram, estimate (*a*) the line current and p.f. at full-load (*b*) the maximum power developed (*c*) the starting torque. Assume the rotor and stator I^2R losses on short-circuit to be equal.

[(*a*) 28.1 A at 0.844 p.f. (*b*) 27.75 kW (*c*) 11.6 synchronous kW/phase] (*London Univ.*)

4. A 40 h.p. (29.84.kW), 440-V, 50-Hz, 3-phase induction motor gave the following test results

No. load : 440 V, 16 A, p.f. = 0.15 S.C. test : 100 V, 55 A, p.f. = 0.225

Ratio of rotor to stator losses on short-circuit = 0.9. Find the full-load current and p.f., the pull-out torque and the maximum output power developed.

[49 A at 0.88 p.f. ; 78.5 synch. kW or 2.575 times F.L. torque ; 701.2 kW] (*I.E.E. London*)

5. A 40 h.p. (29.84 kW), 50-Hz, 6-pole, 420-V, 3-ϕ, slip-ring induction motor furnished the following test figures :

No-load : 420 V, 18 A, p.f. = 0.15 S.C. test : 210 V, 140 A, p.f. = 0.25

The ratio of stator to rotor Cu losses on short-circuit was 7 : 6. Draw the circle diagram and find from it (*a*) the full-load current and power factor (*b*) the maximum torque and power developed.

[(*a*) 70 A at 0.885 p.f. (*b*) 89.7 kg.m ; 76.09 kW] (*I.E.E. London*)

6. A 500 h.p. (373 kW), 8-pole, 3-ϕ, 6,000-V, 50-Hz induction motor gives on test the following figures :

Running light at 6000 V, 14 A/phase, 20,000 W ; Short-circuit at 2000 V, 70 A/phase, 30,500 W

The resistance/phase of the star-connected stator winding is 1.1 Ω, ratio of transformation is 4 :1. Draw the circle diagram of this motor and calculate how much resistance must be connected in each phase of the rotor to make it yield full-load torque at starting. **[0.138 Ω] (*London Univ.*)**

7. A 3-phase induction motor has full-load output of 18.65 kW at 220 V, 720 r.p.m. The full-load p.f. is 0.83 and efficiency is 85%. When running light, the motor takes 5 A at 0.2 p.f. Draw the circle diagram and use it to determine the maximum torque which the motor can exert (*a*) in N-m (*b*) in terms of full-load torque and (*c*) in terms of the starting torque.

[(*a*) 268.7 N-m (*b*) 1.08 (*c*) 7.2 approx.] (*London Univ.*)

8. A 415-V, 40 h.p. (29.84 kW), 50.Hz, Δ-connected motor gave the following test data :

No-load test : 415 V, 21 A, 1250 W ; Locked rotor test : 100 V, 45 A, 2,730 W

Construct the circle diagram and determine

(*a*)the line current and power factor for rated output (*b*) the maximum torque. Assume stator and rotor Cu losses equal at standstill.

[(*a*) 104 A : 0.819 (*b*) 51,980 synch watt] (*A.C. Machines-I, Jadavpur Univ. 1978*)

9. Draw the no-load and short circuit diagram for a 14.92 kW., 400-V, 50-Hz, 3-phase star-connected induction motor from the following data (line values) :

No load test : 400 V, 9 A, cos ϕ = 0.2

Short circuit test : 200 V, 50 A, cos ϕ = 0.4

From the diagram find (*a*) the line current and power factor at full load, and (*b*) the maximum output power. **[(*a*) 32.0 A, 0.85 (*b*) 21.634 kW]**

35.9. Starting of Induction Motors

It has been shown earlier that a plain induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary. Therefore, if normal supply voltage is applied to the stationary motor, then, as in the case of a transformer, a very large initial current is taken by the primary, at least, for a short while. It would be remembered that exactly similar conditions exist in the case of a d.c. motor, if it is thrown directly across the supply lines, because at the time of starting it, there is no back e.m.f. to oppose the initial inrush of current.

Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque. This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines. Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

It was seen in Art. 34.15 that the starting torque of an induction motor can be improved by increasing the resistance of the rotor circuit. This is easily feasible in the case of slip-ring motors but not in the case of squirrel-cage motors. However, in their case, the initial in-rush of current is controlled by applying a reduced voltage to the stator during the starting period, full normal voltage being applied when the motor has run up to speed.

35.10. Direct-switching or Line starting of Induction Motors

It has been shown earlier that

Rotor input $= 2\pi N_s T = kT$ —Art. 34.36

Also, rotor Cu loss $= s \times$ rotor input

$\therefore \quad 3I_2^2R_2 = s \times kT \qquad \therefore \quad T \propto I_2^2/s$ (if R_2 is the same)

Now $\quad I_2 \propto I_1 \qquad \therefore \quad T \propto I_1^2/s$ or $T = KI_1^2/s$

At starting moment $\quad s = 1 \qquad \therefore \quad T_{st} = KI_{st}^2$ where I_{st} = starting current

If $\quad I_f$ = normal full-load current and s_f = full-load slip

then $T_f = KI_f^2/s_f \qquad \therefore \quad \dfrac{T_{st}}{T_f} = \left(\dfrac{I_{st}}{I_f}\right)^2 . s_f$

When motor is direct-switched onto normal voltage, then starting current is the short-circuit current I_{sc}.

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 . s_f = a^2 . s_f \quad \text{where } a = I_{sc}/I_f$$

Suppose in a case, $\quad I_{sc} = 7I_f$, $s_f = 4\% = 0.04$, the $T_{st}/T_f = 7^2 \times 0.04 = 1.96$

$\therefore$ starting torque $= 1.96 \times$ full-load torque

Hence, we find that with a current as great as seven times the full-load current, the motor develops a starting torque which is only 1.96 times the full-load value.

Some of the methods for starting induction motors are discussed below :

Squirrel-cage Motors

(*a*) Primary resistors (or rheostat) or reactors
(*b*) Auto-transformer (or autostarter)
(*c*) Star-delta switches

In all these methods, terminal voltage of the squirrel-cage motor is reduced during starting.

Slip-ring Motors

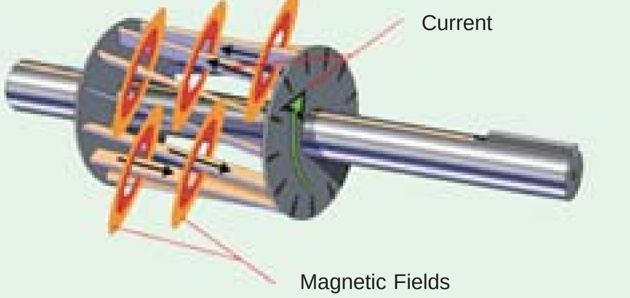
(*a*) Rotor rheostat

35.11. Squirrel-cage Motors

(*a*) Primary resistors

Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way, the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as the voltage, the torque varies as square of applied voltage*

* When applied voltage is reduced, the rotating flux Φ is reduced which, in turn, decreases rotor e.m.f. and hence rotor current I_2. Starting torque, which depends both on Φ and I_2 suffers on two counts when impressed voltage is reduced.



Squirrel cage rotor

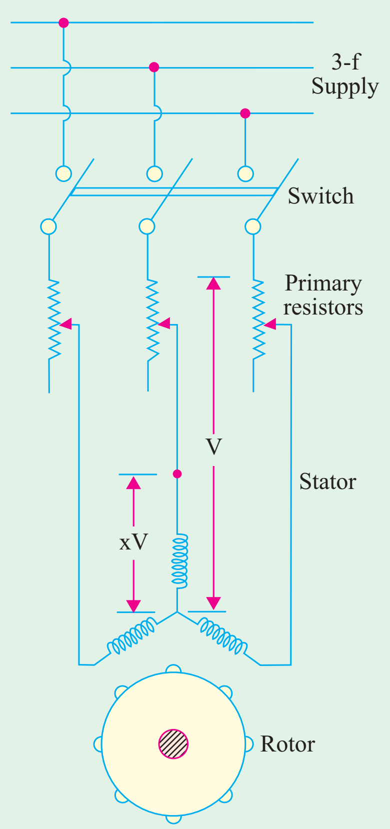


Fig. 35.18

(Art 34.17). If the voltage applied across the motor terminals is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

By using primary resistors (Fig. 35.18), the applied voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x\,I_{sc} \quad \text{and} \quad T_{st} = x^2\,T_{sc}$$

As seen from Art 35.10, above,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . s_f = \left(\frac{x\,I_{sc}}{I_f}\right)^2 s_f$$

$$= x^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f = x^2 . a^2 . s_f$$


Variable auto-transformer

It is obvious that the ratio of the starting torque to full-load torque is x^2 of that obtained with direct switching or across-the-line starting. This method is useful for the smooth starting of small machines only.

(*b*) Auto-transformers

Such starters, known variously as ***auto-starters or compensators***, consist of an auto-transformer, with necessary switches. We may use either two auto-transformers connected as usual [Fig. 35.19 (*b*)] or 3 auto-transformers connected in open delta [Fig. 35.19 (*a*)]. ***This method can be used both for star-and delta-connected motors***. As shown in Fig. 35.20 with starting connections, a reduced voltage is applied across the motor terminals. When the motor has ran up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor. The switch making these changes from 'start' to 'run' may be airbreak (for small motors) or may be oil-immersed (for large motors) to reduce sparking. There is also provision for no-voltage and over-load protection, along with a time-delay device, so that momentary interruption of voltage or momentary over-load do not disconnect the motor from supply line. Most of the auto-starters are provided with 3 sets of taps, so as to reduce voltage to 80, 65 or 50 per cent of the line voltage, to suit the local conditions of supply. The

V-connected auto-transformer is commonly used, because it is cheaper, although the currents are unbalanced during starting period. This is, however, not much objectionable firstly, because the current imbalance is about 15 per cent and secondly, because balance is restored as soon as running conditions are attained.

The quantitative relationships between the motor current, line current, and torque developed can be understood from Fig.35.20.

In Fig 35.20 (*a*) is shown the case when the motor is direct-switched to lines. The motor current is, say, 5 times the full-load current. If V is the line voltage, then voltage/phase across motor is $V/\sqrt{3}$.

$$\therefore \qquad I_{sc} = 5\,I_f = \frac{V}{\sqrt{3}Z} \quad \text{where } Z \text{ is stator impedance/phase.}$$

In the case of auto-transformer, if a tapping of transformation ratio K is used, then phase voltage across motor is $KV/\sqrt{3}$, as marked in Fig. 35.20 (*b*).

$$\therefore \quad \text{motor current at starting } I_2 = \frac{KV}{\sqrt{3}Z} = K \cdot \frac{V}{\sqrt{3}Z} = K \cdot I_{sc} = K \cdot 5\,I_f$$

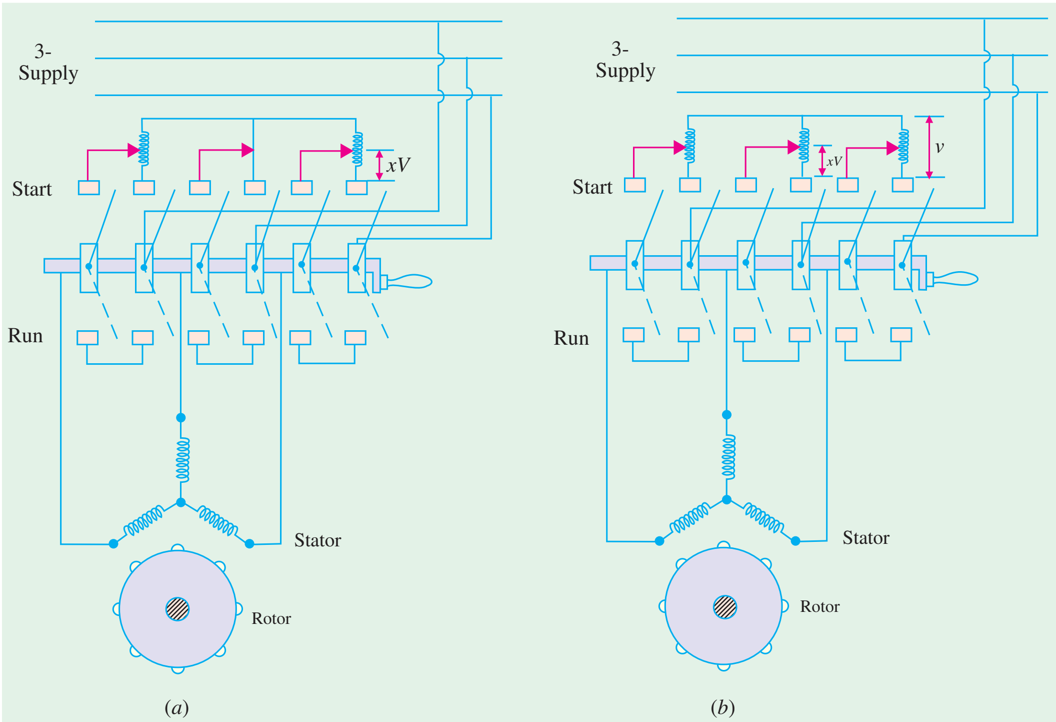


Fig. 35.19

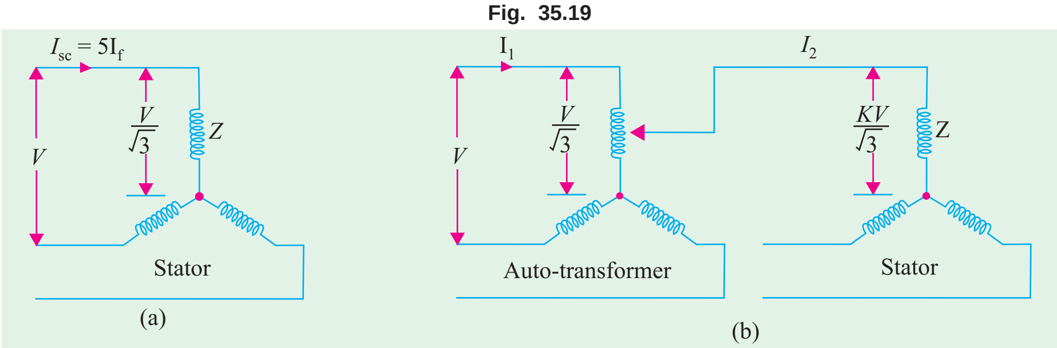


Fig. 35.20

The current taken from supply or by auto-transformer is $I_1 = K I_2 = K^2 \times 5 I_f = K^2 I_{sc}$ if magnetising current of the transformer is ignored. Hence, we find that although motor current per phase is reduced only K times the direct-switching current ($\because K < 1$), the current taken by the line is reduced K^2 times.

Now, remembering that torque is proportional to the square of the voltage, we get

With direct-switching, $T_1 \propto (V/\sqrt{3})^2$; With auto-transformer, $T_2 \propto (KV/\sqrt{3})^2$

$\therefore$ $T_2/T_1 = (KV/\sqrt{3})^2/(V/\sqrt{3})^2$ or $T_2 = K^2 T_1$ or $T_{st} = K^2 . T_{sc}$

$\therefore$ torque with auto-starter $= K^2 \times$ torque with direct-switching.

Relation Between Starting and F.L. Torque

It is seen that voltage across motor phase on direct-switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. With auto-starter, voltage across motor phase is $KV/\sqrt{3}$ and $I_{st} = K I_{sc}$

Now, $T_{st} \propto I_{st}^2 \; (s = 1)$ and $T_f \propto \dfrac{I_f^2}{s_f^2}$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f \quad \text{or} \quad \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = K^2 . a^2 . s_f \qquad (\because I_{st} = K I_{sc})$$

Note that this expression is similar to the one derived in Art. 34.11. (*a*) except that x has been replaced by transformation ratio K.

Example 35.10. *Find the percentage tapping required on an auto-transformer required for a squirrel-cage motor to start the motor against 1/4 of full-load torque. The short-circuit current on normal voltage is 4 times the full-load current and the full-load slip is 3%.*

Solution. $\dfrac{T_{st}}{T_f} = \dfrac{1}{4}, \quad \dfrac{I_{sc}}{I_f} = 4, \quad s_f = 0.03$

$\therefore$ Using $\dfrac{T_{st}}{T_f} = K^2 \left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$, we get $\dfrac{1}{4} = K^2 \times 4^2 \times 0.03$

$\therefore$ $K^2 = \dfrac{1}{64 \times 0.03}$ $\quad\therefore\quad K = 0.722$ or $K =$ **72.2%**

Example 35.11. *A 20 h.p. (14.92 kW), 400-V, 950 r.p.m., 3-φ, 50-Hz, 6-pole cage motor with 400 V applied takes 6 times full-load current at standstill and develops 1.8 times full-load running torque. The full-load current is 30 A.*

(*a*) *what voltage must be applied to produce full-load torque at starting ?*

(*b*) *what current will this voltage produce ?*

(*c*) *if the voltage is obtained by an auto-transformer, what will be the line current ?*

(*d*) *if starting current is limited to full-load current by an auto-transformer, what will be the starting torque as a percentage of full-load torque ?*

Ignore the magnetising current and stator impedance drops.

Solution. (*a*) Remembering that $T \propto V^2$, we have

In the first case, $1.8\, T_f \propto 400^2$, ; In the second case, $T_f \propto V^2$

$$\therefore \quad \left(\frac{V}{400}\right)^2 = \frac{1}{1.8} \quad \text{or} \quad V = \frac{400}{\sqrt{1.8}} = \mathbf{298.1\ V}$$

(*b*) Currents are proportional to the applied voltage.

$$\therefore \quad 6\, I_f \propto 400\,;\; I \propto 298.1 \quad \therefore \quad I = 6 \times \frac{298.1}{400}, I_f = \frac{6 \times 298.1 \times 30}{400} = \mathbf{134.2\ A}$$

(*c*) Here $K = 298.1/400$

Line current $= K^2 I_{sc} = (298.1/400)^2 \times 6 \times 30 =$ **100 A**

(*d*) We have seen in Art. 33.11 (*b*) that line current $= K^2 I_{sc}$

Now, line current = full-load current I_f (given)

$\therefore \quad 30 = K^2 \times 6 \times 30 \quad \therefore \quad K^2 = 1/6$

Now, using $\dfrac{T_{st}}{T_f} = K^2\left(\dfrac{I_{sc}}{I_f}\right)^2 \times s_f$ we get $\dfrac{T_{st}}{T_f} = \dfrac{1}{6}\times\left(\dfrac{6I_f}{I_f}\right)^2 \times 0.05 = 0.3$

Here $N_s = 120 \times 50/6 = 1000$ r.p.m. $N = 950$ r.p.m.; $s_f = 50/1000 = 0.05$

$\therefore \quad T_{st} = 0.3\ T_f$ or **30%** F.L. torque

Example 35.12. *Determine the suitable auto-transformation ratio for starting a 3-phase induction motor with line current not exceeding three times the full-load current. The short-circuit current is 5 times the full-load current and full-load slip is 5%.*

Estimate also the starting torque in terms of the full-load torque.

(Elect. Engg.II, Bombay Univ. 1987)

Solution. Supply line current $= K^2 I_{sc}$

It is given that supply line current at start equals $3\ I_f$ and short-circuit current $I_{sc} = 5\ I_f$ where I_f is the full-load current

$\therefore \quad 3\ I_f = K^2 \times 5\ I_f$ or $K^2 = 0.6 \quad \therefore \quad K = 0.775$ or **77.5%**

In the case of an auto starter,

$$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 \times s_f \quad \therefore \quad \frac{T_{st}}{T_f} = 0.6\times\left(\frac{5I_f}{I_f}\right)^2 \times 0.05 = 0.75$$

$\therefore \quad T_{st} = 0.75\ T_f =$ **75%** of full-load torque.

Example 35.13. *The full-load slip of a 400-V, 3-phase cage induction motor is 3.5% and with locked rotor, full-load current is circulated when 92 volt is applied between lines. Find necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque.*

(Elect. Machines, Banglore Univ. 1991)

Solution. Short-circuit current with full normal voltage applied is

$$I_{sc} = (400/92)\ I_f = (100/23)\ I_f$$

Supply *line* current $= I_{st} = 2\ I_f$

Now, line current $I_{st} = K^2 I_{sc}$

$\therefore \quad 2\ I_f = K^2 \times (100/23)\ I_f \qquad \therefore\ K^2 = 0.46\ ;\ K = 0.678$ or **67.8%**

Also, $\dfrac{T_{st}}{T_f} = K^2\left(\dfrac{I_{sc}}{I_f}\right)^2 \times s_f = 0.46 \times (100/23)^2 \times 0.035 = 0.304$

$\therefore \quad T_{st} =$ **30.4%** of full-load torque

Tutorial Problems 35.2

1. A 3-ϕ motor is designed to run at 5% slip on full-load. If motor draws 6 times the full-load current at starting at the rated voltage, estimate the ratio of starting torque to the full-load torque.
[1.8] (*Electrical Engineering Grad, I.E.T.E. Dec. 1986*)
2. A squirrel-cage induction motor has a short-circuit current of 4 times the full-load value and has a full-load slip of 5%. Determine a suitable auto-transformer ratio if the supply line current is not to

exceed twice the full-load current. Also, express the starting torque in terms of the full-load torque. Neglect magnetising current. **[70.7%, 0.4]**

3. A 3-ϕ, 400-V, 50-Hz induction motor takes 4 times the full-load current and develops twice the full-load torque when direct-switched to 400-V supply. Calculate in terms of full-load values (*a*) the line current, the motor current and starting torque when started by an auto-starter with 50% tap and (*b*) the voltage that has to be applied and the motor current, if it is desired to obtain full-load torque on starting. **[(*a*) 100%, 200%, 50% (*b*) 228 V, 282%]**

(*c*) Star-delta Starter

This method is used in the case of motors which are built to run normally with a delta-connected stator winding. It consists of a two-way switch which connects the motor in star for starting and then in delta for normal running. The usual connections are shown in Fig. 35.21. When star-connected, the applied voltage over each motor phase is reduced by a factor of $1/\sqrt{3}$ and hence the torque developed becomes 1/3 of that which would have been developed if motor were directly connected in delta. The line current is reduced to 1/3. Hence, during starting period when motor is Y-connected, it takes 1/3rd as much starting current and develops 1/3rd as much torque as would have been developed were it directly connected in delta.

Relation Between Starting and F.L. Torque

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} I_{sc} \text{ per phase}$$

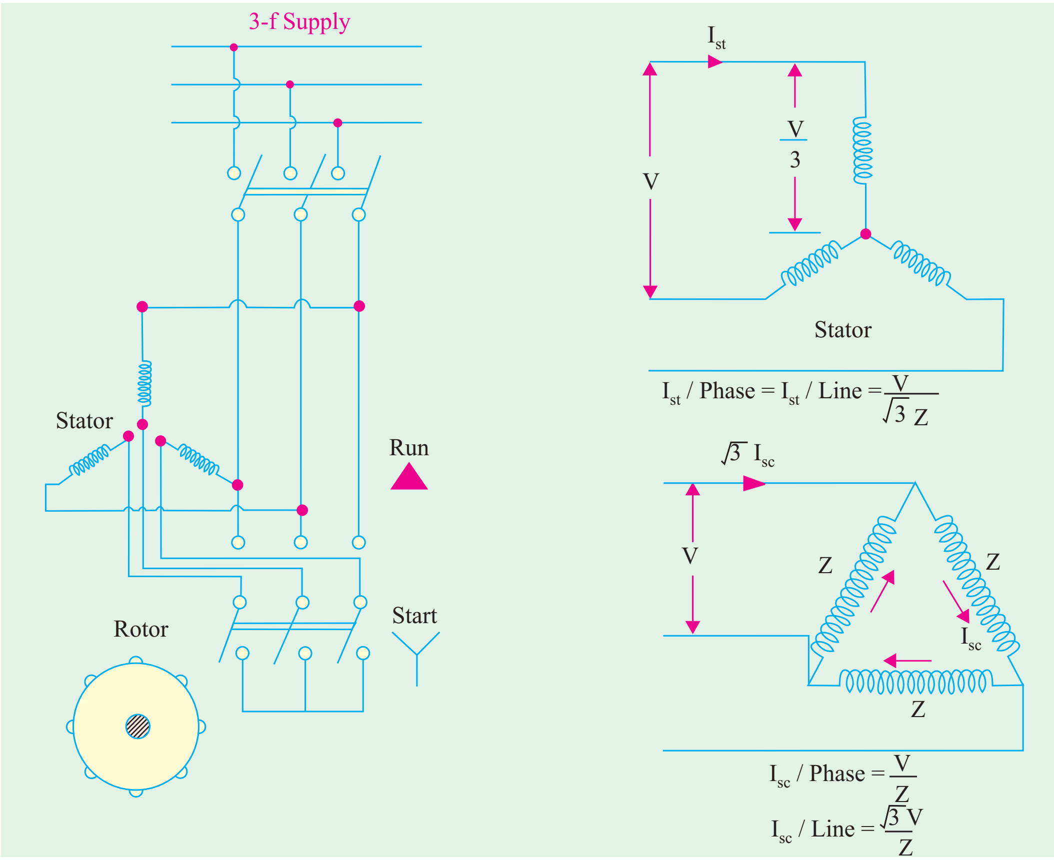


Fig. 35.21

where I_{sc} is the current/phase which Δ-connected motor would have taken if switched on to the supply directly (however, line current at start = 1/3 of line I_{sc})

Now $\quad T_{st} \propto I_{st}^2 \quad (s = 1)$ – Art. 35.10

$T_f \propto I_f^2/s_f$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{I_{sc}}{\sqrt{3}\, I_f}\right)^2 s_f = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} a^2 s_f$$

Here, I_{st} and I_{sc} represent phase values.

It is clear that the star-delta swith is equivalent* to an auto-transformer of ratio 1/$\sqrt{3}$ or 58% approximately.

This method is cheap and effective provided the starting torque is required not to be more than 1.5 times the full-load torque. Hence, it is used for machine tools, pumps and motor-generators etc.

Example 35.14. *The full-load efficiency and power factor of a 12-kW, 440-V, 3-phase induction motor are 85% and 0.8 lag respectively. The blocked rotor line current is 45 A at 220 V. Calculate the ratio of starting to full-load current, if the motor is provided with a star-delta starter. Neglect magnetising current.* **(Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. Blocked rotor current with full voltage applied

$$I_{sc} = 45 \times 440/220 = 90 \text{ A}$$

Now, $\quad \sqrt{3} \times 440 \times I_f \times 0.8 = 12{,}000/0.85, \quad \therefore \quad I_f = 23.1$ A

In star-delta starter, $\quad I_{st} = I_{sc}/\sqrt{3} = 90/\sqrt{3} = 52$ A

$\therefore \quad I_{st}/I_f = 52/23.1 =$ **2.256**

Example 35.15. *A 3-phase, 6-pole, 50-Hz induction motor takes 60 A at full-load speed of 940 r.p.m. and develops a torque of 150 N-m. The starting current at rated voltage is 300 A. What is the starting torque? If a star/delta starter is used, determine the starting torque and starting current.* **(Electrical Machinery-II, Mysore Univ. 1988)**

Solution. As seen from Art. 33.10, for direct-switching of induction motors

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f. \qquad \text{Here,} \quad I_{st} = I_{sc} = 300 \text{ A (line value)} ;\ I_f = 60 \text{ A (line value)},$$

$s_f = (1000 - 940)/1000 = 0.06 ;\ T_f = 150$ N-m

$\therefore \quad T_{st} = 150(300/60)^2 \times 0.06 =$ **225 N-m**

When star/delta starter is used

Starting current = 1/3 × starting current with direct starting = 300/3 = **100 A**

Starting torque = 225/3 = **75 N-m** – Art 35-11 (*c*)

Example 35.16. *Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of* **(*a*)** *a star-delta switch* **(*b*)** *an auto-transformer with 70.7 % tapping. The short-circuit current of the motor at normal voltage is 6 times the full-load current and the full-load slip is 4%. Neglect the magnetising current.*

(Electrotechnics, M.S. Univ. Baroda 1986)

Solution. (*a*) $\quad \dfrac{T_{st}}{T_f} = \dfrac{1}{3}\left(\dfrac{I_{sc}}{I_f}\right)^2 s_f = \dfrac{1}{3} \times 6^2 \times 0.04 = 0.48$

$\therefore \quad T_{st} = 0.48\, T_f \quad$ or **48% of F.L. value**

(*b*) Here $\quad K = 0.707 = 1/\sqrt{2} ;\ K^2 = 1/2$

* By comparing it with the expression given in Art. 35.11 (*b*)

Now, $$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{2}\times 6^2 \times 0.04 = 0.72$$

$\therefore$ $T_{st} = 0.72\ T_f$ or **72% of T_f**

Example 35.17. *A 15 h.p. (11.2 kW), 3-φ, 6-pole, 50-HZ, 400-V, Δ-connected induction motor runs at 960 r.p.m. on full-load. If it takes 86.4 A on direct starting, find the ratio of starting torque to full-load torque with a star-delta starter. Full-load efficiency and power factor are 88% and 0.85 respectively.*

Solution. Here, $I_{sc}/\text{phase} = 86.4/\sqrt{3}$ A

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}}\cdot I_{sc} \text{ per phase} = \frac{86.4}{\sqrt{3}\times\sqrt{3}} = 28.8 \text{ A}$$

Full-load input line current may be found from

$$\sqrt{3}\times 400\times I_L\times 0.85 = 11.2\times 10^3/0.88 \qquad \therefore \text{ Full-load } I_L = 21.59 \text{ A}$$

$$\text{F.L. } I_{ph} = 21.59/\sqrt{3} \text{ A}; \qquad I_f = 21.59/\sqrt{3} \text{ A per phase}$$

$$N_s = 120\times 50/6 = 1000 \text{ r.p.m.}, \qquad N = 950;\ s_f = 0.05$$

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{28.8\times\sqrt{3}}{21.59}\right)^2\times 0.05 \qquad \therefore\ T_{st} = 0.267\ T_f \text{ or } \textbf{26.7\% F.L. torque}$$

Example 35.18. *Find the ratio of starting to full-load current in a 10 kW (output), 400-V, 3-phase induction motor with star/delta starter, given that full-load p.f. is 0.85, the full-load efficiency is 0.88 and the blocked rotor current at 200 V is 40 A. Ignore magnetising current.*

(Electrical Engineering, Madras Univ. 1985)

Solution. F.L. line current drawn by the Δ-connected motor may be found from

$$\sqrt{3}\times 400\times I_L\times 0.85 = 10\times 1000/0.88 \qquad \therefore\ I_L = 19.3 \text{ A}$$

Now, with 200 V, the line value of S.C. current of the Δ-connected motor is 40 A. If full normal voltage were applied, the line value of S.C. current would be = 40 × (400/200) = 80 A.

$\therefore$ I_{sc} (line value) = 80 A ; I_{sc} (phase value) = $80/\sqrt{3}$ A

When connected in star across 400 V, the starting current per phase drawn by the motor stator during starting is

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}}\times I_{sc} \text{ per phase} = \frac{1}{\sqrt{3}}\times\frac{80}{\sqrt{3}} = \frac{80}{3} \text{ A} \qquad \text{–Art. 35.10}$$

Since during starting, motor is star-connected, I_{st} per phase = line value of I_{sc} = 80/3A

$$\therefore\ \frac{\text{line value of starting current}}{\text{line value of F.L. current}} = \frac{80/3}{19.3} = \textbf{1.38}$$

Example 35.19. *A 5 h.p. (3.73 kW), 400-V, 3-φ, 50-Hz cage motor has a full-load slip of 4.5%. The motor develops 250% of the rated torque and draws 650% of the rated current when thrown directly on the line. What would be the line current, motor current and the starting torque if the motor were started (i) be means of a star/delta starter and (ii) by connecting across 60% taps of a starting compensator.* **(Elect. Machines-II, Indore Univ. 1989)**

Solution. (*i*) Line current = (1/3) × 650 = **216.7%**

Motor being star-connected, line current is equal to phase current.

$\therefore$ motor current = 650/3 = **216.7%**

As shown earlier, starting torque developed for star-connection is one-third of that developed on direct switching with delta-connection $\therefore$ T_{st} = 250/3 = **83.3%**

(*ii*) Line current = $K^2\times I_{sc} = (60/100)^2\times 650$ = **234%**

$$\text{Motor current} = K \times I_{sc} = (60/100) \times 650 = \mathbf{390\%}$$

$$T_{st} = K^2 \times T_{sc} = (60/100)^2 \times 250 = \mathbf{90\%}$$

Example. 35.20. *A squirrel-cage type induction motor when started by means of a star/delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculated the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed.* **(Utilization of Elect. Power, A.M.I.E. 1987)**

Solution. With star-delta starter, $\dfrac{T_{st}}{T_f} = \dfrac{1}{3}\left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$

Line current on line-start $I_{sc} = 3 \times 180\%$ of $I_f = 3 \times 1.8\, I_f = 5.4\, I_f$

Now, $T_{st}/T_f = 0.35$ (given) ; $I_{sc}/I_f = 5.4$

$\therefore$ $0.35 = (1/3) \times 5.4^2\, s_f$ or $5.4^2\, s_f = 1.05$

Autostarter : Here, $K = 0.75$

Line starting current $= K^2\, I_{sc} = (0.75)^2 \times 5.4\, I_f =$ **3.04 I_f = 304% of F.L. current**

$$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f\,;\quad \frac{T_{st}}{T_f} = (0.75)^2 \times 5.4^2\, s_f$$

$$= (0.75)^2 \times 1.05 = 0.59$$

$T_{st} = 0.59\, T_f =$ **59% F.L. torque**

Example. 35.21. *A 10 h.p. (7.46 kW) motor when started at normal voltage with a star-delta switch in the star position is found to take an initial current of 1.7 × full-load current and gave an initial starting torque of 35% of full-load torque. Explain what happens when the motor is started under the following conditions : (a) an auto-transformer giving 60% of normal voltage (b) a resistance in series with the stator reducing the voltage to 60% of the normal and calculate in each case the value of starting current and torque in terms of the corresponding quantities at full-load.*

(Elect. Machinery-III, Kerala Univ. 1987)

Solution. If the motor were connected in delta and direct-switched to the line, then it would take a line current three times that which it takes when star-connected.

$\therefore$ line current on line start or $I_{sc} = 3 \times 1.7\, I_f = 5.1\, I_f$

We know $\dfrac{T_{st}}{T_f} = \dfrac{1}{3}\left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$

Now $\dfrac{T_{st}}{T_f} = 0.35$ ...given ; $\dfrac{I_{sc}}{I_f} = 5.1$ ...calculated

$\therefore$ $0.35 = (1/3) \times 5.1^2 \times s_f$

We can find s_f from $5.1^2 \times s_f = 1.05$

(*a*) When it is started with an auto-starter, then $K = 0.6$

Line starting current $= K^2 \times I_{sc} = 0.6^2 \times 5.1\, I_f =$ **0.836 I_f**

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.6^2 \times 1.05 = 0.378$ $\therefore$ $T_{st} =$ **37.8%** of F.L. torque

(*b*) Here, voltage across motor is reduced to 60% of normal value. In this case *motor* current is the same as line current but it decreases in proportion to the decrease in voltage.

As voltage across motor = 0.6 of normal voltage

$\therefore$ line starting current $= 0.6 \times 5.1\, I_f =$ **3.06 I_f**

Torque at starting would be the same as before.

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.378$ $\therefore$ $T_{st} =$ **37.8%** of F.L. torque.

Tutorial Problems 35.3

1. A 3-phase induction motor whose full-load slip is 4 per cent, takes six times full-load current when switched directly on to the supply. Calculate the approximate starting torque in terms of the full-load torque when started by means of an auto-transformer starter, having a 70 percent voltage tap. **[0.7 T_f]**
2. A 3-phase, cage induction motor takes a starting current at normal voltage of 5 times the full-load value and its full-load slip is 4 per cent. What auto-transformer ratio would enable the motor to be started with not more than twice full-load current drawn from the supply ?

   What would be the starting torque under these conditions and how would it compare with that obtained by using a stator resistance starter under the same limitations of line current ? **[63.3% tap ; 0.4 T_f ; 0.16 T_f]**
3. A 3-phase, 4-pole, 50-Hz induction motor takes 40 A at a full-load speed of 1440 r.p.m. and develops a torque of 100 N-m at full-load. The starting current at rated voltage is 200 A. What is the starting torque ? If a star-delta starter is used, what is the starting torque and starting current ? Neglect magnetising current. **[100 N-m; 33.3 N-m; 66.7 A] (*Electrical Machines-IV, Bangalore Univ. Aug. 1978*)**
4. Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (*a*) a star-delta switch (*b*) an auto-transformer with 50% tapping. Ignore magnetising current. The short-circuit current of the motor at normal voltage is 5 times the full-load current and the full-load slip is 4 per cent. **[(*a*) 0.33 (*b*) 0.25] (*A.C. Machines, Madras Univ. 1976*)**
5. Find the ratio of starting to full-load current for a 7.46 kW, 400-V, 3-phase induction motor with star/delta starter, given that the full-load efficiency is 0.87, the full-load p.f. is 0.85 and the short-circuit current is 15 A at 100 V. **[1.37] (*Electric Machinery-II, Madras Univ. April 1978*)**
6. A four-pole, 3-phase, 50-Hz, induction motor has a starting current which is 5 times its full-load current when directly switched on. What will be the percentage reduction in starting torque if (*a*) star-delta switch is used for starting (*b*) auto-transformer with a 60 per cent tapping is used for starting ? **(*Electrical Technology-III, Gwalior Univ. Nov. 1917*)**
7. Explain how the performance of induction motor can be predicted by circle diagram. Draw the circle diagram for a 3-phase, mesh-connected, 22.38 kW, 500-V, 4-pole, 50-Hz induction motor. The data below give the measurements of line current, voltage and reading of two wattmeters connected to measure the input :

No load	500 V	8.3 A	2.85 kW	– 1.35 kW
Short circuit	100 V	32 A	– 0.75 kW	2.35 kW

From the diagram, find the line current, power factor, efficiency and the maximum output. **[83 A, 0.9, 88%, 50.73 kW] (*Electrical Machines-II, Vikram Univ. Ujjan 1977*)**

35.12. Starting of Slip-ring Motors

These motors are practically always started with full line voltage applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. The controlling resistance is in the form of a rheostat, connected in star (Fig. 35.22), the resistance being gradually cut-out of the rotor circuit, as the motor gathers speed. It has been already shown that by increasing the rotor resistance, not only is the rotor (and hence stator) current reduced at starting, but at the same time, the starting torque is also increased due to improvement in power factor.

Slip-ring electric motor

The controlling rheostat is either of stud or contactor type and

Rheostat

may be hand-operated or automatic. The starter unit usually includes a line switching contactor for the stator along with no-voltage (or low- voltage) and over-current protective devices. There is some form of interlocking to ensure proper sequential operation of the line contactor and the starter. This interlocking prevents the closing of stator contactor unless the starter is 'all in'.

As said earlier, the introduction of additional external resistance in the rotor circuit enables a slip-ring motor to develop a high starting torque with reasonably moderate starting current. Hence, such motors can be started under load. This additional resistance is for starting purpose only. It is gradually cut out as the motor comes up to speed.

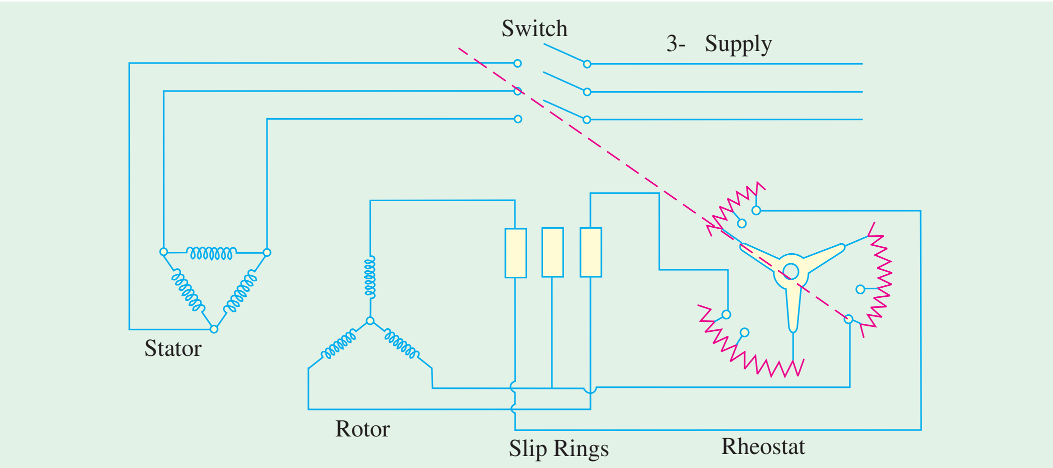


Fig. 35.22

The rings are, later on, short-circuited and brushes lifted from them when motor runs under normal conditions.

35.13. Starter Steps

Let it be assumed, as usually it is in the case of starters, that (*i*) the motor starts against a constant torque and (*ii*) that the rotor current fluctuates between fixed maximum and minimum values of I_{2max} and I_{2min} respectively.

In Fig. 35.23 is shown one phase of the 3-phase rheostat AB having n steps and the rotor circuit. Let R_1, R_2 ....etc. be the *total* resistances of the rotor circuit on the first, second step...etc. respectively. The resistances R_1, R_2 ..., etc. consist of rotor resistance per phase r_2 and the external resistances ρ_1, ρ_2 .... etc. Let the corresponding values of slips be s_1, s_2 ...etc. at stud No.1, 2...etc. At the commencement of each step, the current is I_{2max} and at the instant of leaving it, the current is I_{2min} . Let E_2 be the standstill e.m.f. induced in each phase of the rotor. When the handle touches first stud, the current rises to a maximum value I_{2max}, so that

$$I_{2max} = \frac{s_1 E_2}{\sqrt{[R_1^2 + (s_1 X_2)^2]}} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}}$$

where s_1 = slip at starting *i.e.* unity and X_2 = rotor reactance/phase

Then, *before* moving to stud No. 2, the current is reduced to I_{2min} and slip changes to s_2 such that

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}}$$

As we now move to stud No. 2, the speed momentarily remains the same, but current rises to I_{2max} because some resistance is cut out.

Hence, $$I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}}$$

After some time, the current is again reduced to I_{2min} and the slip changes to s_3 such that

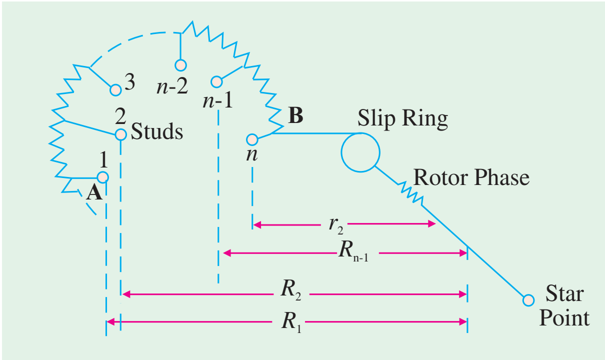


Fig. 35.23

$$I_{2min} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}}$$

As we next move over to stud No.3, again current rises to I_{2max} although speed remains momentarily the same.

$$\therefore \quad I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} \qquad \text{Similarly} \quad I_{2min} = \frac{E_2}{\sqrt{[(R_3/s_4)^2 + X_2^2]}}$$

At the last stud *i.e.* nth stud, $I_{2max} = \dfrac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$ where s_{max} = slip under normal running conditions, when external resistance is completely cut out.

It is found from above that

$$I_{2max} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}} = ... = \frac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$$

$$\text{or} \quad \frac{R_1}{s_1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = ... = \frac{R_{n-1}}{s_{n-1}} = \frac{R_n}{s_n} = \frac{r_2}{s_{max}} \qquad ...(i)$$

Similarly,

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} = ... = \frac{E_2}{\sqrt{[(R_{n-1}/s_{max})^2 + X_2^2]}}$$

$$\text{or} \quad \frac{R_1}{s_2} = \frac{R_2}{s_3} = \frac{R_3}{s_4} = ... = \frac{R_{n-1}}{s_{max}} \qquad ...(ii)$$

From ***(i)*** and ***(ii)***, we get

$$\frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = ... = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = ... = \frac{r_2}{R_{n-1}} = K \text{ (say)} \qquad ...(iii)$$

Now, from (*i*) it is seen that $R_1 = \dfrac{s_1 \times r_2}{s_{max}}$.

Now, $s_1 = 1$ at starting, when rotor is stationary.

$\therefore \quad R_1 = r_2/s_{max}$. Hence, R_1 becomes known in terms of rotor resistance/phase and normal slip. From ***(iii)***, we obtain

$$R_2 = KR_1 \,;\, R_3 = KR_2 = K^2R_1 \,;\, R_4 = KR_3 = K^3R_1 \text{ and } r_2 = KR_{n-1} = K^{n-1}\,.\,R_1$$

$$\text{or} \quad r_2 = K^{n-1}\,.\,\frac{r_2}{s_{max}} \qquad \text{(putting the value of } R_1\text{)}$$

$K = (s_{max})^{1/n-1}$ where n is the number of starter studs.

The resistances of various sections can be found as given below :

$\rho_1 = R_1 - R_2 = R_1 - KR_1 = (1 - K)\, R_1\ ;\ \rho_2 = R_2 - R_3 = KR_1 - K^2R_1 = K\rho_1$

$\rho_3 = R_3 - R_4 = K^2\, \rho_1$ etc.

Hence, it is seen from above that if s_{max} is known for the assumed value I_{2max} of the starting current, then n can be calculated.

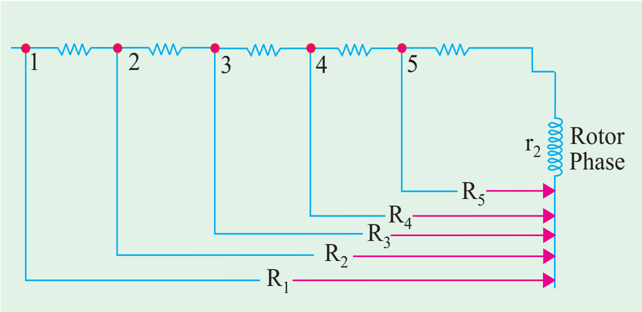


Fig. 35.24

Example 35.22. *Calculate the steps in a 5-step rotor resistance starter for a 3 phase induction motor. The slip at the maximum starting current is 2% with slip-ring short-circuited and the resistance per rotor phase is 0.02 Ω.*

Solution. Here, s_{max} = 2%

= 0.02; r_2 = 0.02 Ω, n = 6

R_1 = total resistance in rotor circuit/phase on first stud

= r_2/s_{max} = 0.02/0.02 = 1 Ω

Now, $K = (s_{max})^{1/n-1} = (0.02)^{1/5} = 0.4573$

$R_1 = 1\ \Omega;\ R_2 = KR_1 = 0.4573 \times 1 = 0.4573\ \Omega$

$R_3 = KR_2 = 0.4573 \times 0.4573 = 0.2091\ \Omega;\ R_4 = KR_3 = 0.4573 \times 0.2091 = 0.0956\ \Omega$

$R_5 = KR_4 = 0.4573 \times 0.0956 = 0.0437\ \Omega;\ r_2 = KR_5 = 0.4573 \times 0.0437 = 0.02\ \Omega$ (as given)

The resistances of various starter sections are as found below :

$\rho_1 = R_1 - R_2 = 1 - 0.4573 =$ **0.5427 Ω**; $\rho_2 = R_2 - R_3 = 0.4573 - 0.2091 =$ **0.2482 Ω**

$\rho_3 = R_3 - R_4 = 0.2091 - 0.0956 =$ **0.1135 Ω**; $\rho_4 = R_4 - R_5 = 0.0956 - 0.0437 =$ **0.0519 Ω**

$\rho_5 = R_5 - r_2 = 0.0437 - 0.02 =$ **0.0237 Ω**

The resistances of various sections are shown in Fig. 35.24.

35.14. Crawling

It has been found that induction motors, particularly the squirrel-cage type, sometimes exhibit a tendency to run stably at speeds as low as one-seventh of their synchronous speed N_s. This phenomenon is known as crawling of an induction motor.

This action is due to the fact that the a.c. winding of the stator produces a flux wave, which is not a pure sine wave. It is a complex wave consisting of a fundamental wave, which revolves synchronously and odd harmonics like 3rd, 5th, and 7th etc. which rotate either in the forward or backward direction at N_s / 3, N_s / 5 and N_s / 7 speeds respectively. As a result, in addition to the fundamental torque, harmonic torques are also developed, whose synchronous speeds are 1/nth of the speed for the fundamental torque *i.e.* N_s / n, where n is the order of the harmonic torque. Since 3rd harmonic currents are absent in a balanced 3-phase system, they produce no rotating field and, therefore, no torque. Hence, total motor torque has three components : (*i*) the fundamental torque, rotating with the synchronous speed N_s (*ii*) 5th harmonic torque* rotating at N_s / 5 speed and (*iii*) 7th harmonic torque, having a speed of N_s / 7.

* The magnitude of the harmonic torques is $1/n^2$ of the fundamental torque.

Now, the 5th harmonic currents have a phase difference of $5 \times 120° = 600° = -120°$ in three stator windings. The revolving field, set up by them, rotates in the *reverse* direction at $N_s / 5$. The forward speed of the rotor corresponds to a slip greater than 100%. The small amount of 5th harmonic reverse torque produces a braking action and may be neglected.

The 7th harmonic currents in the three stator windings have a phase difference of $7 \times 120° = 2 \times 360° + 120° = 120°$. They set up a forward rotating field, with a synchronous speed equal to 1/7th of the synchronous speed of the fundamental torque.

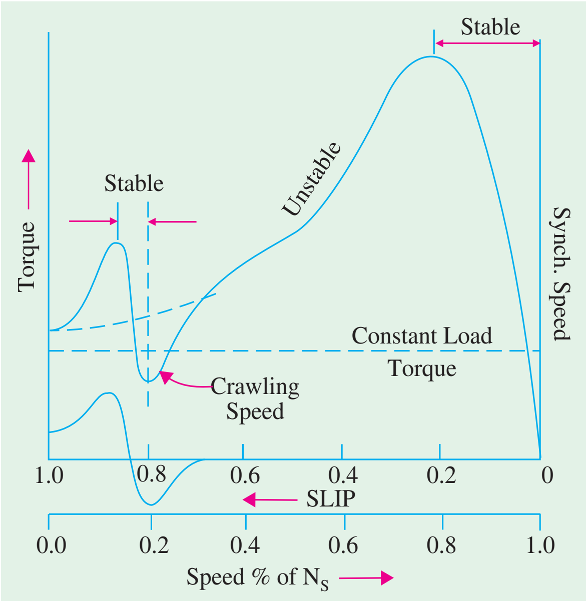
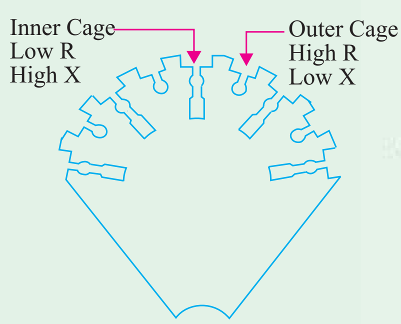


Fig. 35.25

If we neglect all higher harmonics, the resultant torque can be taken as equal to the sum of the fundamental torque and the 7th harmonic torque, as shown in Fig. 35.25. It is seen that the 7th harmonic torque reaches its maximum positive value just before 1/7th synchronous speed N_s, beyond which it becomes negative in value. Consequently, the resultant torque characteristic shows a dip which may become very pronounced with certain slot combinations. If the mechanical load on the shaft involves a constant load torque, it is possible that the torque developed by the motor may fall below this load torque. When this happens, the motor will not accelerate upto its normal speed but will remain running at a speed, which is nearly 1/7th of its full-speed. This is referred to as crawling of the motor.

35.15. Cogging or Magnetic Locking



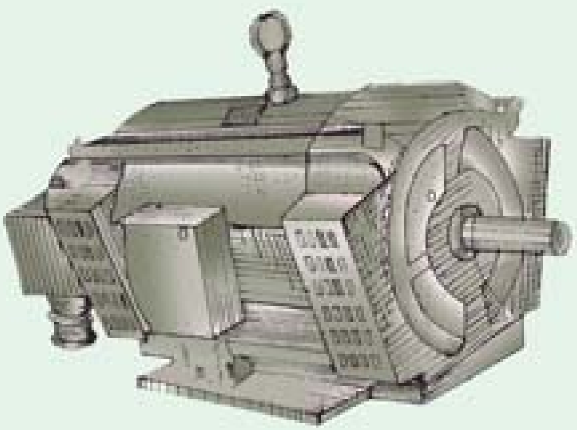


Fig. 35.26

Fig. 35.27

Double-cage, 30-kW, 400/440-V, 3-f, 960 r.p.m. squirrel-cage motor. (*Courtesy : Jyoti Ltd., Baroda*)

The rotor of a squirrel-cage motor sometimes refuses to start at all, particularly when the voltage is low. This happens when the number of stator teeth S_1 is equal to the number of rotor teeth S_2 and is due to the magnetic locking between the stator and rotor teeth. That is why this phenomenon is sometimes referred to as teeth-locking.

It is found that the reluctance of the magnetic path is minimum when the stator and rotor teeth face each other rather than when the teeth of one element are opposite to the slots on the other. It is in such positions of minimum reluctance, that the rotor tends to remain fixed and thus cause serious trouble during starting. Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

35.16. Double Squirrel Cage Motor

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance. The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions (because there will be more rotor Cu losses). The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period. Many efforts have been made to build a squirrel-cage motor which should have a high starting torque without sacrificing its electrical efficiency, under normal running conditions. The result is a motor, due to Boucheort, which has two independent cages on the same rotor, one inside the other. A punching for such a double cage rotor is shown in Fig. 35.26.

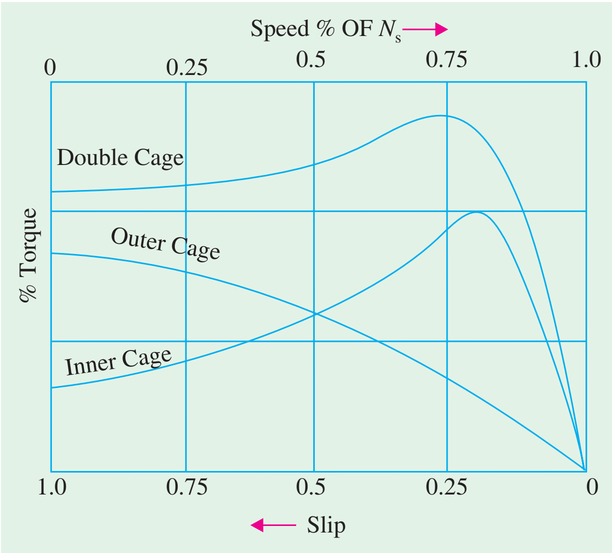


Fig. 35.28

The ***outer*** cage consists of bars of a ***high-resistance*** metal, whereas the inner cage has low-resistance copper bars.

Hence, outer cage has high resistance and low ratio of reactance-to-resistance, whereas the inner cage has low resistance but, being situated deep in the rotor, has a large ratio of reactance-to-resistance. Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

As said earlier, at starting and at large slip values, frequency of induced e.m.f in the rotor is high. So the reactance of the inner cage (= $2\pi f L$) and therefore, its impedance are both high. Hence, very little current flows in it. Most of the starting current is confined to outer cage, despite its high resistance. Hence, the motor develops a high starting torque due to high-resistance outer cage. Double squirrel-cage motor is shown in Fig. 35.27.

As the speed increases, the frequency of the rotor e.m.f. decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions. Most of the current then flows through it and hence it develops the greater part of the motor torque.

In fact, when speed is normal, frequency of rotor e.m.f. is so small that the reactance of both cages is practically negligible. The current is carried by two cages in parallel, giving a low combined resistance.

Hence, it has been made possible to construct a single machine, which has a good starting torque with reasonable starting current and which maintains high efficiency and good speed regulation, under normal operating conditions.

The torque-speed characteristic of a double cage motor may be approximately taken to be the sum of two motors, one having a high-resistance rotor and the other a low-resistance one (Fig. 35.28).

Such motors are particularly useful where frequent starting under heavy loads is required.

35.17. Equivalent Circuit

The two rotor cages can be considered in parallel, if it is assumed that both cages completely link the main flux. The equivalent circuit for one phase of the rotor, as referred to stator, is shown in Fig. 35.29. If the magnetising current is neglected, then the figure is simplified to that shown in Fig. 35.30. Hence, R_0'/s and R_i'/s are resistances of outer and inner rotors as referred to stator respectively and X_0' and X_i' their reactances

Total impedance as referred to primary is given by

$$Z_{01} = R_1 + jX_1 + \frac{1}{1/Z_1' + 1/Z_0'} = R_1 + jX_1 + \frac{Z_i' Z_0'}{Z_0' + Z_i'}$$

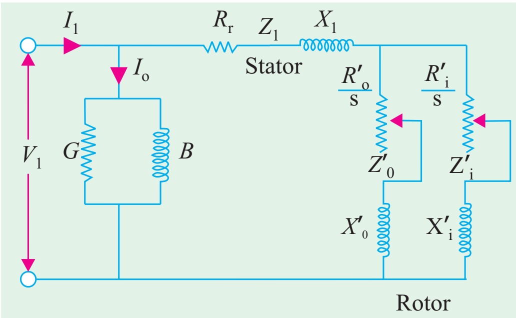


Fig. 35.29

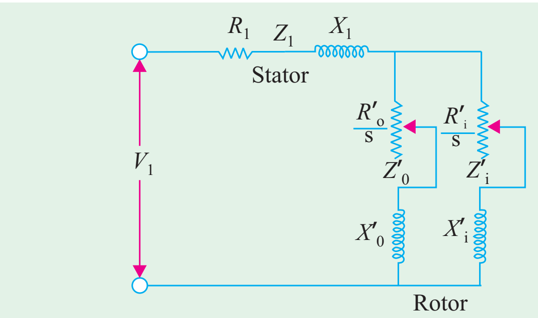


Fig. 35.30

Example 35.23. *A double-cage induction motor has the following equivalent circuit parameters, all of which are phase values referred to the primary :*

Primary	$R_1 = 1\ \Omega$	$X_1 = 3\ \Omega$
Outer cage	$R_0' = 3\ \Omega$	$X_0' = 1\ \Omega$
Inner cage	$R_i' = 0.6\ \Omega$	$X_i' = 5\ \Omega$

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4%. The magnetising current may be neglected.

Solution.

Refer to Fig. 35.31

(*i*) At start, s = 1

$$\mathbf{Z_{01}} = 1 + j3 + \frac{1}{1/(3 + j1) + 1/(0.6 + j5)}$$

$$= 2.68 + j\,4.538\ \Omega$$

$$\text{Current / phase} = 440 / (2.68^2 + 4.538^2)^{1/2} = 83.43\ \text{A}$$

$$\text{Torque} = 3 \times 83.43^2 \times (2.68 - 1)$$

$$= \mathbf{35{,}000\ synch\ watt.}$$

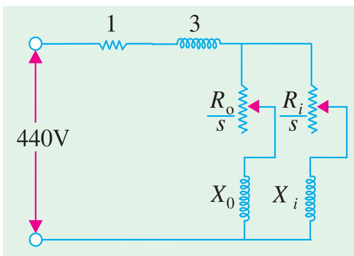


Fig. 35.31

(*ii*) when *s* = 4 per cent

In this case, approximate torque may be found by neglecting the outer cage impedance altogether. However, it does carry some current, which is almost entirely determined by its resistance.

$$\mathbf{Z_{01}} = 1 + j\,3 + \frac{1}{1/(3/0.04) + 1/(0.6/0.04 + j\,5)} = 13.65 + j\,6.45$$

$$\text{Current/phase} = 440/(13.65^2 + 6.45^2)^{1/2} = 29.14 \text{ A}$$

$$\text{Torque} = 3 \times 29.14^2 \times (13.65 - 1) = \mathbf{32{,}000 \text{ synch watt.}}$$

Example 35.24. *At standstill, the equivalent impedance of inner and outer cages of a double-cage rotor are* (0.4 + *j*2) Ω *and* (2+ *j*0.4) Ω *respectively. Calculate the ratio of torques produced by the two cages* (*i*) *at standstill* (*ii*) *at 5% slip.* **(Elect. Machines-II, Punjab Univ. 1989)**

Solution. The equivalent circuit for one phase is shown in Fig. 35.32.

(*i*) At standstill, *s* = 1

Impedance of inner cage $= Z_i = \sqrt{0.4^2 + 2^2} = 2.04\ \Omega$

Impedance of outer cage $= Z_0 = \sqrt{2^2 + 0.4^2} = 2.04\ \Omega$

If I_0 and I_i are the current inputs of the two cages, then

power input of inner cage, $P_i = I_i^2 R_i = 0.4\, I_i^2$ watt

power input of outer cage, $P_0 = I_0^2 R_0 = 2\, I_0^2$

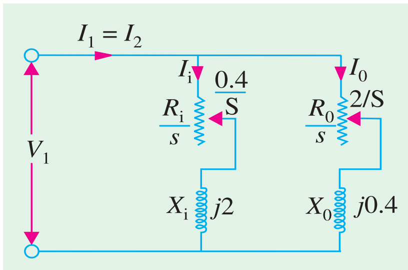


Fig. 35.32

$$\therefore \quad \frac{\text{torque of outer cage, } T_0}{\text{torque of inner cage, } T_i} = \frac{P_0}{P_i} = \frac{2\,I_0^2}{0.4\,I_i^2} = 5\left(\frac{I_0}{I_i}\right)^2$$

$$= 5\left(\frac{Z_i}{Z_0}\right)^2 = 5\left(\frac{2.04}{2.04}\right)^2 = 5 \qquad \therefore \quad T_0 : T_i :: 5 : 1$$

(*ii*) When $\quad s = 0.05$

$$Z_0 = \sqrt{[(R_0/s)^2 + X_0^2]} = \sqrt{(2/0.05)^2 + 0.4^2} = 40\ \Omega$$

$$Z_i = \sqrt{[(R_i/s)^2 + X_i^2]} = \sqrt{(0.4/0.05)^2 + 2^2} = 8.25\ \Omega$$

$$\therefore \quad \frac{I_o}{I_i} = \frac{Z_i}{Z_o} = \frac{8.25}{40} = 0.206$$

$$P_0 = I_0^2 R_0 / s = 40\, I_0^2;\ P_i = I_i^2 R_i / s = 8\, I_i^2$$

$$\therefore \quad \frac{T_o}{T_i} = \frac{P_o}{P_i} = \frac{40\,I_o^2}{8\,I_i^2} = 5\left(\frac{I_o}{I_i}\right)^2 = 5\,(0.206)^2 = 0.21$$

$\therefore$ $\mathbf{T_0 : T_i :: 0.21 : 1}$

It is seen from above, that the outer cage provides maximum torque at starting, whereas inner cage does so later.

Example 35.25. *A double-cage rotor has two independent cages. Ignoring mutual coupling between cages, estimate the torque in synchronous watts per phase* (*i*) *at standstill and at 5 per cent slip, given that the equivalent standstill impedance of the inner cage is* (0.05 + *j* 0.4) *ohm per phase and of the outer cage* (0.5 + *j* 0.1) *ohm per phase and that the rotor equivalent induced e.m.f. per phase is 100 V at standstill.*

Solution. The equivalent circuit of the double-cage rotor is shown in Fig. 35.33.

(*i*) At standstill, s = 1

The combined impedance of the two cages is

$$Z = \frac{Z_0 Z_i}{Z_0 + Z_i}$$

where Z_0 = impedance of the outer cage

Z_i = impedance of the inner cage

$$Z = \frac{(0.5 + j\,0.1)\,(0.05 + j\,0.4)}{(0.55 + j\,0.5)}$$

$$= 0.1705 + j\,0.191 \text{ ohm}$$

$$\therefore \quad Z = \sqrt{0.1705^2 + 0.191^2} = 0.256\ \Omega$$

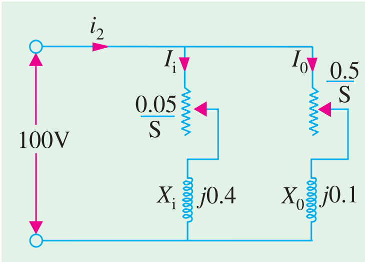


Fig. 35.33

Rotor current I_2 = 100/0.256 A; Combined resistance R_2 = 0.1705 Ω

Torque at standstill in synchronous watts per phase is

$$= I_2^2 R_2 = \left(\frac{100}{0.256}\right)^2 \times 0.1705 = \textbf{26,000 synch watts}$$

(*ii*) Here s = 0.05

$$Z = \frac{\left(\frac{0.5}{0.05} + j\,0.1\right)\left(\frac{0.05}{0.05} + j\,0.4\right)}{\left(\frac{0.05}{0.5} + \frac{0.5}{0.05} + j\,0.5\right)} = 1.01 + j\,0.326 \text{ ohm}$$

$$Z = \sqrt{(1.01)^2 + 0.326^2} = 1.06\ \Omega$$

Combined resistance R_2 = 1.01 Ω ; rotor current = 100/1.06 A

Torque in synchronous watts per phase is

$$= I_2^2 R_2 = (100/1.06)^2 \times 1.1 = \textbf{9,000 synch.watt.}$$

Example 35.26. *In a double-cage induction motor, if the outer cage has an impedance at standstill of (2 + j 1.2) ohm, determine the slip at which the two cages develop equal torques if the inner cage has an impedance of (0.5 + j 3.5) ohm at standstill.*

(Electric Machines, Osmania Univ. 1991)

Solution. Let s be the slip at which two cage develop equal torques.

Then $\quad Z_1 = \sqrt{(2/s)^2 + 1.2^2}$ and $Z_2 = \sqrt{(0.5/s)^2 + 3.5^2}$

$$\therefore \quad \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{Z_2}{Z_1}\right)^2 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44}$$

Power input to outer cage $\quad P_1 = I_1^2 R_1/s$

$$\therefore \quad P_1 = I_1^2 \times \frac{2}{s}\ ; \qquad P_2 = I_2^2 \times \frac{0.5}{s}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{P_1}{P_2} = \left(\frac{I_1}{I_2}\right)^2 \times 4 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44} \times 4$$

As $\quad T_1 = T_2$

$$\therefore \quad \frac{4}{s^2} + 1.44 = \left(\frac{0.25}{s^2} + 12.25\right) \times 4 \qquad \therefore \quad s = 0.251 = \textbf{25.1\%}$$

Example 35.27. *The resistance and reactance (equivalent) values of a double-cage induction motor for stator, outer and inner cage are 0.25, 1.0 and 0.15 ohm resistance and 3.5, zero and 3.0 ohm reactance respectively. Find the starting torque if the phase voltage is 250 V and the synchronous speed is 1000 r.p.m.* **(I.E.E. London)**

Solution. The equivalent circuit is shown in Fig. 35.34 where magnetising current has been neglected. At starting, s = 1

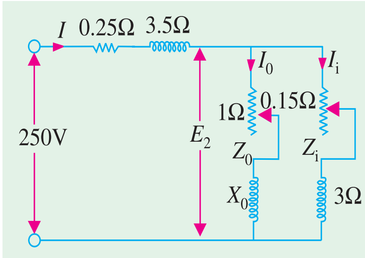


Fig. 35.34

Impedance of outer cage $Z_0' = (1 + j\,0)$

Impedance of inner cage $\quad Z_i' \;=\; (0.15 + j\,3)$

The two impedances are in parallel. Hence, their equivalent impedance

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(1 + j\,0)\,(0.15 + j\,3)}{(1 + j\,0) + (0.15 + j\,3)}$$

$$= 0.889 + j\,0.29$$

Stator impedance $= (0.25 + j\,3.5)$

$\therefore$ total impedance $Z_{01} = (0.889 + j\,0.29) + (0.25 + j\,3.5) = (1.14 + j\,3.79)\ \Omega$ (approx.)

$$\text{Current } I = \frac{\text{phase voltage}}{\text{total phase impedance}} = \frac{250 + j\,0}{1.14 + j\,3.79} = 18.2 - j\,60.5 = 66.15\text{ A}$$

Rotor Cu loss/phase = (current)$^2 \times$ total resistance/phase of two rotors

$$= 66.15^2 \times 0.889 = 3{,}890\text{ W}$$

Total Cu loss in 3-phases $= 3 \times 3890 = 11{,}670$ W

Now, rotor input $= \dfrac{\text{rotor Cu loss}}{s}$; At starting, $s = 1$ $\quad\therefore\quad$ rotor input = 11,670 W

$\therefore \qquad T_{start} = 11{,}670$ synchronous watts

Also $T_{start} \times 2\pi\,N_S = 11{,}670$ or $T_{start} = \dfrac{11{,}670}{2\pi \times (1000/60)}$ = **111.6 N-m (approx.)**

Note. If torques developed by the two rotors separately are required, then find E_2 (Fig. 35.34), then I_1 and I_2. Knowing these values, T_1 and T_2 can be found as given in previous example.

Example 35.28. *A double-cage induction motor has the following equivalent circuit parameters all of which are phase values referred to the primary :*

Primary : $\quad R_1 = 1$ *ohm* $\quad X_1 = 2.8$ *ohm*

Outer cage : $\quad R_0' = 3$ *ohm* $\quad X_0' = 1.0$ *ohm*

Inner cage : $\quad R_i' = 0.5$ *ohm* $\quad X_i' = 5$ *ohm*

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4 per cent. The magnetizing branch can be assumed connected across the primary terminals.

(Electrical Machines-II, South Gujarat Univ. 1987)

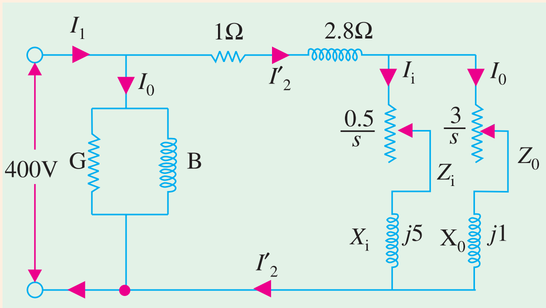


Fig. 35.35

Solution. The equivalent circuit for one phase is shown in Fig. 35.35. It should be noted that magnetising impedance Z_0 has no bearing on the torque and speed and hence, can be neglected so far as these two quantities are concerned

(i) At standstill $\quad s = 1$

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(3 + j1.0)\,(0.5 + j\,5)}{(3.5 + j\,6)}$$

$$= 1.67 + j\,1.56$$

$$Z_{01} = Z_1 + Z_2' = (1 + j\,2.8) + (1.67 + j\,1.56)$$

$$= (2.67 + j\,4.36) = 5.1 \angle 58.5°$$

Voltage per phase, V_1 = 440 V. Since stator is delta-connected.

$\mathbf{I_2'} = \mathbf{V_1/Z_{01}} = 440/5.1 \angle 58.5^\circ = 86.27 \angle -58.5^\circ$

Combined resistance R_2 = 1.6 Ω

∴ starting torque per phase $= I_2'^2 R_2 = 86.27^2 \times 1.67$ = **12,430 synch. watt.**

***(ii)* when s = 0.04**

$\mathbf{Z_0'} = (3/0.04) + j\,1.0 = 75 + j\,1.0;\quad \mathbf{Z_i'} = (0.5/0.04) + j\,5 = 12.5 + j\,5$

$$\mathbf{Z_2'} = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(75 + j1.0)(12.5 + j5)}{(87.5 + j6)} = 10.3 + j\,3.67$$

$\mathbf{Z_{01}} = Z_1 + Z_2' = (1 + j\,2.8) + (10.3 + j\,6.47) = 11.3 + j\,6.47 = 13.03 \angle 29.8^\circ$

$\mathbf{I_2'} = \mathbf{V_1/Z_{01}} = 440/13.03 \angle 29.8^\circ = 33.76 \angle -29.8^\circ$

combined resistance R_2 = 10.3 Ω

∴ full-load torque per phase $= I_2' R_2 = 33.76^2 \times 10.3$ = **11.740 synch. watts**

Obviously, starting torque is higher than full-load torque.

Tutorial Problems 35.4

1. Calculate the steps in a 5-section rotor starter of a 3-phase induction motor for which the starting current should not exceed the full-load current, the full-load slip is 0.018 and the rotor resistance is 0.015 Ω per phase

ρ_1 = **0.46 Ω** ; ρ_2 = **0.206 Ω** ; ρ_3 = **0.092 Ω** ; ρ_4 = **0.042 Ω** ; ρ_5 = **0.0185 Ω**

(Electrical Machinery-III, Kerala Univ. Apr. 1976)

2. The full-load slip of a 3-phase double-cage induction motor is 6% and the two cages have impedances of (3.5 + *j* 1.5) Ω and (0.6 + *j* 7.0) Ω respectively. Neglecting stator impedances and magnetising current, calculate the starting torque in terms of full-load torque. **[79%]**

3. In a double-cage induction motor, if the outer cage has an impedance at standstill of (2 + *j* 2) ohm and the inner cage an impedance of (0.5 + *j* 5) Ω, determine the slip at which the two cages develop equal torques. **[17.7%]**

4. The two independent cages of a rotor have the respective standstill impedance of (3 + *j* 1) ohm and (1 + *j* 4) ohm. What proportion of the total torque is due to the outer cage (*a*) at starting and (*b*) at a fractional slip of 0.05 ?**[(*a*) 83.6% (*b*) 25.8%]** ***(Principle of Elect. Engg.I, Jadavpur Univ. 1975)***

5. An induction motor has a double cage rotor with equivalent impedance at standstill of (1.0 + *j* 1.0) and (0.2 + *j* 4.0) ohm. Find the relative value of torque given by each cage at a slip of 5%.

[(*a*) 40.1 (*b*) 0.4 : 1] ***(Electrical Machines-I, Gwalior Univ. Nov. 1977)***

35.18. Speed Control of Induction Motors*

A 3-phase induction motor is practically a constant-speed machine, more or less like a d.c. shunt motor. The speed regulation of an induction motor (having low resistance) is usually less than 5% at full-load. However, there is one difference of practical importance between the two. Whereas d.c. shunt motors can be made to run at any speed within wide limits, with good efficiency and speed regulation, merely by manipulating a simple field rheostat, the same is not possible with induction motors. In their case, speed reduction is accompanied by a corresponding loss of efficiency and good speed regulation. That is why it is much easier to build a good adjustable-speed d.c. shunt motor than an adjustable speed induction motor.

Different methods by which speed control of induction motors is achieved, may be grouped under two main headings :

* For Electronic Control of AC Motors, please consult the relevant chapter of this book, in vol. III.

1. Control from stator side

(*a*) by changing the applied voltage (*b*) by changing the applied frequency

(*c*) by changing the number of stator poles

2. Control from rotor side

(*d*) rotor rheostat control

(*e*) by operating two motors in concatenation or cascade

(*f*) by injecting an e.m.f. in the rotor circuit.

A brief description of these methods would be given below :

(*a*) Changing Applied Voltage

This method, though the cheapest and the easiest, is rarely used because

(*i*) a large change in voltage is required for a relatively small change in speed

(*ii*) this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

(*b*) Changing the Applied Frequency

This method is also used very rarely. We have seen that the synchronous speed of an induction motor is given by $N_s = 120\, f/P$. Clearly, the synchronous speed (and hence the running speed) of an induction motor can be changed by changing the supply frequency f. However, this method could only be used in cases where the induction motor happens to be the only load on the generators, in which case, the supply frequency could be controlled by controlling the speed of the prime movers of the generators. But, here again the range over which the motor speed may be varied is limited by the economical speeds of the prime movers. This method has been used to some extent on electrically-driven ships.

(*c*) Changing the Number of Stator Poles

This method is ***easily applicable to squirrel-cage motors*** because the squirrel-cage rotor adopts itself to any reasonable number of stator poles.

From the above equation it is also clear that the synchronous (and hence the running)speed of an induction motor could also be changed by changing the number of stator poles. This change of number of poles is achieved by having two or more entirely independent stator windings in the same slots. Each winding gives a different number of poles and hence different synchronous speed. For example, a 36-slot stator may have two 3-ϕ windings, one with 4 poles and the other with 6-poles. With a supply frequency of 50-Hz, 4-pole winding will give $N_s = 120 \times 50/4 = 1500$ r.p.m. and the 6-pole winding will give $N_s = 120 \times 50/6 = 1000$ r.p.m. Motors with four independent stator winding are also in use and they give four different synchronous (and hence running) speeds. Of course, one winding is used at a time, the others being entirely disconnected.

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Speeds in the ratio of 2:1 can be produced by a single winding if wound on the consequent-pole principle. In that case, each of the two stator windings can be connected by a simple switch to give two speeds, each, which means four speeds in all. For example, one stator winding may give 4 or 8-poles and the other 6 or 12-poles. For a supply frequency of 50-Hz, the four speeds will be 1500, 750, 1000 and 500 r.p.m. Another combination, commonly used, is to group 2- and 4-pole winding with a 6- and 12-pole winding, which gives four synchronous speeds of 3000, 1500, 1000 and 500 r.p.m.

(*d*) Rotor Rheostat Control

In this method (Fig. 35.36), which is applicable to slip-ring motors alone, the motor speed is reduced by introducing an external resistance in the rotor circuit. For this purpose, the rotor starter may be used, provided it is continuously rated. ***This method is, in fact, similar to the armature rheostat control method of d.c. shunt motors.***

It has been shown in Art 34.22 that near synchronous speed (*i.e.* for very small slip value), $T \propto s/R_2$.

It is obvious that for a given torque, slip can be increased *i.e.* speed can be decreased by increasing the rotor resistance R_2.

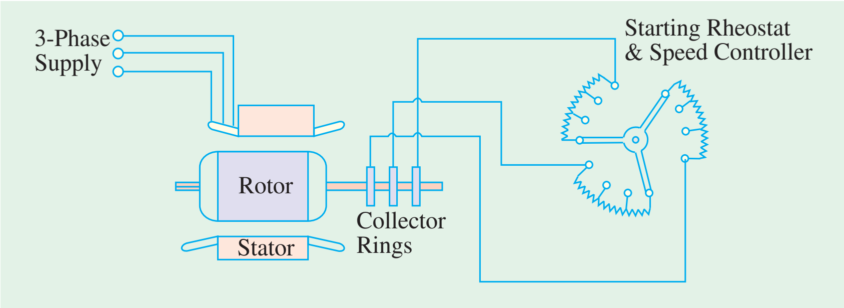


Fig. 35. 36

One serious disadvantage of this method is that with increase in rotor resistance, I^2R losses also increase which decrease the operating efficiency of the motor. In fact, the loss is directly proportional to the reduction in the speed.

The second disadvantage is the double dependence of speed, not only on R_2 but on load as well.

Because of the wastefulness of this method, it is used where speed changes are needed for short periods only.

Example 35.29. *The rotor of a 4-pole, 50-Hz slip-ring induction motor has a resistance of 0.30 Ω per phase and runs at 1440 rpm. at full load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before.*

(Advanced Elect. Machines AMIE Sec.E1992)

Solution. The motor torque is given by $T = \dfrac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2}$

Since, X_2 is not given, $T = \dfrac{K\,s\,R_2}{R_2^2} = \dfrac{K\,s}{R_2}$

In the first case, $T_1 = Ks_1/R_2$; in the second case, $T_2 = Ks_2 / (R_2 + r)$

where r is the external resistance per phase, added to the rotor circuit

Since $T_1 = T_2$ $\quad\therefore\quad K\,s_1/R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r) / R_2 = s_2 / s_1$

Now, $N_s = 120 \times 50/4 = 1500$ rpm; $N_1 = 1440$ rpm; $N_2 = 1320$ rpm

$\therefore s_1 = (1500 - 1440)/1500 = 0.04$; $s_2 = (1500 - 1320)/1500 = 0.12$

$$\therefore \quad \frac{0.3 + r}{0.3} = \frac{0.12}{0.04} \qquad \therefore \quad r = \mathbf{0.6\ \Omega}$$

Example 35.30. *A certain 3-phase, 6-pole, 50-Hz induction motor when fully-loaded, runs with a slip of 3%. Find the value of the resistance necessary in series per phase of the rotor to reduce the speed by 10%. Assume that the resistance of the rotor per phase is 0.2 ohm.*

(Electrical Engineering-II (M), Bangalore Univ. 1989)

Solution. $$T = \frac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2} = \frac{K_s\,R_2}{R_2^2} = \frac{K\,s}{R_2} \qquad \text{- neglecting } (s\,X_2)$$

$\therefore \quad T_1 = Ks_1 / R_2$ and $T_2 = K\,s_2 / (R_2 + r)$ where r is the external resistance per phase added to the rotor circuit.

Since $\quad T_1 = T_2$, $K\,s_1 / R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r)/R_2 = s_2 / s_1$

Now, $\quad N_s = 120 \times 50/6 = 1000$ rpm., $s_1 = 0.03$, $N_1 = 1000\,(1 - 0.030) = 970$ rpm.

$$N_2 = 970 - 10\% \text{ of } 970 = 873 \text{ rpm., } s_2 = (1000 - 873)/1000 = 0.127$$

$$\therefore \quad \frac{0.2 + r}{0.2} = \frac{0.127}{0.03}; \quad r = \mathbf{0.65\ \Omega.}$$

(e) Cascade or Concatenation or Tandem Operation

In this method, two motors are used (Fig. 35.37) and are ordinarily mounted on the same shaft, ***so that both run at the same speed*** (or else they may be geared together).

The stator winding of the main motor A is connected to the mains in the usual way, while that of the auxiliary motor B is fed from the rotor circuit of motor A. For satisfactory operation, the main motor A should be phase-wound *i.e.* of slip-ring type with stator to rotor winding ratio of 1 : 1, so that, in addition to concatenation, each motor may be run from the supply mains separately.

There are at least three ways (and sometimes four ways) in which the combination may be run.

1. Main motor A may be run separately from the supply. In that case, the synchronous speed is $N_{sa} = 120\, f / P_a$ where P_a = Number of stator poles of motor A.

2. Auxiliary motor B may be run separately from the mains (with motor A being disconnected). In that case, synchronous speed is $N_{sb} = 120 \times f / P_b$ where P_b = Number of stator poles of motor B.

3. The combination may be connected in cumulative cascade *i.e.* in such a way that the phase rotation of the stator fields of both motors is in the same direction. The synchronous speed of the cascaded set, in this case, is $N_{sc} = 120\, f/(P_a + P_b)$.

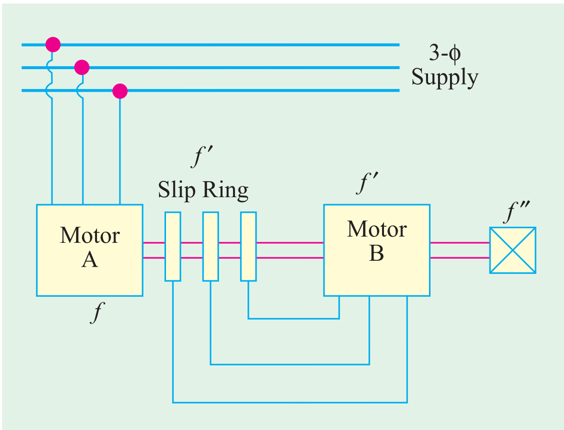


Fig. 35. 37

Proof

Let N = actual speed of concatenated set ;

N_{sa} = synchronous speed of motor A, it being independent of N.

Clearly, the relative speed of rotor A with respect to its stator field is $(N_{sa} - N)$. Hence, the frequency f' of the induced e.m.f. in rotor A is given by

$$f' = \frac{N_{sa} - N}{N_{sa}} \times f$$

This is also the frequency of the e.m.f. applied to the stator of motor B. Hence, the synchronous speed of motor B with this input frequency is

$$N' = 120 \frac{f'}{P_b} = \frac{120\,(N_{sa} - N)\, f}{P_b \times N_{sa}} \quad \text{...}(i)$$

(Note that N' is not equal to N_{sb} which is the synchronous speed of motor B with supply frequency f).

This will induce an e.m.f. of frequency, say, f'' in the rotor B. Its value is found from the fact that the stator and rotor frequencies are proportional to the speeds of stator field and the rotor

$$\therefore \quad f'' = \frac{N' - N}{N'} f$$

Now, on no-load, the speed of rotor B is almost equal to its synchronous speed, so that the frequency of induced e.m.f. is, to a first approximation, zero.

$$f'' = 0, \quad \text{or} \quad \frac{N' - N}{N'} f = 0 \quad \text{or} \quad N' = N \qquad \text{...}(ii)$$

From (i) above
$$N' = \frac{120\ f\ (N_{sa} - N)}{P_b \times N_{sa}} = \frac{120\ f}{P_b}\left(1 - \frac{N}{N_{sa}}\right)$$

Hence, from (ii) above,
$$\frac{120\ f}{P_b}\left(1 - \frac{N}{N_{sa}}\right) = N \quad \text{or} \quad \frac{120\ f}{P_b} = N\left(1 + \frac{1}{N_{sa}} \times \frac{120\ f}{P_b}\right)$$

Putting $N_{sa} = 120\ f/P_a$, we get
$$\frac{120 f}{P_b} = N\left(1 + \frac{P_a}{120 f} \times \frac{120\ f}{P_b}\right) = N\left(1 + \frac{P_a}{P_b}\right) \quad \therefore N = \frac{120\ f}{(P_a + P_b)}$$

Concatenated speed of the set = $120\ f / (P_a + P_b)$

How the Set Starts ?

When the cascaded set is started, the voltage at frequency f is applied to the stator winding of machine A. An induced e.m.f. of the same frequency is produced in rotor A which is supplied to auxiliary motor B. Both the motors develop a forward torque. As the shaft speed rises, the rotor frequency of motor A falls and so does the synchronous speed of motor B. The set settles down to a stable speed when the shaft speed becomes equal to the speed of rotating field of motor B.

Considering load conditions, we find that the electrical power taken in by stator A is partly used to meet its I^2R and core losses and the rest is given to its rotor. The power given to rotor is further divided into two parts : one part, proportional to the speed of set *i.e.* N is converted into mechanical power and the other part proportional to $(N_{sa} - N)$ is developed as electrical power at the slip frequency, and is passed on to the auxliary motor B, which uses it for producing mechanical power and losses. Hence, approximately, the mechanical outputs of the two motors are in the ratio $N : (N_{sa} - N)$. ***In fact, it comes to that the mechanical outputs are in the ratio of the number of poles of the motors***.

It may be of interest to the reader to know that it can be proved that

(*i*) $s = f''/f$ where s = slip of the set referred to its synchronous speed N_{sc}.
$= (N_{sc} - N)/N_{sc}$

(*ii*) $s = s_a\, s_b$

where s_a and s_b are slips of two motors, referred to their respective stators *i.e*

$$s_a = \frac{N_{sa} - N}{N_{sa}} \quad \text{and} \quad s_b = \frac{N' - N}{N}$$

Conclusion

We can briefly note the main conclusions drawn from the above discussion :

(*a*) the mechanical outputs of the two motors are in the ratio of their number of poles.

(*b*) $s = f''/f$ **(*c*)** $s = s_a . s_b$

4. The fourth possible connection is the ***differential cascade***. In this method, the phase rotation of stator field of the motor B is opposite to that of the stator of motor A. This reversal of phase rotation of stator of motor B is obtained by interchanging any of its two leads. It can be proved in the same way as above, that for this method of connection, the synchronous speed of the set is

$$N_{sc} = 120\ f / (P_a - P_b)$$

As the differentially-cascaded set has a very small or zero starting torque, this method is rarely used. Moreover, the above expression for synchronous speed becomes meaningless for $P_a = P_b$.

Example 35.31. *Two 50-Hz, 3-ϕ induction motors having six and four poles respectively are cumulatively cascaded, the 6-pole motor being connected to the main supply. Determine the* *frequencies of the rotor currents and the slips referred to each stator field if the set has a slip of 2 per cent.* **(Elect. Machinery-II Madras Univ. 1987)**

Solution. Synchronous speed of set $N_{sc} = 120 \times 50/10 = 600$ r.p.m.

Actual rotor speed $N = (1 - s)\, N_{sc} = (1 - 0.02)\, 600 = 588$ r.p.m.

Synchronous speed of the stator field of 6-pole motor, $N_{sa} = 120 \times 50/6 = 1000$ r.p.m.
Slip referred to this stator field is

$$s_a = \frac{N_{sa} - N}{N_{sa}} = \frac{1000 - 588}{1000} = 0.412 \quad \text{or} \quad \mathbf{41.2\%}$$

Frequency of the rotor currents of 6-pole motor $f' = s_a f = 0.412 \times 50 =$ **20.6 Hz**

This is also the frequency of stator currents of the four pole motor. The synchronous speed of the stator of 4-pole motor is

$$N' = 120 \times 20.6/4 = 618 \text{ r.p.m.}$$

This slip, as referred to the 4-pole motor, is $s_b = \dfrac{N' - N}{N'} = \dfrac{618 - 588}{618}$

$= 0.0485$ or **4.85 %**

The frequency of rotor current of 4-pole motor is

$$f'' = s_b f' = 0.0485 \times 20.6 = \mathbf{1.0\ Hz\ (approx)}$$

As a check, $f'' = sf = 0.02 \times 50 = 1.0$ Hz

Example 35.32. *A 4-pole induction motor and a 6-pole induction motor are connected in cumulative cascade. The frequency in the secondary circuit of the 6-pole motor is observed to be 1.0 Hz. Determine the slip in each machine and the combined speed of the set. Take supply frequency as 50 Hz.* **(Electrical Machinery-II, Madras Univ. 1986)**

Solution. With reference to Art. 35.18 (*e*) and Fig. 35.38

$N_{sc} = 120 \times 50/(4 + 6) = 600$ r.p.m.

$s = f''/f = 1/50 = 0.02$

N = actual speed of the concatenated set

$$\therefore \quad 0.02 = \frac{600 - N}{600} \quad \text{or} \quad N = \mathbf{588\ r.p.m.}$$

$N_{sa} = 120 \times 50/4 = 1500$ r.p.m.

$s_a = (1500 - 588)/1500$

$= 0.608$ or **60.8%**

$f' = s_a f = 0.608 \times 50 = 30.4$ Hz

N' = synchronous speed of 6-pole motor with frequency f'

$= 120 \times 30.4/6 = 608$ r.p.m.

$$s_b = \frac{N' - N}{N'} = \frac{608 - 588}{608} = 0.033 \quad \text{or} \quad \mathbf{3.3\%}$$

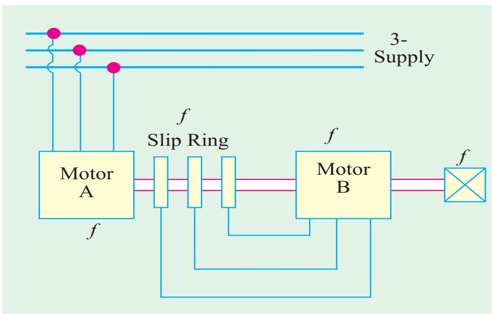


Fig. 35.38

Example 35.33. *The stator of a 6-pole motor is joined to a 50-Hz supply and the machine is mechanically coupled and joined in cascade with a 4-pole motor, Neglecting all losses, determine the speed and output of the 4-pole motor when the total load on the combination is 74.6 kW.*

Solution. As all losses are neglected, the actual speed of the rotor is assumed to be equal to the synchronous speed of the set.

Now, $N_{sc} = 120 \times 50/10 =$ **600 r.p.m.**

As said earlier, mechanical outputs are in the ratio of the number of poles of the motors.

∴ output of 4-pole motor = 74.6 × 4/10 = **29.84 kW**

Example 35.34. *A cascaded set consists of two motors A and B with 4 poles and 6 poles respectively. The motor A is connected to a 50-Hz supply. Find*

(*i*) the speed of the set

(*ii*) *the electric power transferred to motor B when the input to motor A is 25 kW. Neglect losses.* **(Electric Machines-I, Utkal Univ. 1990)**

Solution. Synchronous speed of the set is*

$N_{sc} = 120\, f/(P_a + P_b) = 120 \times 50/(6 + 4) =$ **600 r.p.m.**

(*ii*) The outputs of the two motors are proportional to the number of their poles.

∴ output of 4-pole motor $B = 25 \times 4/10 =$ **10 kW**

(*f*) Injecting an e.m.f. in the Rotor Circuit

In this method, the speed of an induction motor is controlled by injecting a voltage in the rotor circuit, it being of course, necessary for the injected voltage to have the same frequency as the slip frequency. There is, however, no restriction as to the phase of the injected e.m.f.

When we insert a voltage which is in ***phase opposition*** to the induced rotor e.m.f., it amounts to increasing the rotor resistance, whereas inserting a voltage which is in phase with the induced rotor e.m.f., is equivalent to ***decreasing*** its resistance. Hence, by changing the phase of the injected e.m.f. and hence the rotor resistance, the speed can be controlled.

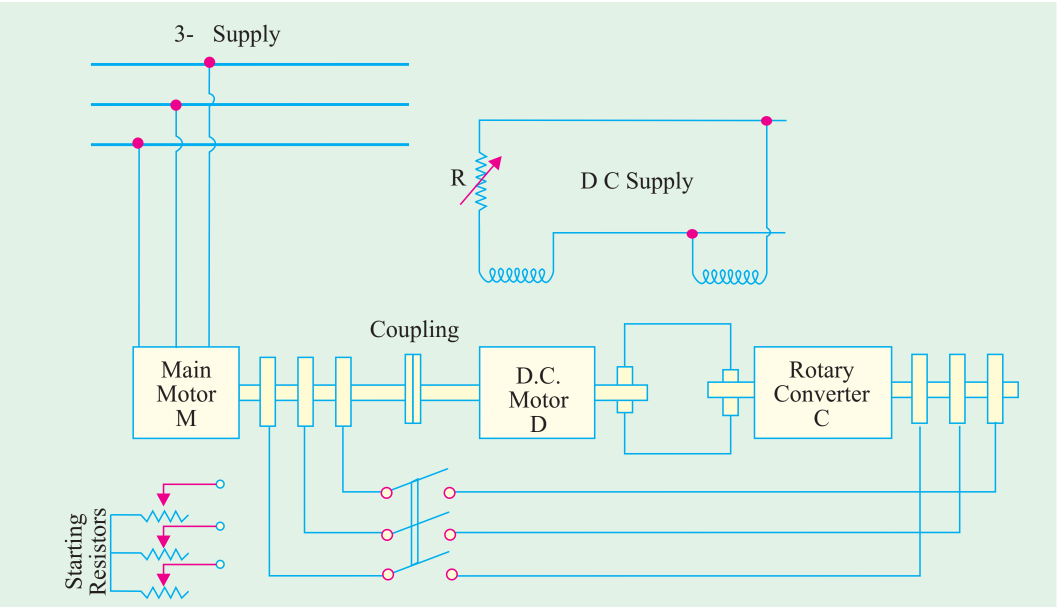


Fig. 35.39

One such practical method of this type of speed control is Kramer system, as shown in Fig. 35.39, which is used in the case of large motors of 4000 kW or above. It consists of a rotary converter *C* which converts the low-slip frequency a.c. power into d.c. power, which is used to drive a d.c. shunt motor *D*, mechanically coupled to the main motor *M*.

The main motor is coupled to the shaft of the d.c. shunt motor *D*. The slip-rings of *M* are connected to those of the rotary converter *C*. The d.c. output of *C* is used to drive *D*. Both *C* and *D*

* It is assumed that the two motors are connected in cumulative cascade.

are excited from the d.c. bus-bars or from an exciter. There is a field regulator which governs the back e.m.f. E_b of D and hence the d.c. potential at the commutator of C which further controls the slip-ring voltage and therefore, the speed of M.

One big advantage of this method is that any speed, within the working range, can be obtained instead of only two or three, as with other methods of speed control.

Yet another advantage is that if the rotary converter is over-excited, it will take a leading current which compensates for the lagging current drawn by main motor M and hence improves the power factor of the system.

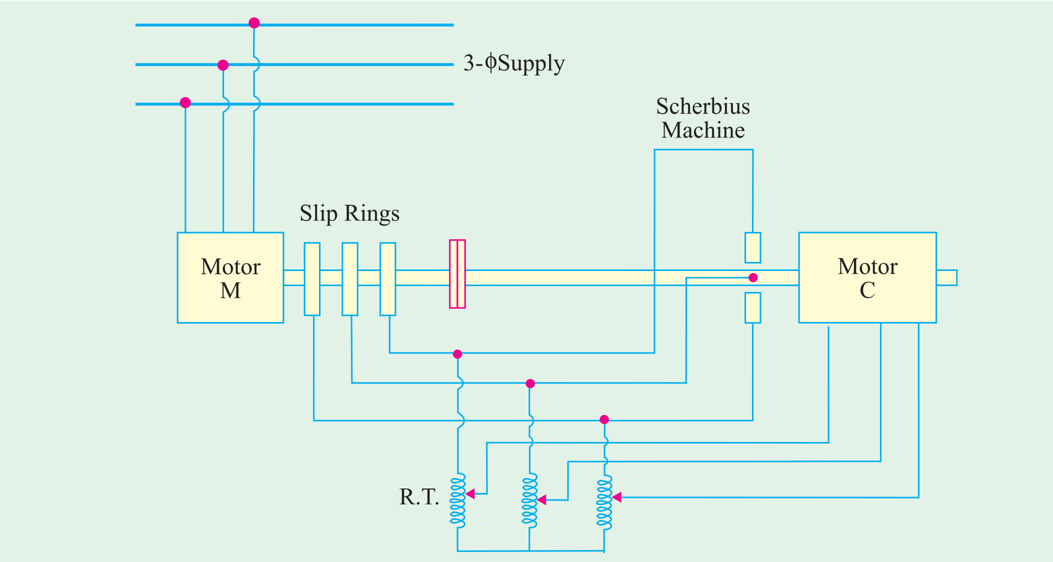


Fig. 35.40

In Fig. 35.40 is shown another method, known as Scherbius system, for controlling the speed of large induction motors. The slip energy is not converted into d.c. and then fed to a d.c. motor, rather it is fed directly to a special 3-phase (or 6-phase) a.c. commutator motor-called a, Scherbius machine.

The polyphase winding of machine C is supplied with the low-frequency output of machine M through a regulating transformer RT. The commutator motor C is a variable-speed motor and its speed (and hence that of M) is controlled by either varying the tappings on RT or by adjusting the position of brushes on C.

Tutorial Problems 35.5

1. An induction motor has a double-cage rotor with equivalent impedances at standstill of $(1.0 + j\,1.0)$ and $(0.2 + j\,4.0)$ Ω. Find the relative values of torque given by each cage (*a*) at starting and (*b*) at 5 % slip [(*a*) **40:1** (*b*) **0.4:1**] ***(Adv. Elect. Machines AMIE Sec. B 1991)***
2. The cages of a double-cage induction motor have standstill impedances of $(3.5 + j\,1.5)$ Ω and $(0.6 + j\,7.0)$ Ω respectively. The full-load slip is 6%. Find the starting torque at normal voltage in terms of full-load torque. Neglect stator impedance and magnetizing current.
   [300%] ***(Elect. Machines-I, Nagpur Univ. 1993)***
3. The rotor of a 4 pole, 50 Hz, slip ring induction motor has a resistance of 0.25 ohm per phase and runs at 1440 rpm at full-load. Calculate the external resistance per phase, which must be added to lower the speed to 1200 rpm, the torque being same as before.
   [1 Ω] ***(Utilisation of Electric Power (E-8) AMIE Sec. B Summer 1992)***

35.19. Three-phase A.C. Commutator Motors

Such motors have shunt speed characteristics *i.e.* change in their speed is only moderate, as compared to the change in the load. They are ideally suited for drives, requiring a uniform accelerating torque and continuously variable speed characteristics over a wide range. Hence, they find wide use in high-speed lifts, fans and pumps and in the drives for cement kilns, printing presses, pulverised fuel plants, stokers and many textile machines. Being more complicated, they are also more expensive than single-speed motors. Their efficiency is high over the whole speed range and their power factor varies from low value at synchronous speed to unity at maximum (supersynchronous) speed.

The speed control is obtained by injecting a variable voltage at ***correct*** frequency into the secondary winding of the motor via its commutator. If injected voltage assists the voltage induced in the secondary winding, the speed is increased but if it is in the opposing direction, then motor speed is reduced. The commutator acts as a frequency changer because it converts the supply frequency of the regulating voltage to the slip frequency corresponding to the speed required.

Following are the two principal types of such motors :

(*i*) Schrage or rotor-fed or brush shift motor and **(*ii*)** stator-fed or induction-regulator type motor.

35.20. Schrage Motor*

It is a rotor-fed, shunt-type, brush-shifting, 3-phase commutator induction motor which has built-in arrangement both for ***speed control and power factor improvement***. In fact, it is an induction motor with a built-in slip-regulator. It has **three** windings:two in rotor and one in stator as shown in Fig. 35.41 and 35.42 (*a*) The three windings are as under:

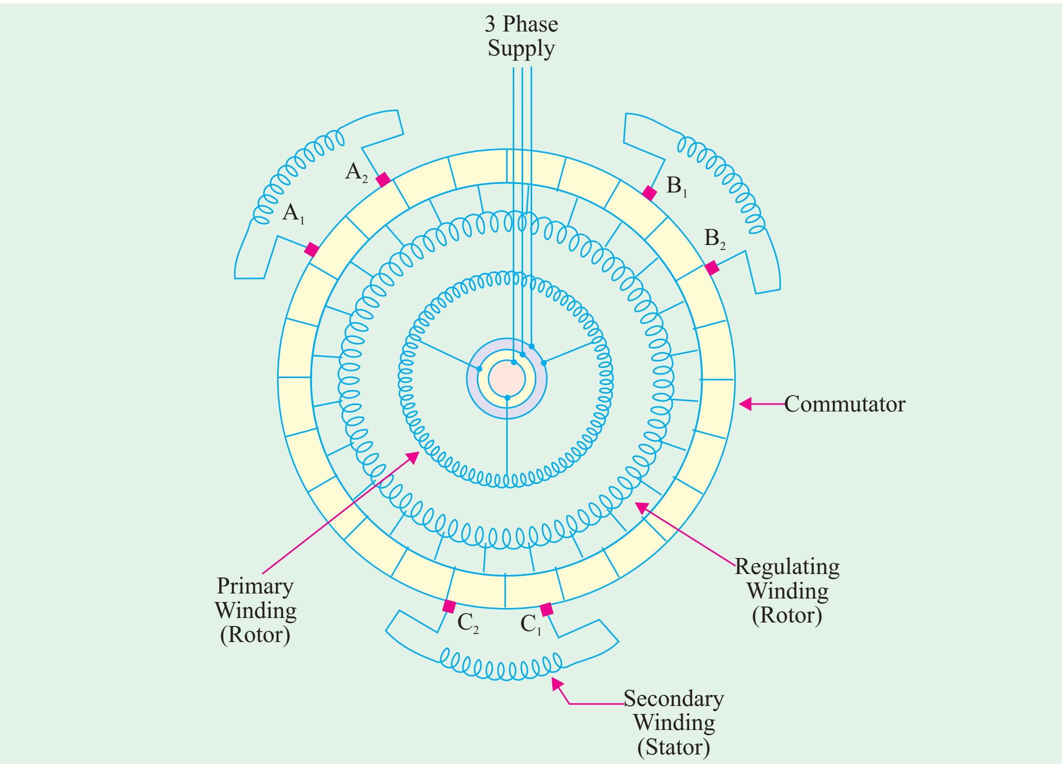


Fig. 35.41

(*i*) Primary winding. It is housed in the lower part of the rotor slots (not stator) and is supplied through slip-rings and brushes at ***line*** frequency. It generates the working flux in the machine.

* After the name of its inventor K.H. Schrage of Sweden.

(ii)* Regulating winding.** It is variously known as ***compensating winding or ***tertiary*** winding. It is also housed in ***rotor slots*** (in the upper part) and is connected to the commutator in a manner similar to the armature of a d.c. motor.

(iii)* Secondary winding.** It is contained in the ***stator slots, but end of each phase winding is connected to one of the pair of brushes arranged on the commutator. These brushes are mounted on two separate brush rockers, which are designed to move in opposite directions relative to the centre line of the corresponding stator phase (usually by a rack and pinion mechanism). Brushes A_1, B_1, and C_1 move together and are 120 electrical degrees apart. Similarly, brushes A_2, B_2 and C_2 move together and are also 120 electrical degrees apart. A sectional drawing of the motor is shown in Fig. 35.44.

(*a*) Working

When primary is supplied at line frequency, there is transformer action between primary and regulating winding and normal induction motor action between primary and secondary winding. Hence, voltage at line frequency is induced in the regulating winding by transformer action. The commutator, acting as a frequency changer, converts this line-frequency voltage of the regulating winding to the slip frequency for feeding it into the secondary winding on the stator. The voltage across brush pairs A_1A_2, B_1B_2 and C_1C_2 increases as brushes are separated. In fact, magnitude of the voltage injected into the secondary winding depends on the angle of separation of the brushes A_1 and A_2, B_1 and B_2 and C_1 and C_2. How slip-frequency e.m.f. is induced in secondary winding is detailed below:

When 3-ϕ power is connected to slip-rings, synchronously rotating field is set up in the rotor core. Let us suppose that this field revolves in the clockwise direction. Let us further suppose that brush pairs are on ***one commutator segment***, which means that secondary is short-circuited. With rotor still at rest, this field cuts the secondary winding, thereby inducing voltage and so producing currents in it which react with the field to produce ***clockwise (CW)*** torque in the ***stator***. Since stator cannot rotate, as a reaction, it makes the rotor rotate in the ***counterclockwise (CCW)*** direction.

Suppose that the rotor speed is N rpm. Then

1. rotor flux is still revolving with synchronous speed relative to the primary and regulating winding.

2. however, this rotor flux will rotate at slip speed $(N_s - N)$ relative to the stator. It means that the revolving rotor flux will rotate at slip speed in space.

3. if rotor could rotate at synchronous speed *i.e.* if $N = N_s$, then flux would be stationary in space (*i.e.* relative to stator) so that there would be no cutting of the secondary winding by the flux and, consequently, no torque would be developed in it.

As seen from above, in a Schrage motor, the flux rotates at synchronous speed, relative to rotor but with slip speed relative to space (*i.e.* stator), whereas in a normal induction motor, flux rotates synchronously relative to stator (*i.e.* space) but with slip speed relative to the rotor. (Art. 34.11).

Another point worth noting is that since at ***synchronous*** speed, magnetic field is stationary in space, the regulating winding acts as a d.c. armature and the direct current taken from the commutator flows in the secondary winding. Hence, Schrage motor then operates like a synchronous motor.

(*b*) Speed Control

It is quite easy to obtain speeds above as well as below synchronism in a Schrage motor. As shown in Fig. 35.42 (*b*) (*i*) when brush pairs are together on the same commutator segment (*i.e.* are electrically connected via commutator), the secondary winding is short-circuited and the machine

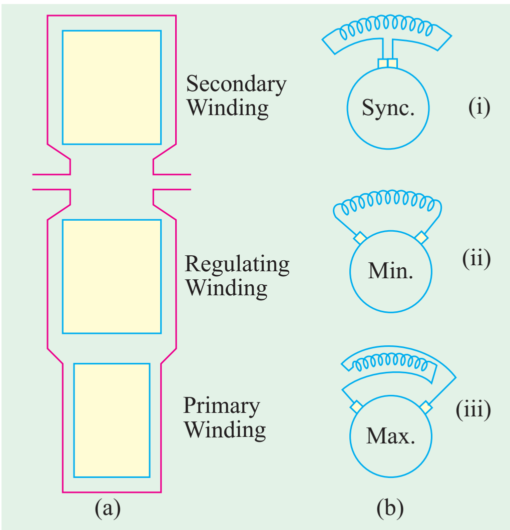


Fig. 35.42

operates as an inverted* plain squirrel-cage induction motor, running with a small positive slip. Parting the brushes in one direction, as shown in Fig. 35.42 (*b*) (*ii*) produces subsynchronous speeds, because in this case, regulating voltage injected into the secondary winding ***opposes*** the voltage induced in it from primary winding. However, when movement of brushes is reversed and they are parted in opposite directions, the direction of the regulating voltage is reversed and so motor speed ***increases*** to super synchronous (maximum) value, as shown in Fig. 35.41 (*b*) (*iii*) The commutator provides maximum voltage when brushes are separated by one pole pitch.

No-load motor speed is given by $N \cong N_s\,(1 - \mathrm{K}\sin 0.5\beta)$ where β is brush separation in electrical degrees and K is a constant whose value depends on turn ratio of the secondary and regulating windings.

Maximum and minimum speeds are obtained by changing the magnitude of the regulating voltage. Schrage motors are capable of speed variations from zero to nearly twice the synchronous speed, though a speed range of 3:1 is sufficient for most applications. It is worth noting that Schrage motor is essentially a shunt machine, because for a particular brush separation, speed remains approximately constant as the load torque is increased as happens with dc shunt motors (Art 29.14).

(*c*) Power Factor Improvement

Power factor improvement can be brought about by changing the phase angle of the voltage injected into the secondary winding. As shown in Fig. 35.43, when one set of brushes is advanced more rapidly than the other is retarted, then injected voltage has a quadrature component which leads the rotor induced voltage. Hence, it results in the improvement of motor power factor. This differential movement of brush sets is obtained by coupling the racks driving the brush rockers to the hand wheel with gears having differing ratios. In Schrage motor, speed depends on angular distance between the individual brush sets (A_1 and A_2 in Fig. 35.41) but p.f. depends on the angular positions of the brushes as a whole.

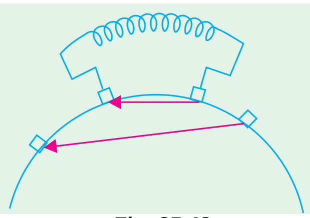


Fig. 35.43

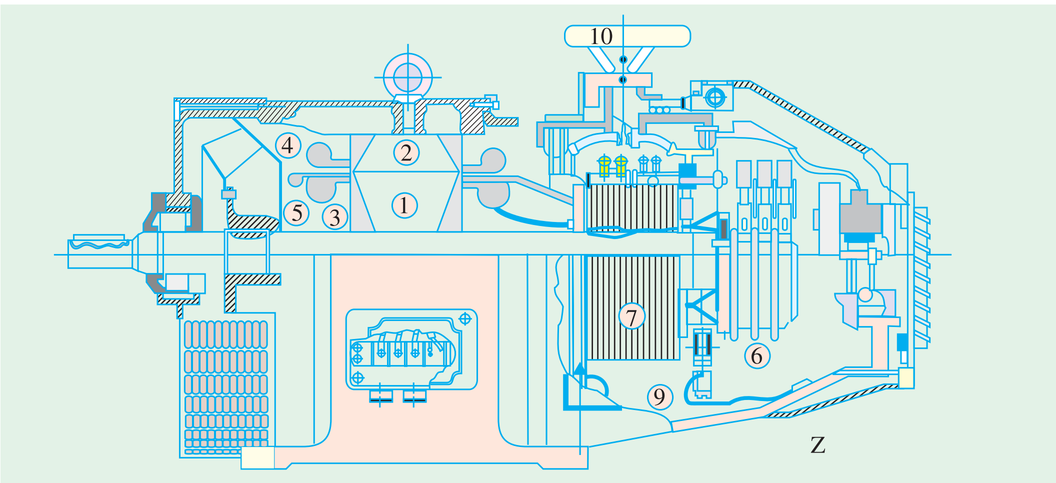


Fig. 35.44. Sectional drawing of a Schrage motor (*Courtesy : Elekrta Faurandou, Germany*)

(*d*) Starting

Schrage motors are usually started with brushes in the lowest speed position by direct-on contactor starters. Usually, interlocks are provided to prevent the contactor getting closed on the line when brushes are in any other position. One major disadvantage of this motor is that its operating voltage is limited to about 700 V because a.c. power has to be fed through slip-rings. It is available in sizes upto 40 kW and is designed to operate on 220, 440 and 550 V. It is ordinarily wound for four or six poles.

* Inverted in the sense that primary is in the rotor and secondary in the stator - just opposite of that in the normal induction motor (Art. 34.3).

Fig. 35.44 shows a sectional drawing of a Schrage motor. The details of different parts labelled in the diagram are as under:

1. rotor laminations **2.** stator laminations **3.** primary winding **4.** secondary winding **5.** regulating winding **6.** slip-ring unit **7.** commutator **8.** cable feed for outer brush yoke **9.** cable feed for inner brush yoke **10.** hand wheel.

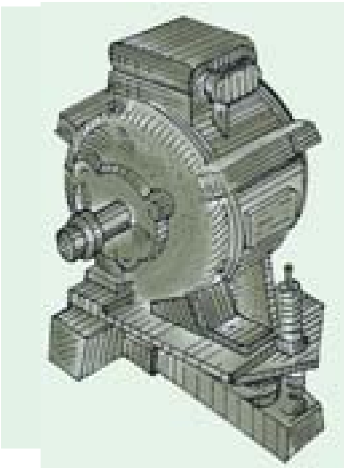


Fig. 35.45. Totally-enclosed surface cooled 750 W, 750 r.pm. high torque induction motor (*Courtesy : Jyoti Limited*)

35.21. Motor Enclosures

Enclosed and semi-enclosed motors are practically identical with open motors in mechanical construction and in their operating characteristics. Many different types of frames or enclosures are available to suit particular requirements. Some of the common type enclosures are described below:

(*i*) Totally-enclosed, Non-ventilated Type

Such motors have solid frames and end- shields, but no openings for ventilation. They get cooled by surface radiation only (Fig. 35.45). Such surface-cooled motors are seldom furnished in sizes above two or three kW, because higher ratings require frames of much larger sizes than fan-cooled motors of corresponding rating.

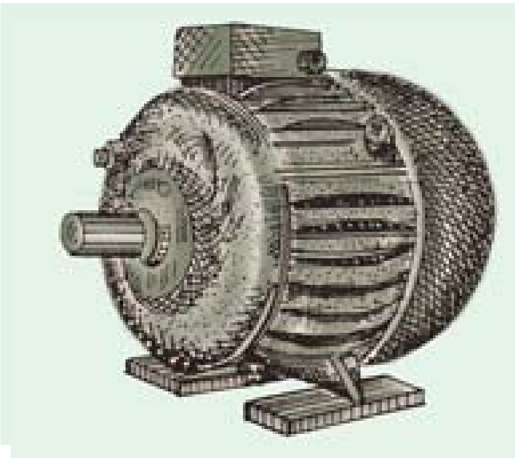


Fig. 35.46. Totally-enclosed, fan-cooled 10-kW 440/400-V, 1000 r.p.m. 50-Hz induction motor (*Courtesy : Jyoti Limited*)

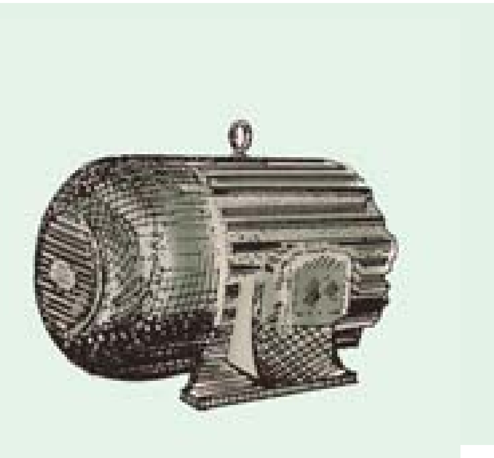


Fig. 35.47. Squirrel-cage motor, showing cowl over the external fan. (*Courtesy : General Electric Co. of India*)

(*ii*) Splash-proof Type

In the frames of such motors, the ventilating openings are so constructed that the liquid drops or dust particles falling on the motor are coming towards it in a straight line at any angle not greater than 100º from the vertical are not able to enter the motor either directly or by striking and running along the surface.

(*iii*) Totally-enclosed, Fan-cooled Type

In such motors (Fig. 35.46), cooling air is drawn into the motor by a fan mounted on the shaft. This air is forced through the motor between the inner fully-enclosed frame and an outer shell, over the end balls and the stator laminations and is then discharged through openings in the opposite side. An internal fan carries the generated heat to the totally enclosing frame, from where it is conducted to the outside. Because of totally enclosing frame, all working parts are protected against corrosive or abrasive effects of fumes, dust, and moisture.

(*iv*) Cowl-covered Motor

These motors are simplified form of fan-cooled motors (Fig. 35.47). These consist of totally-enclosed frame with a fan and cowl mounted at the end opposite to the driving end. The air is drawn into the cowl with the help of fan and is then forced over the frame. The contours of the cowl guide the cooling air in proper directions. These motors are superior to the usual fan-cooled motors for operation in extremely dusty atmosphere *i.e.* gas works, chemical works, collieries and quarries etc. because there are no air passages which will become clogged with dust.

Fig. 35.48. Squirrel cage motor

(*v*) Protected Type

This construction consists of perforated covers for the openings in both end shields

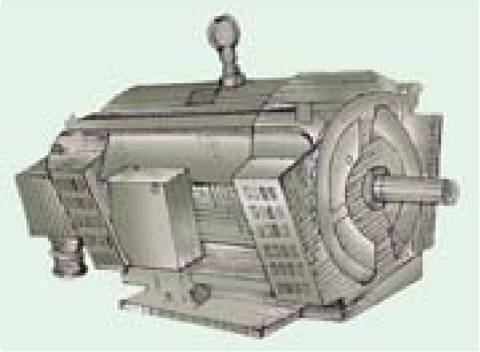


Fig. 35.49. Protected slip-ring motor with totally enclose slip -rings. (*Courtesy : General Electric Co. of India*)

(*vi*) Drip-proof Motors

The frames of such motors are so constructed that liquid drops or dust particles, falling on the machine at any angle greater than 15º from the vertical, cannot enter the motor, either directly or by striking and running along a horizontal or inwardly inclined smooth surface (Fig. 35.51).

(*vii*) Self (Pipe) Ventilated Type

The construction of such motors consists of enclosed shields with provision for pipe connection on both the shields. The motor fan circulates sufficient air through pipes which are of ample section.

(*viii*) Separately (Forced) Ventilated Type

These motors are similar to the self-ventilated type except that ventilation is provided by a separated blower.

35.22. Standard Types of Squirrel-cage Motors

Different types of 3-phase squirrel-cage motors have been standardized, according to their electric characteristics, into six types, designated as design *A*, *B*, *C*, *D*, *E* and *F* respectively. The original commercial squirrel-cage induction motors which were of shallow-slot type are designated as class *A*. For this reason, Class *A* motors are used as a reference and are referred to as 'normal starting-torque, normal starting-current, normal slip' motors.

(*i*) Class *A* - Normal starting torque, normal starting current, normal slip

(*ii*) Class *B* - Normal starting torque, low starting current, normal slip

(*iii*) Class *C* - High starting torque, low starting current, normal slip

Fig. 35.50. Squirrel cage A C induction motor

(*iv*) Class *D* - High starting torque, low starting current, high slip

(*v*) Class *E* - Low starting torque, normal starting current, low slip

(*vi*) Class *F* - Low starting torque, low starting current, normal slip

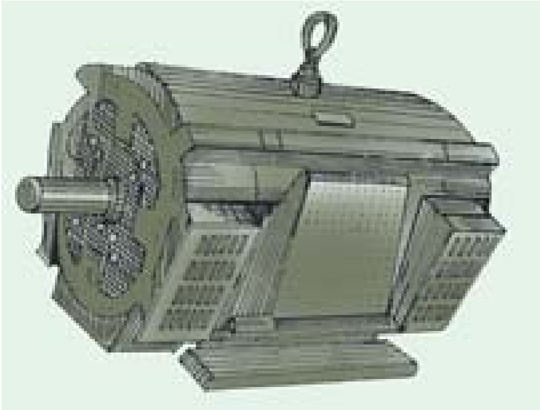


Fig. 35.51. Drip-proof slip-ring 50 up, 440/400-V, 50HZ, 1000 r.p.m. motor (courtesy : Jyoti Limited)

35.23. Class A Motors

It is the most popular type and employs squirrel cage having relatively low resistance and reactance. Its locked-rotor current with full voltage is generally more than 6 times the rated full-load current. For smaller sizes and number of poles, the starting torque with full voltage is nearly twice the full-load torque whereas for larger sizes and number of poles, the corresponding figure is 1.1 times the full-load torque. The full-load slip is less than 5 per cent. The general configuration of slot construction of such motors is shown in Fig. 35.52. As seen, the rotor bars are placed close to the surface so as to reduce rotor reactance.

Such motors are used for fans, pumps, compressors and conveyors etc. which are started and stopped in frequently and have low inertia loads so that the motor can accelerate in a few seconds.

35.24. Class B Motors

These motors are so built that they can be started at full-load while developing normal starting torque with relatively low starting current. Their locked-rotor current with full voltage applied is generally 5 to 5 1/2 times the full-load current. Their cages are of high reactance as seen from Fig. 35.53. The rotor is constructed with deep and narrow bars so as to obtain high reactance during starting.

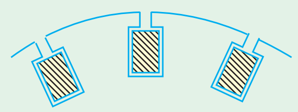


Fig. 35.52

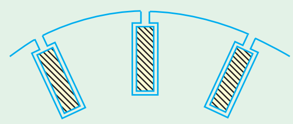


Fig. 35.53

Such motors are well-suited for those applications where there is limitation on the starting current or if the starting current is still in excess of what can be permitted, then reduced voltage starting is employed. One of the common applications of such motors is large fans most of which have high moment of inertia. It also finds wide use in many machine tool applications, for pumps of centrifugal type and for driving electric generators.

35.25. Class C Motors

Such motors are usually of double squirrel-cage type (Fig. 35.54) and combine high starting torque with low starting current. Their locked-rotor currents and slip with full voltage applied are nearly the same as for class *B* motors. Their starting torque with full voltage applied is usually 2.75 times the full-load torque.

For those applications where reduced voltage starting does not give sufficient torque to start the load with either class *A* or *B* motor, class C motor, with its high inherent starting torque along with reduced starting current supplied by reduced-voltage starting may be used. Hence, it is frequently used for crushers, compression pumps, large refrigerators, coveyor equipment, textile machinery, boring mills and wood-working equipment etc.

35.26. Class D Motors

Such motors are provided with a high-resistance squirrel cage giving the motor a high starting torque with low starting current. Their locked-rotor currents with full voltage applied are of the same order as for class *C* motors. Their full-load slip varies from 5% to 20 per cent depending on the application. Their slot structure is shown in Fig. 35.55. For obtaining high starting torque with low starting current, thin rotor bars are used which make the leakage flux of the rotor low and the useful flux high.

Since these motors are used where extremely high starting torque is essential, they are usually used for bulldozers, shearing machines, punch presses, foundry equipment, stamping machines, hoists, laundry equipment and metal drawing equipment etc.

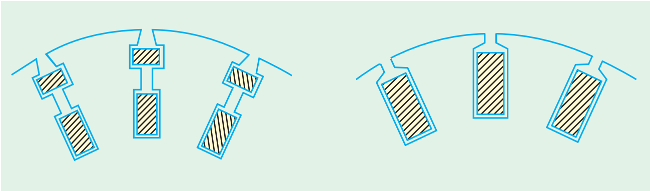
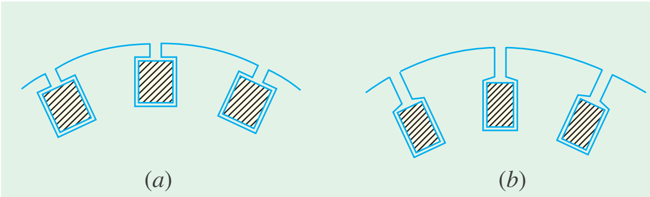


Fig. 35.54 **Fig. 35.55**

35.27. Class E Motors

These motors have a relatively low slip at rated load. For motors above 5 kW rating, the starting current may be sufficiently high as to require a compensator or resistance starter. Their slot structure is shown in Fig. 35.56 (*a*).



(*a*) (*b*)

Fig. 35.56

35.28. Class F Motors

Such motors combine a low starting current with a low starting torque and may be started on full voltage. Their low starting current is due to the design of rotor which has high reactance during starting [Fig. 35.56 (*b*)]. The locked rotor currents with full voltage applied and the full-load slip are in the same range as those for class *B* and *C* motors. The starting torque with full voltage applied is nearly 1.25 times the full-torque.

QUESTIONS AND ANSWERS ON THREE-PHASE INDUCTION MOTORS

Q. 1. How do changes in supply voltage and frequency affect the performance of an induction motor ?

Ans. High voltage decreases both power factor and slip, but increases torque. Low voltage does just the opposite. Increase in frequency increases power factor but decreases the torque. However, per cent slip remains unchanged. Decrease in frequency decreases power factor but increases torque leaving per cent slip unaffected as before.

Q. 2. What is, in brief, the basis of operation of a 3-phase induction motor ?

Ans. The revolving magnetic field which is produced when a 3-phase stator winding is fed from a 3-phase supply.

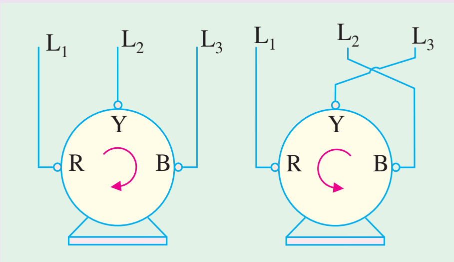


Fig. 35.57

Q. 3. What factors determine the direction of rotation of the motor ?

Ans. The phase sequence of the supply lines and the order in which these lines are connected to the stator winding.

Q. 4. How can the direction of rotation of the motor be reversed ?

Ans. By transposing or changing over any two line leads, as shown in Fig. 35.57.

Q. 5. Why are induction motors called asynchronous ?

Ans. Because their rotors can never run with the synchronous speed.

Q. 6. How does the slip vary with load ?

Ans. The greater the load, greater is the slip or slower is the rotor speed.

Q. 7. What modifications would be necessary if a motor is required to operate on voltage different from that for which it was originally designed ?

Ans. The number of conductors per slot will have to be changed in the same ratio as the change in voltage. If the voltage is doubled, the number of conductors per slot will have to be doubled.

Q. 8. Enumerate the possible reasons if a 3-phase motor fails to start.

Ans. Any one of the following reasons could be responsible :

1. one or more fuses may be blown.
2. voltage may be too low.
3. the starting load may be too heavy.
4. worn bearings due to which the armature may be touching field laminae, thus introducing excessive friction.

Q. 9. A motor stops after starting *i.e.* it fails to carry load. What could be the causes ?

Ans. Any one of the following:

1. hot bearings, which increase the load by excessive friction.
2. excessive tension on belt, which causes the bearings to heat.
3. failure of short cut-out switch.
4. single-phasing on the running position of the starter.

Q. 10. Which is the usual cause of blow-outs in induction motors ?

Ans. The commonest cause is single-phasing.

Q. 11. What is meant by 'single-phasing' and what are its causes ?

Ans. By single-phasing is meant the opening of one wire (or leg) of a three-phase circuit whereupon the remaining leg at once becomes single-phase. When a three-phase circuit functions normally, there are three distinct currents flowing in the circuit. As is known, any two of these currents use the third wire as the return path *i.e.* one of the three phases acts as a return path for the other two. Obviously, an open circuit in one leg kills two of the phases and there will be only one current or phase working, even though two wires are left intact. The remaining phase attempts to carry all the load. The usual cause of single-phasing is, what is generally referred to as ***running fuse***, which is a fuse whose current-carrying capacity is equal to the full-load current of the motor connected in the circuit. This fuse will blow-out whenever there is overload (either momentary or sustained) on the motor.

Q. 12. What happens if single-phasing occurs when the motor is running ? And when it is stationary ?

Ans. (*i*) If already running and carrying half load or less, the motor will continue running as a single-phase motor on the remaining single-phase supply, without damage because half loads do not blow normal fuses.

(*ii*) If motor is very heavily loaded, then it will stop under single-phasing and since it can neither restart nor blow out the remaining fuses, the burn-out is very prompt.

A stationary motor will not start with one line broken. In fact, due to heavy standstill current, it is likely to burn-out quickly unless immediately disconnected.

Q. 13. Which phase is likely to burn-out in a single-phasing delta-connected motor, shown in Fig. 35.58.

Ans. The Y-phase connected across the live or operative lines carries nearly three times its normal current and is the one most likely to burn-out.

The other two phases R and B, which are in series across L_2 and L_3 carry more than their full-load currents.

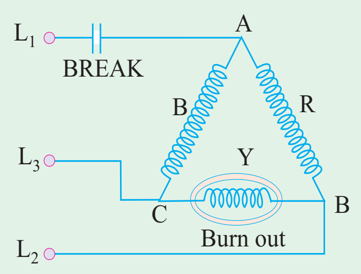


Fig. 35.58

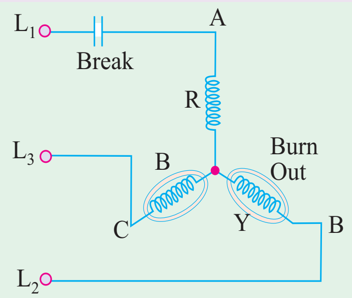


Fig. 35.59

Q. 14. What currents flow in single-phasing star-connected motor of Fig. 35.59.

Ans. With L_1 disabled, the currents flowing in L_2 and L_3 and through phases Y and B in series will be of the order of 250 per cent of the normal full-load current, 160 per cent on 3/4 load and 100 per cent on 1/2 load.

Q. 15. How can the motors be protected against single-phasing ?

Ans. (*i*) By incorporating a combined overload and single-phasing relay in the control gear.

(*ii*) By incorporating a phase-failure relay in the control gear. The relay may be either voltage or current-operated.

Q. 16. Can a 3-phase motor be run on a single-phase line ?

Ans. Yes, it can be. But a phase-splitter is essential.

Q. 17. What is a meant by a phase-splitter ?

Ans. It is a device consisting of a number of capacitors so connected in the motor circuit that it produces, from a single input wave, three output waves which differ in phase from each other.

Q. 18. What is the standard direction of rotation of an induction motor ?

Ans. Counterclockwise, when looking from the front end *i.e.* non-driving end of the motor.

Q. 19. Can a wound-motor be reversed by transposing any two leads from the slip-rings ?

Ans. No. There is only one way of doing so *i.e.* by transposing any two line leads.

Q. 20. What is jogging ?

Ans. It means inching a motor i.e. make it move a little at a time by constant starting and stopping.

Q. 21. What is meant by plugging ?

Ans. It means stopping a motor by instantaneously reversing it till it stops.

Q. 22. What are the indications of winding faults in an induction motor ?

Ans. Some of the indications are as under:

(*i*) excessive and unbalanced starting currents

(*ii*) some peculiar noises and (*iii*) overheating.

OBJECTIVE TESTS – 35

1. In the circle diagram for a 3-ϕ induction motor, the diameter of the circle is determined by
(*a*) rotor current
(*b*) exciting current
(*c*) total stator current
(*d*) rotor current referred to stator.

2. Point out the WRONG statement.
Blocked rotor test on a 3-ϕ induction motor helps to find
(*a*) short-circuit current with normal voltage
(*b*) short-circuit power factor
(*c*) fixed losses
(*d*) motor resistance as referred to stator.

3. In the circle diagram of an induction motor, point of maximum input lies on the tangent drawn parallel to
(*a*) output line
(*b*) torque line
(*c*) vertical axis
(*d*) horizontal axis.

4. An induction motor has a short-circuit current 7 times the full-load current and a full-load slip of 4 per cent. Its line-starting torque is .... times the full-load torque.
(*a*) 7 (*b*) 1.96
(*c*) 4 (*d*) 49

5. In a SCIM, torque with autostarter is .... times the torque with direct-switching.
(*a*) K^2 (b) K
(c) $1/K^2$ (d) $1/K$
where K is the transformation ratio of the autostarter.

6. If stator voltage of a SCIM is reduced to 50 per cent of its rated value, torque developed is reduced by .... per cent of its full-load value.
(*a*) 50 (*b*) 25
(c) 75 (*d*) 57.7

7. For the purpose of starting an induction motor, a Y-Δ switch is equivalent to an auto-starter of ratio.......per cent.
(*a*) 33.3 (*b*) 57.7
(*c*) 73.2 (*d*) 60.

8. A double squirrel-cage motor (DSCM) scores over SCIM in the matter of
(*a*) starting torque
(b) high efficiency under running conditions
(c) speed regulation under normal operating conditions
(*d*) all of the above.

9. In a DSCM, outer cage is made of high resistance metal bars primarily for the purpose of increasing its
(*a*) speed regulation
(*b*) starting torque
(*c*) efficiency
(*d*) starting current.

10. A SCIM with 36-slot stator has two separate windings : one with 3 coil groups/ phase/pole and the other with 2 coil groups/phase/pole. The obtainable two motor speeds would be in the ratio of
(*a*) 3 : 2 (*b*) 2 : 3
(c) 2 : 1 (*d*) 1 : 2

11. A 6-pole 3-ϕ induction motor taking 25 kW from a 50-Hz supply is cumulatively-cascaded to a 4-pole motor. Neglecting all losses, speed of the 4-pole motor would be .... r.p.m.
(*a*) 1500 (*b*) 1000
(c) 600 (*d*) 3000.
and its output would be .... kW.
(*e*) 15 (*f*) 10
(g) 50/3 (*h*) 2.5.

12. Which class of induction motor will be well suited for large refrigerators?
(*a*) Class E (*b*) Class B
(*c*) Class F (*d*) Class C

13. In a Schrage motor operating at supersynchronous speed, the injected emf and the standstill secondary induced emf
(*a*) are in phase with each other
(*b*) are at 90º in time phase with each other
(*c*) are in phase opposition
(*d*) none of the above.
(Power App.-III, Delhi Univ. July 1987)

14. For starting a Schrage motor, 3-ϕ supply is connected to
(*a*) stator
(*b*) rotor via slip-rings
(*c*) regulating winding
(*d*) secondary winding via brushes.

15. Two separate induction motors, having 6 poles and 5 poles respectively and their cascade combination from 60 Hz, 3-phase supply can give the following synchronous speeds in rpm
(*a*) 720, 1200, 1500 and 3600
(*b*) 720, 1200 1800
(*c*) 600, 1000, 15000
(*d*) 720 and 3000
(Power App.-II, Delhi Univ.Jan 1987)

16. Mark the WRONG statement.
A Schrage motor is capable of behaving as a/ an
(*a*) inverted induction motor
(*b*) slip-ring induction motor
(*c*) shunt motor
(*d*) series motor
(*e*) synchronous motor.

17. When a stationary 3-phase induction motor is switched on with one phase disconnected
(*a*) it is likely to burn out quickly unless immediately disconnected
(b) it will start but very slowly
(c) it will make jerky start with loud growing noise
(d) remaining intact fuses will be blown out due to heavy inrush of current

18. If single-phasing of a 3-phase induction motor occurs under running conditions, it
(*a*) will stall immediately
(*b*) will keep running though with slightly increased slip
(*c*) may either stall or keep running depending on the load carried by it
(*d*) will become noisy while it still keeps running.

ANSWERS

1. *c* **2.** *c* **3.** *d* **4.** *b* **5.** *a* **6.** *c* **7.** *b* **8.** *d* **9.** *b* **10.** *a* **11.** *c*, *f* **12.** *d* **13.** *a* **14.** *b* **15.** *a* **16.** *d* **17.** *a* **18.** *c*