

# CHAPTER 38

## Learning Objectives

- Synchronous Motor-General
- Principle of Operation
- Method of Starting
- Motor on Load with Constant Excitation
- Power Flow within a Synchronous Motor
- Equivalent Circuit of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Synchronous Motor with Different Excitations
- Effect of increased Load with Constant Excitation
- Effect of Changing Excitation of Constant Load
- Different Torques of a Synchronous Motor
- Power Developed by a Synchronous Motor
- Alternative Expression for Power Developed
- Various Conditions of Maxima
- Salient Pole Synchronous Motor
- Power Developed by a Salient Pole Synchronous Motor
- Effects of Excitation on Armature Current and Power Factor
- Constant-Power Lines
- Construction of V-curves
- Hunting or Surging or Phase Swinging
- Methods of Starting
- Procedure for Starting a Synchronous Motor
- Comparison between Synchronous and Induction Motors
- Synchronous Motor Applications

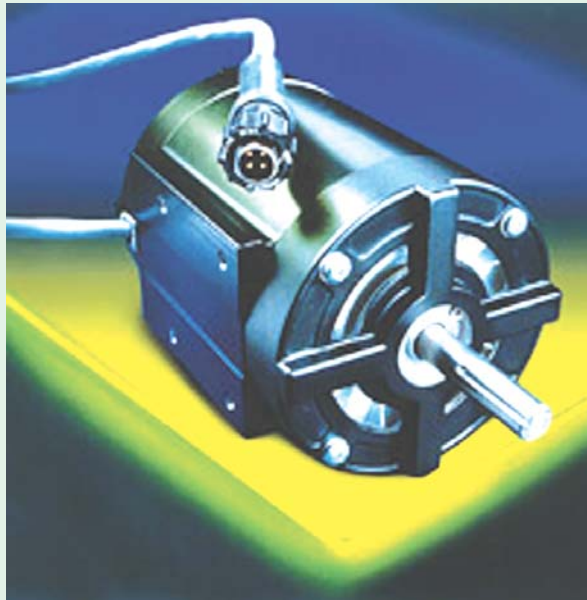
## SYNCHRONOUS MOTOR



Rotary synchronous motor for lift applications

### 38.1. Synchronous Motor—General

A synchronous motor (Fig. 38.1) is electrically identical with an alternator or a.c. generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of d.c. machines. Most



Synchronous motor

synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m.

Some characteristic features of a synchronous motor are worth noting :

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency (because  $N_s = 120f/P$ ).
2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.
3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

### 38.2. Principle of Operation

As shown in Art. 34.7, when a 3- $\phi$  winding is fed by a 3- $\phi$  supply, then a magnetic flux of constant magnitude but *rotating* at synchronous speed, is produced. Consider a two-pole stator of Fig. 38.2, in which are shown two stator poles (marked  $N_s$  and  $S_s$ ) rotating at synchronous speed, say, in clockwise direction. With the rotor position as shown, suppose the stator poles are at that instant situated at points *A* and *B*. The two similar poles, *N* (of rotor) and  $N_s$  (of stator) as well as *S* and  $S_s$  will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

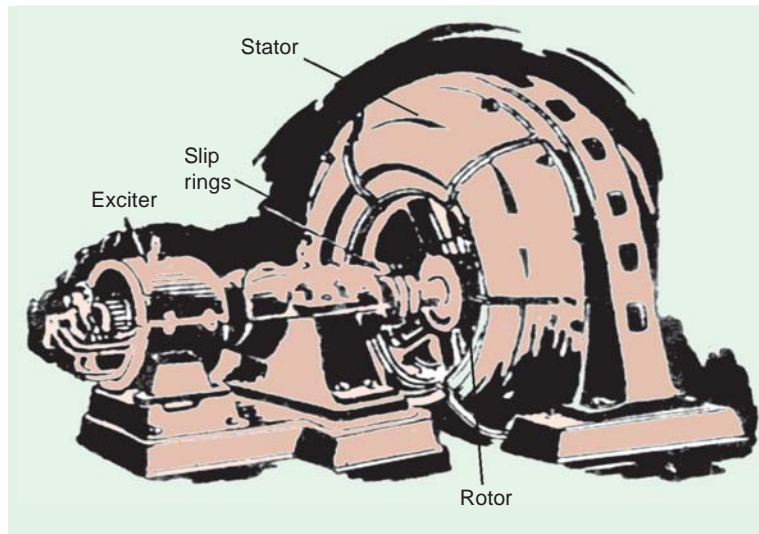


Fig. 38.1

But half a period later, stator poles, having rotated around, interchange their positions *i.e.*  $N_s$  is at point  $B$  and  $S_s$  at point  $A$ . Under these conditions,  $N_s$  attracts  $S$  and  $S_s$  attracts  $N$ . Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.

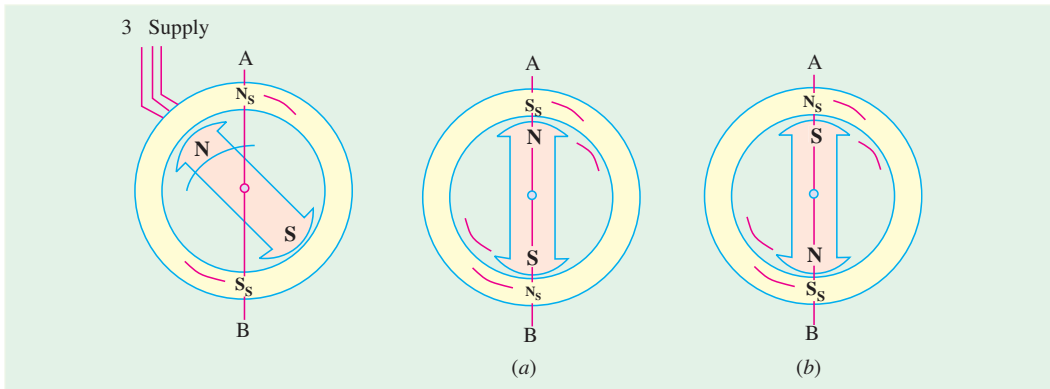


Fig. 38.2

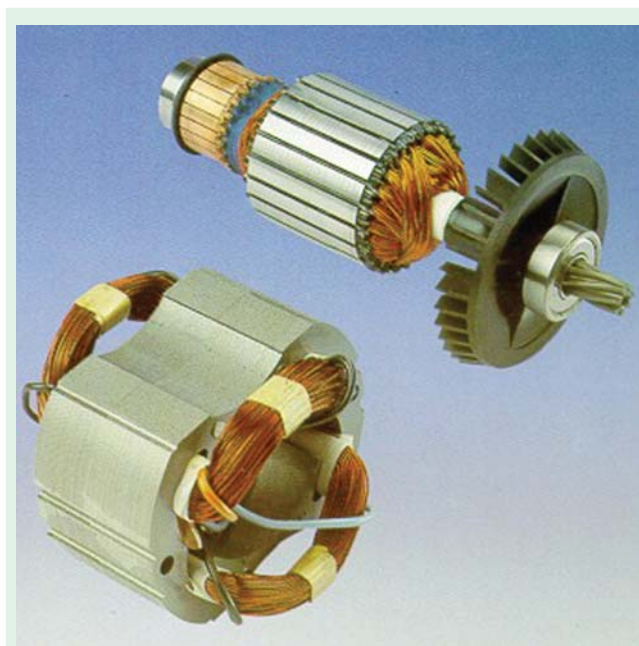
Fig. 38.3

Fig. 38.3

Now, consider the condition shown in Fig. 38.3 (a). The stator and rotor poles are attracting each other. Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig. 38.3 (b). Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e.*, clockwise torque, as shown in Fig. 38.3.

### 38.3. Method of Starting

The rotor (which is as yet unexcited) is speeded up to synchronous / near synchronous speed by some arrangement and then excited by the d.c. source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. The synchronous speed is given by the usual relation  $N_s = 120f / P$ .



The rotor and the stator parts of motor.

However, it is important to understand that the arrangement between the stator and rotor poles is *not an absolutely rigid one*. As the load on the motor is increased, the rotor progressively tends to fall back *in phase* (but *not* in speed as in d.c. motors) by some angle (Fig. 38.4) *but it still continues to run synchronously*. The value of this load angle or coupling angle (as it is called) depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, say,  $\alpha$ .

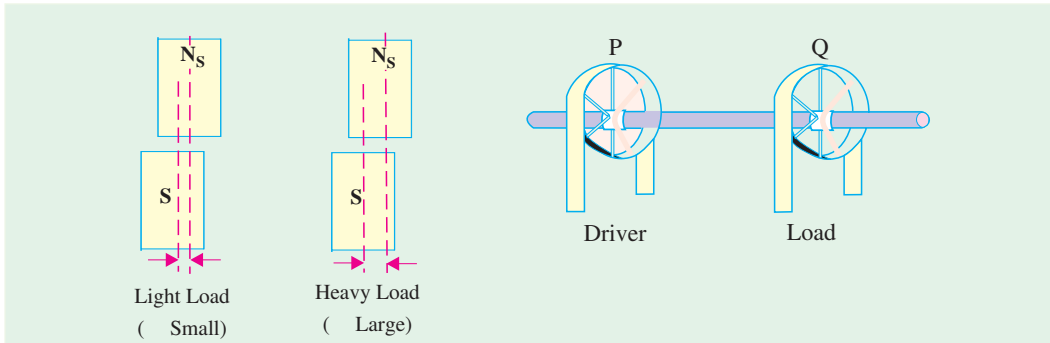


Fig. 38.4

Fig. 38.5

The working of a synchronous motor is, in many ways, similar to the transmission of mechanical power by a shaft. In Fig. 38.5 are shown two pulleys  $P$  and  $Q$  transmitting power from the driver to the load. The two pulleys are assumed to be keyed together (just as stator and rotor poles are interlocked) hence they run at exactly the same (average) speed. When  $Q$  is loaded, it slightly falls behind owing to the twist in the shaft (twist angle corresponds to  $\alpha$  in motor), the angle of twist, in fact, being a measure of the torque transmitted. It is clear that unless  $Q$  is so heavily loaded as to break the coupling, both *pulleys must run at exactly the same (average) speed*.

### 38.4. Motor on Load with Constant Excitation

Before considering as to what goes on inside a synchronous motor, it is worthwhile to refer briefly to the d.c. motors. We have seen (Art. 29.3) that when a d.c. motor is running on a supply of, say,  $V$  volts then, on rotating, a back e.m.f.  $E_b$  is set up in its armature conductors. The resultant voltage across the armature is  $(V - E_b)$  and it causes an armature current  $I_a = (V - E_b) / R_a$  to flow where  $R_a$  is armature circuit resistance. The value of  $E_b$  depends, among other factors, on the speed of the rotating armature. The mechanical power developed in armature depends on  $E_b I_a$  ( $E_b$  and  $I_a$  being in opposition to each other).

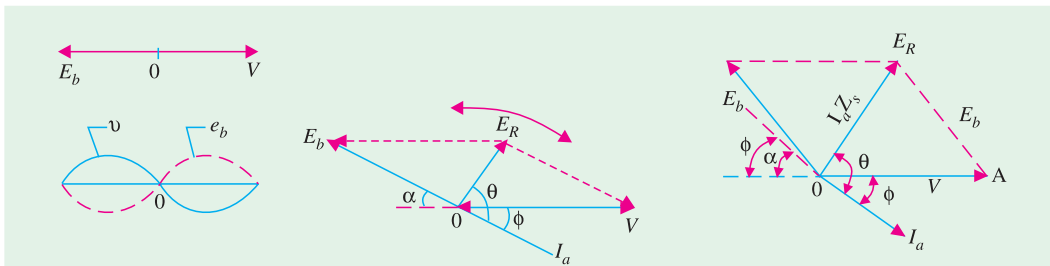


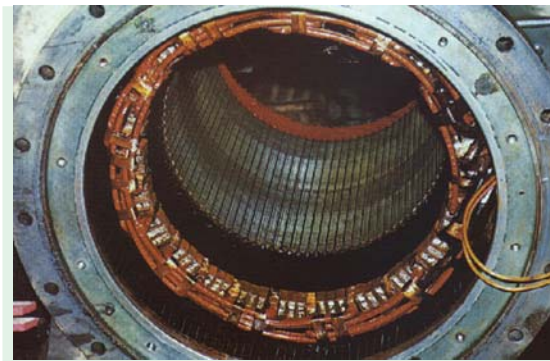
Fig. 38.6

Fig. 38.7

Fig. 38.8

Similarly, in a synchronous machine, a back e.m.f.  $E_b$  is set up in the armature (stator) by the rotor flux which opposes the applied voltage  $V$ . This back e.m.f. depends on rotor excitation only (and not on speed, as in d.c. motors). The net voltage in armature (stator) is the *vector difference* (not arithmetical, as in d.c. motors) of  $V$  and  $E_b$ . Armature current is obtained by dividing this *vector* difference of voltages by armature impedance (not resistance as in d.c. machines).

Fig. 38.6 shows the condition when the motor (properly synchronized to the supply) is running on *no-load* and has *no losses*.\* and is having field excitation which makes  $E_b = V$ . It is seen that vector difference of  $E_b$  and  $V$  is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be met by it. In other words, the motor just floats.



Stator of synchronous motor

If motor is on no-load, but it has losses, then the vector for  $E_b$  falls back (vectors are rotating anti-clockwise) by a certain small angle  $\alpha$  (Fig. 38.7), so that a resultant voltage  $E_R$  and hence current  $I_a$  is brought into existence, which supplies losses.\*\*

If, now, the motor is loaded, then its rotor will further fall back *in phase* by a greater value of angle  $\alpha$  – called the load angle or coupling angle (corresponding to the twist in the shaft of the pulleys). The resultant voltage  $E_R$  is increased and motor draws an increased armature current (Fig. 38.8), though at a slightly decreased power factor.

### 38.5. Power Flow within a Synchronous Motor

Let  $R_a$  = armature resistance / phase ;  $X_S$  = synchronous reactance / phase

then  $Z_S = R_a + jX_S$  ;  $I_a = \frac{E_R}{Z_S} = \frac{V - E_b}{Z_S}$  ; Obviously,  $V = E_b + I_a Z_S$

The angle  $\theta$  (known as internal angle) by which  $I_a$  lags behind  $E_R$  is given by  $\tan \theta = X_S / R_a$ .

If  $R_a$  is negligible, then  $\theta = 90^\circ$ . Motor input =  $V I_a \cos \phi$  —per phase

Here,  $V$  is applied voltage / phase.

Total input for a star-connected, 3-phase machine is,  $P = \sqrt{3} V_L \cdot I_L \cos \phi$ .

The mechanical power developed in the rotor is

$$P_m = \text{back e.m.f.} \times \text{armature current} \times \text{cosine of the angle between the two } i.e., \text{ angle between } I_a \text{ and } E_b \text{ reversed.}$$

$$= E_b I_a \cos (\alpha - \phi) \text{ per phase} \quad \dots \text{Fig. 38.8}$$

Out of this power developed, some would go to meet iron and friction and excitation losses. Hence, the power available at the shaft would be less than the developed power by this amount.

Out of the input power / phase  $V I_a \cos \phi$ , and amount  $I_a^2 R_a$  is wasted in armature\*\*\*, the rest ( $V \cdot I_a \cos \phi - I_a^2 R_a$ ) appears as mechanical power in rotor; out of it, iron, friction and excitation losses are met and the rest is available at the shaft. If power input / phase of the motor is  $P$ , then

$$P = P_m + I_a^2 R_a$$

or mechanical power in rotor  $P_m = P - I_a^2 R_a$  —per phase

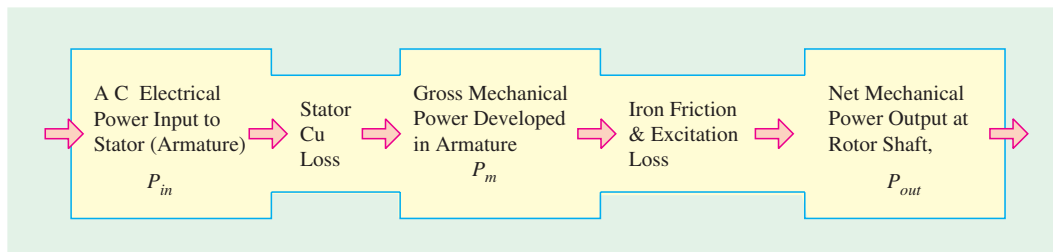
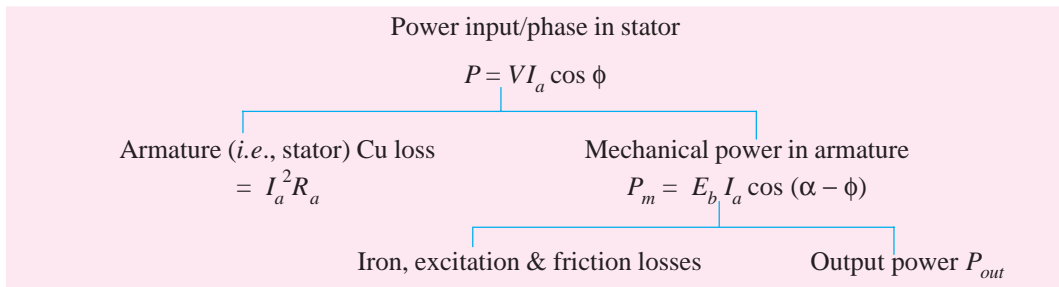
For three phases  $P_m = \sqrt{3} V_L I_L \cos \phi - 3 I_a^2 R_a$

The per phase power development in a synchronous machine is as under :

\* This figure is exactly like Fig. 37.74 for alternator except that it has been shown horizontally rather than vertically.

\*\* It is worth noting that magnitude of  $E_b$  does not change, only its phase changes. Its magnitude will change only when rotor dc excitation is changed *i.e.*, when magnetic strength of rotor poles is changed.

\*\*\* The Cu loss in rotor is not met by motor ac input, but by the dc source used for rotor excitation.



### 38.6. Equivalent Circuit of a Synchronous Motor

Fig. 38.9 (a) shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. 38.9 (b) that the phase applied voltage  $V$  is the vector sum of reversed back e.m.f. *i.e.*,  $-E_b$  and the impedance drop  $I_a Z_s$ . In other words,  $V = (-E_b + I_a Z_s)$ . The angle  $\alpha^*$  between the phasor for  $V$  and  $E_b$  is called the load angle or power angle of the synchronous motor.

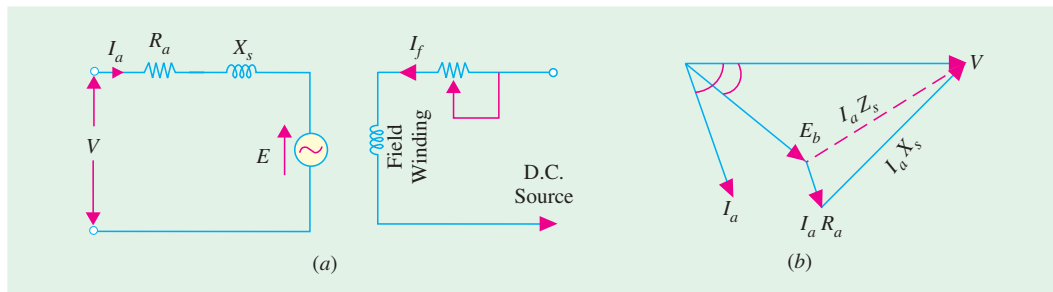


Fig. 38.9

### 38.7. Power Developed by a Synchronous Motor

Except for very small machines, the armature resistance of a synchronous motor is negligible as compared to its synchronous reactance. Hence, the equivalent circuit for the motor becomes as shown in Fig. 38.10 (a). From the phasor diagram of Fig. 38.10 (b), it is seen that

$$AB = E_b \sin \alpha = I_a X_s \cos \phi$$

$$\text{or } VI_a \cos \phi = \frac{E_b V}{X_s} \sin \alpha$$

Now,  $VI_a \cos \phi =$  motor power input/phase

\* This angle was designated as  $\delta$  when discussing synchronous generators.

$$\begin{aligned} \therefore P_{in} &= \frac{E_b V}{X_s} \sin \alpha && \dots \text{per phase}^* \\ &= 3 \frac{E_b V}{X_s} \sin \alpha && \dots \text{for three phases} \end{aligned}$$

Since stator Cu losses have been neglected,  $P_{in}$  also represents the gross mechanical power  $\{P_m\}$  developed by the motor.

$$\therefore P_m = \frac{3E_b V}{X_s} \sin \alpha$$

The gross torque developed by the motor is  $T_g = 9.55 P_m / N_s \text{ N-m} \dots \text{Ns in rpm.}$

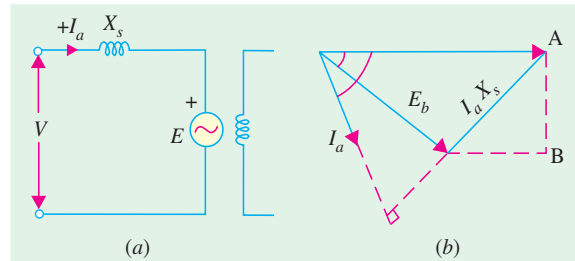


Fig. 38.10

**Example 38.1.** A 75-kW, 3- $\phi$ , Y-connected, 50-Hz, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency excluding field and stator losses, is 95% and  $X_s = 2.5 \Omega$ . Calculate (i) mechanical power developed (ii) armature current (iii) back e.m.f. (iv) power angle and (v) maximum or pull-out torque of the motor.

**Solution.**  $N_s = 120 \times 50/4 = 1500 \text{ rpm} = 25 \text{ rps}$

(i)  $P_m = P_{in} = P_{out} / \eta = 75 \times 10^3 / 0.95 = 78,950 \text{ W}$

(ii) Since power input is known

$$\therefore \sqrt{3} \times 440 \times I_a \times 0.8 = 78,950; \quad I_a = 129 \text{ A}$$

(iii) Applied voltage/phase =  $440/\sqrt{3} = 254 \text{ V}$ . Let  $V = 254 \angle 0^\circ$  as shown in Fig. 38.11.

Now,  $V = E_b + j I X_s$  or  $E_b = V - j I_a X_s = 254 \angle 0^\circ - 129 \angle 36.9^\circ \times 2.5 \angle 90^\circ = 250 \angle 0^\circ - 322 \angle 126.9^\circ = 254 - 322 (\cos 126.9^\circ + j \sin 126.9^\circ) = 254 - 322 (-0.6 + j 0.8) = 516 \angle -30^\circ$

(iv)  $\therefore \alpha = -30^\circ$

(v) pull-out torque occurs when  $\alpha = 90^\circ$

$$\text{maximum } P_m = 3 \frac{E_b V}{X_s} \sin \delta = 3 \frac{256 \times 516}{2.5} = \sin 90^\circ = 157,275 \text{ W}$$

$$\therefore \text{pull-out torque} = 9.55 \times 157,275 / 1500 = 1,000 \text{ N-m}$$

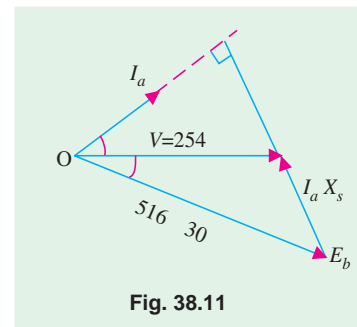


Fig. 38.11

### 38.8. Synchronous Motor with Different Excitations

A synchronous motor is said to have normal excitation when its  $E_b = V$ . If field excitation is such that  $E_b < V$ , the motor is said to be **under-excited**. In both these conditions, it has a lagging power factor as shown in Fig. 38.12.

On the other hand, if d.c. field excitation is such that  $E_b > V$ , then motor is said to be **over-excited** and draws a leading current, as shown in Fig. 38.13 (a). There will be some value of excitation for which armature current will be in phase with  $V$ , so that power factor will become unity, as shown in Fig. 38.13 (b).

\* Strictly speaking, it should be  $P_m = \frac{-E_b V}{X_s} \sin \alpha$

The value of  $\alpha$  and back e.m.f.  $E_b$  can be found with the help of vector diagrams for various power factors, shown in Fig. 38.14.

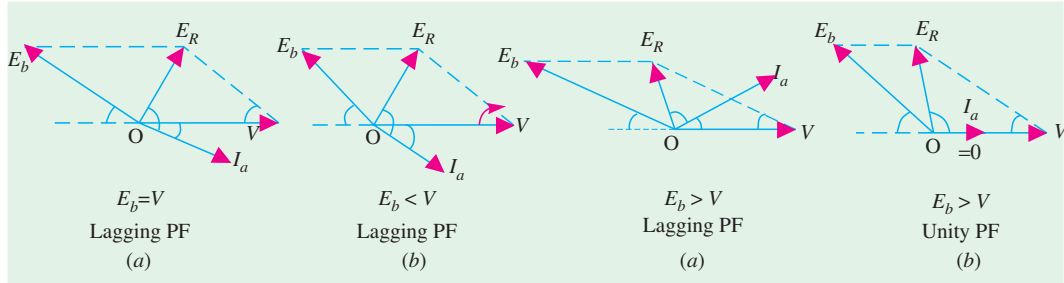


Fig. 38.12

Fig. 38.13

(i) **Lagging p.f.** As seen from Fig. 38.14 (a)

$$AC^2 = AB^2 + BC^2 = [V - E_R \cos(\theta - \phi)]^2 + [E_R \sin(\theta - \phi)]^2$$

$$\therefore E_b = \sqrt{[V - I_a Z_S \cos(\theta - \phi)]^2 + [I_a Z_S \sin(\theta - \phi)]^2}$$

$$\text{Load angle } \alpha = \tan^{-1} \left( \frac{BC}{AB} \right) = \tan^{-1} \left[ \frac{I_a Z_S \sin(\theta - \phi)}{V - I_a Z_S \cos(\theta - \phi)} \right]$$

(ii) **Leading p.f.** [38.14 (b)]

$$E_b = V + I_a Z_S \cos[180^\circ - (\theta + \phi)] + j I_a Z_S \sin[180^\circ - (\theta + \phi)]$$

$$\alpha = \tan^{-1}$$

(iii) **Unity p.f.** [Fig. 38.14 (c)]

Here,  $OB = I_a R_a$  and  $BC = I_a X_S$

$$\therefore E_b = (V - I_a R_a) + j I_a X_S; \alpha = \tan^{-1}$$

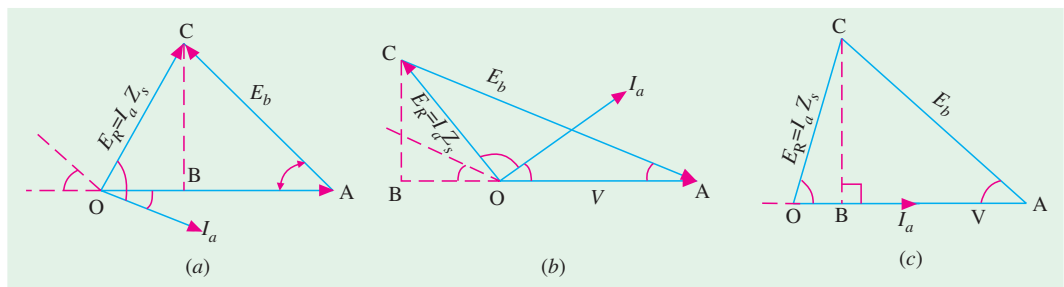


Fig. 38.14

### 38.9. Effect of Increased Load with Constant Excitation

We will study the effect of increased load on a synchronous motor under conditions of normal, under and over-excitation (ignoring the effects of armature reaction). With normal excitation,  $E_b = V$ , with under excitation,  $E_b < V$  and with over-excitation,  $E_b > V$ . Whatever the value of excitation, it would be kept **constant** during our discussion. It would also be assumed that  $R_a$  is negligible as compared to  $X_S$  so that phase angle between  $E_R$  and  $I_a$  i.e.,  $\theta = 90^\circ$ .

(i) **Normal Excitation**

Fig. 38.15. (a) shows the condition when motor is running with light load so that (i) torque angle



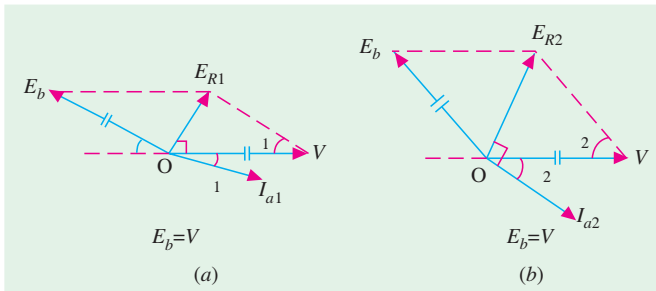


Fig. 38.15

$\alpha_1$  is small (ii) so  $E_{R1}$  is small (iii) hence  $I_{a1}$  is small and (iv)  $\phi_1$  is small so that  $\cos \phi_1$  is large.

Now, suppose that load on the motor is **increased** as shown in Fig. 38.15 (b). For meeting this extra load, motor must develop more torque by drawing more armature current. Unlike a d.c. motor, a synchronous motor cannot increase its  $I_a$  by

decreasing its speed and hence  $E_b$  because both are constant in its case. What actually happens is as under :

1. rotor falls back **in phase i.e.**, load angle increases to  $\alpha_2$  as shown in Fig. 38.15 (b),
2. the resultant voltage in armature is increased **considerably** to new value  $E_{R2}$ ,
3. as a result,  $I_{a1}$  increases to  $I_{a2}$ , thereby increasing the torque developed by the motor,
4.  $\phi_1$  increases to  $\phi_2$ , so that power factor **decreases** from  $\cos \phi_1$  to the new value  $\cos \phi_2$ .

Since increase in  $I_a$  is much greater than the **slight** decrease in power factor, the torque developed by the motor is **increased** (on the whole) to a new value sufficient to meet the extra load put on the motor. It will be seen that essentially it is by increasing its  $I_a$  that the motor is able to carry the extra load put on it.



Geared motor added to synchronous servo motor line offers a wide range of transmission ratios, and drive torques.

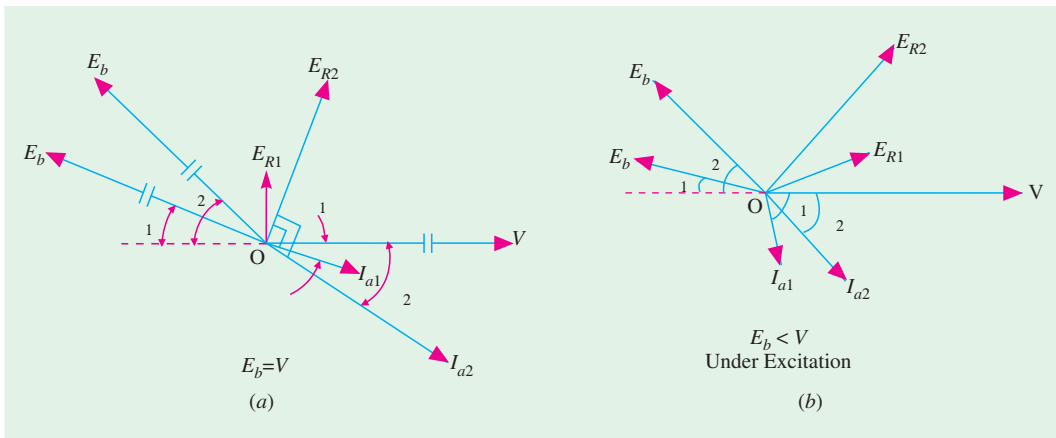


Fig. 38.16

A phase summary of the effect of increased load on a synchronous motor at **normal excitation** is shown in Fig. 38.16 (a) It is seen that there is a comparatively much greater **increase** in  $I_a$  than in  $\phi$ .

**(ii) Under-excitation**

As shown in Fig. 38.16 (b), with a small load and hence, small torque angle  $\alpha_1$ ,  $I_{a1}$  lags behind  $V$  by a **large** phase angle  $\phi_1$  which means poor power factor. Unlike normal excitation, a much larger armature current must flow for developing the same power because of poor power factor. That is why  $I_{a1}$  of Fig. 38.16 (b) is larger than  $I_{a1}$  of Fig. 38.15 (a).

As load increases,  $E_{R1}$  increases to  $E_{R2}$ , consequently  $I_{a1}$  increases to  $I_{a2}$  and p.f. angle **decreases** from  $\phi_1$  to  $\phi_2$  or p.f. **increases** from  $\cos \phi_1$  to  $\cos \phi_2$ . Due to increase both in  $I_a$  and p.f., power generated by the armature increases to meet the increased load. As seen, in this case, **change in power factor is more than the change in  $I_a$ .**

**(iii) Over-excitation**

When running on light load,  $\alpha_1$  is small but  $I_{a1}$  is comparatively larger and **leads  $V$**  by a larger angle  $\phi_1$ . Like the under-excited motor, as more load is applied, the power factor improves and **approaches** unity. The armature current also increases thereby producing the necessary increased armature power to meet the increased applied load (Fig. 38.17). However, it should be noted that in this case, power factor angle  $\phi$  decreases (or p.f. increases) at a faster rate than the armature current thereby producing the necessary increased power to meet the increased load applied to the motor.

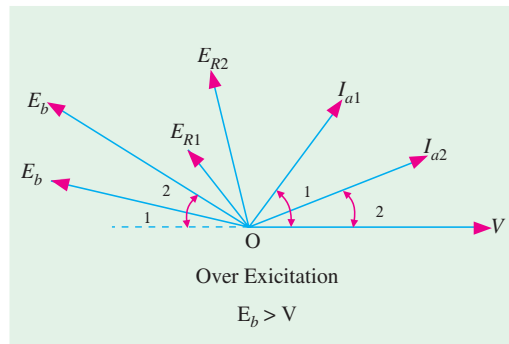


Fig. 38.17

**Summary**

The main points regarding the above three cases can be summarized as under :

1. As load on the motor increases,  $I_a$  **increases regardless of excitation.**
2. For under-and over-excited motors, p.f. tends to approach unity with increase in load.
3. Both with under-and over-excitation, change in p.f. is greater than in  $I_a$  with increase in load.
4. With normal excitation, when load is increased change in  $I_a$  is greater than in p.f. which tends to become increasingly lagging.

**Example 38.2.** A 20-pole, 693-V, 50-Hz, 3- $\phi$ ,  $\Delta$ -connected synchronous motor is operating at no-load with normal excitation. It has armature resistance per phase of zero and synchronous reactance of 10  $\Omega$ . If rotor is retarded by 0.5° (mechanical) from its synchronous position, compute.

- |  |                                 |
|--|---------------------------------|
| (i) rotor displacement in electrical degrees | (iii) armature current / phase  |
| (ii) armature emf / phase                    | (v) power developed by armature |
| (iv) power drawn by the motor                |                                 |

How will these quantities change when motor is loaded and the rotor displacement increases to 5° (mechanical) ?

(Elect. Machines, AMIE Sec. B, 1993)

**Solution.** (a) 0.5° (mech) Displacement [Fig 38.18 (a)]

$$\begin{aligned}
 (i) \quad \alpha \text{ (elect.)} &= \frac{P}{2} \times \alpha \text{ (mech)} \\
 \therefore \alpha \text{ (elect)} & \\
 &= \frac{20}{2} \times 0.5 = 5^\circ \text{ (elect)}
 \end{aligned}$$

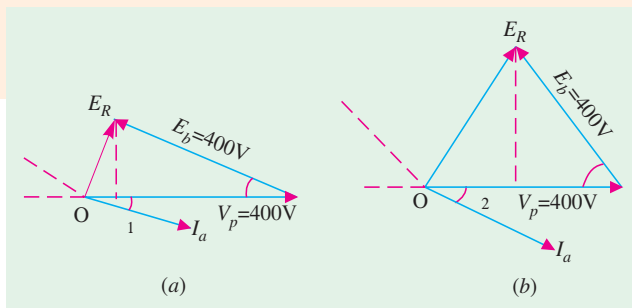


Fig. 38.18

- (ii)  $V_p = V_L / \sqrt{3} = 693 / \sqrt{3}$   
 $= 400 \text{ V}$ ,  
 $E_b = V_p = 400 \text{ V}$   
 $\therefore E_R = (V_p - E_b \cos \alpha) + j E_b \sin \alpha = (400 - 400 \cos 5^\circ + j 400 \sin 5^\circ)$   
 $= 1.5 + j 35 = \mathbf{35 \angle 87.5 \text{ V/phase}}$
- (iii)  $Z_S = 0 + j10 = 10 \angle 90^\circ$ ;  $I_a = E_R / Z_S = 35 \angle 87.5^\circ / 10 \angle 90^\circ = \mathbf{3.5 \angle -2.5^\circ \text{ A/phase}}$   
 Obviously,  $I_a$  lags behind  $V_p$  by  $2.5^\circ$
- (iv) Power input/phase  $V_p I_a \cos \phi = 400 \times 3.5 \times \cos 2.5^\circ = 1399 \text{ W}$   
 Total input power  $= 3 \times 1399 = \mathbf{4197 \text{ W}}$
- (v) Since  $R_a$  is negligible, armature Cu loss is also negligible. Hence 4197 W also represent power developed by armature.
- (b)  $5^\circ$  (mech) Displacement – Fig. 38.18 (b)
- (i)  $\alpha$  (elect)  $= \frac{20}{2} \times 5^\circ = \mathbf{50^\circ}$
- (ii)  $E_R = (400 - 400 \cos 50^\circ) + j400 \sin 50^\circ = 143 + j 306.4 = \mathbf{338.2 \angle 64.9^\circ}$
- (iii)  $I_a = 338.2 \times 64.9^\circ / 10 \angle 90^\circ = \mathbf{33.8 \angle -25.1^\circ \text{ A/phase}}$
- (iv) motor power/phase  $= V_p I_a \cos \phi = 400 \times 33.8 \cos 25.1^\circ = 12,244 \text{ W}$   
 Total power  $= 3 \times 12,244 = 36,732 \text{ W} = \mathbf{36.732 \text{ kW}}$

It is seen from above that as motor load is increased

1. rotor displacement increases from  $5^\circ$  (elect) to  $50^\circ$  (elect) *i.e.*  $E_b$  falls back in phase considerably.
2.  $E_R$  increases from 35 V to 338 V/phase
3.  $I_a$  increases from 3.5 A to 33.8 A
4. angle  $\phi$  increases from  $2.5^\circ$  to  $25.1^\circ$  so that p.f. *decreases* from 0.999 (lag) to 0.906 (lag)
5. increase in power is almost *directly proportional to increase in load angle*.

Obviously, increase in  $I_a$  is much more than decrease in power factor.

It is interesting to note that not only power but even  $I_a$ ,  $E_R$  and  $\phi$  also increase almost as many times as  $\alpha$ .

### Special Illustrative Example 38.3

#### Case of Cylindrical Rotor Machine :

A 3-Phase synchronous machine is worked as follows: Generator - mode : 400 V/Ph, 32 A/Ph, Unity p.f.  $X_S = 10$  ohms. Motoring - mode : 400 V/Ph, 32 A/Ph, Unity p.f. ,  $X_S = 10$  ohms. Calculate  $E$  and  $\delta$  in both the cases and comment.

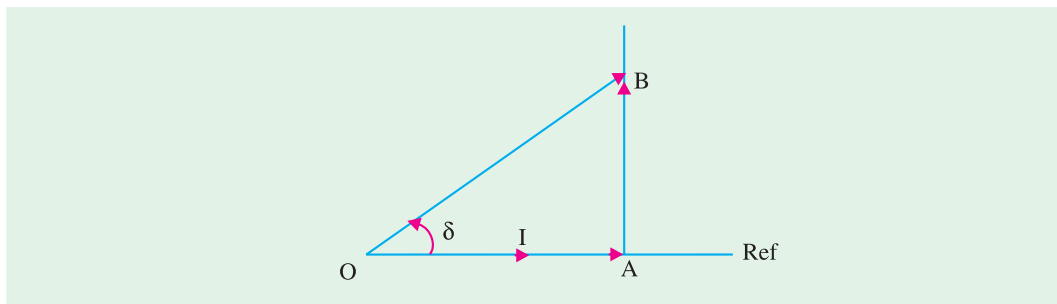


Fig. 38.19 (a) Generator-mode

**Solution.** In Fig. 38.19 (a),  $V = OA = 400$ ,  $IX_S = AB = 320$  V

$$E = OB = 512.25, \delta = \tan^{-1} \frac{320}{400} = 38.66^\circ$$

Total power in terms of parameters measurable at terminals (i.e.,  $V$ ,  $I$ , and  $\phi$ )  
 $= 3 V_{ph} I_{ph} \cos \phi = 3 \times 400 \times 32 = 38.4$  kW

Total power using other parameters  $= 3 \times \left[ \frac{VE}{X_S} \sin \delta \right] \times 10^{-3}$  kW  
 $= 3 \times \frac{400 \times 512.25}{10} \times (\sin 38.66^\circ) \times 10^{-3} = 38.4$  kW

Since losses are neglected, this power is the electrical output of generator and also is the required mechanical input to the generator.

**For motoring mode :**  $V = OA = 400$ ,  $-IX_S = AB = 320$   
 $E = OB = 512.25$ , as in Fig. 38.19 (b)

Hence,  $|\delta| = 38.66^\circ$ , as before.

**Comments :** The change in the sign of  $\delta$  has to be noted in the two modes. It is +ve for generator and -ve for motor.  $E$  happens to be equal in both the cases due to unity p.f. At other p.f., this will be different.

As before, power can be calculated in two ways and it will be electrical power input to motor and also the mechanical output of the motor.

Naturally, Power = 38.4 kW

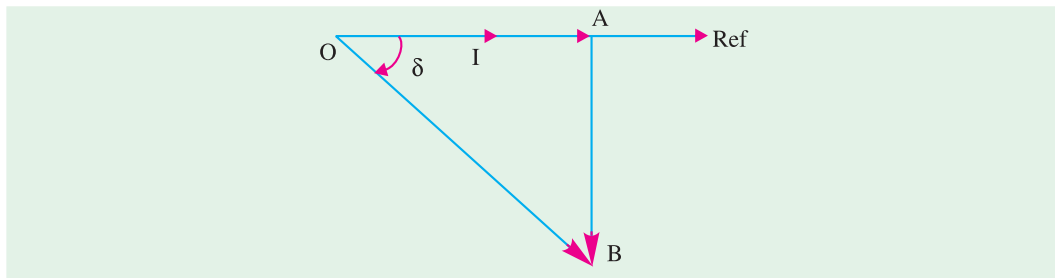


Fig. 38.19 (b) Motoring mode

### 38.10. Effect of Changing Excitation on Constant Load

As shown in Fig. 38.20 (a), suppose a synchronous motor is operating with normal excitation ( $E_b = V$ ) at unity p.f. with a given load. If  $R_a$  is negligible as compared to  $X_S$ , then  $I_a$  lags  $E_R$  by  $90^\circ$  and is in phase with  $V$  because p.f. is unity. The armature is drawing a power of  $V.I_a$  per phase which is enough to meet the mechanical load on the motor. Now, let us discuss the effect of decreasing or increasing the field excitation when the load applied to the motor **remains constant**.

#### (a) Excitation Decreased

As shown in Fig. 38.20 (b), suppose due to decrease in excitation, back e.m.f. is reduced to  $E_{b1}$  at the **same load angle**  $\alpha_1$ . The resultant voltage  $E_{R1}$  causes a lagging armature current  $I_{a1}$  to flow. Even though  $I_{a1}$  is larger than  $I_a$  in magnitude it is incapable of producing necessary power  $V I_{a1}$  for carrying the **constant** load because  $I_{a1} \cos \phi_1$  component is less than  $I_a$  so that  $V I_{a1} \cos \phi_1 < V I_a$ .

Hence, it becomes necessary for load angle to **increase** from  $\alpha_1$  to  $\alpha_2$ . It increases back e.m.f. from  $E_{b1}$  to  $E_{b2}$  which, in turn, increases resultant voltage from  $E_{R1}$  to  $E_{R2}$ . Consequently, armature current increases to  $I_{a2}$  whose in-phase component produces enough power ( $V I_{a2} \cos \phi_2$ ) to meet the constant load on the motor.

**(b) Excitation Increased**

The effect of increasing field excitation is shown in Fig. 38.20 (c) where increased  $E_{b1}$  is shown at the original load angle  $\alpha_1$ . The resultant voltage  $E_{R1}$  causes a **leading** current  $I_{a1}$  whose in-phase component is larger than  $I_a$ . Hence, armature develops more power than the load on the motor. Accordingly, load angle **decreases** from  $\alpha_1$  to  $\alpha_2$  which decreases resultant voltage from  $E_{R1}$  to  $E_{R2}$ . Consequently, armature current decreases from  $I_{a1}$  to  $I_{a2}$  whose in-phase component  $I_{a2} \cos \phi_2 = I_a$ . In that case, armature develops power sufficient to carry the constant load on the motor.

Hence, we find that variations in the excitation of a synchronous motor running with a **given** load produce variations in its **load angle only**.

### 38.11. Different Torques of a Synchronous Motor

Various torques associated with a synchronous motor are as follows:

1. starting torque
2. running torque
3. pull-in torque and
4. pull-out torque

**(a) Starting Torque**

It is the torque (or turning effort) developed by the motor when full voltage is applied to its stator (armature) winding. It is also sometimes called **breakaway** torque. Its value may be as low as 10% as in the case of centrifugal pumps and as high as 200 to 250% of full-load torque as in the case of loaded reciprocating two-cylinder compressors.

**(b) Running Torque**

As its name indicates, it is the torque developed by the motor under running conditions. It is determined by the horse-power and speed of the **driven** machine. The peak horsepower determines the maximum torque that would be required by the driven machine. The motor must have a breakdown or a maximum running torque greater than this value in order to avoid stalling.

**(c) Pull-in Torque**

A synchronous motor is started as induction motor till it runs 2 to 5% below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously-rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

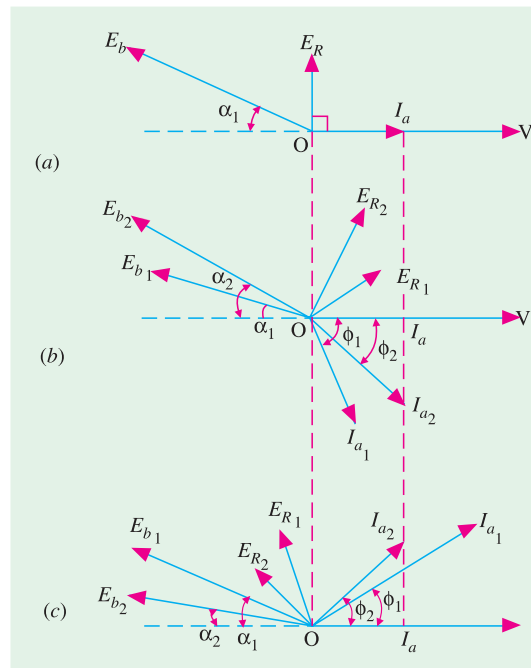


Fig. 38.20



Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions

(d) Pull-out Torque

The maximum torque which the motor can develop without pulling out of step or synchronism is called the pull-out torque.

Normally, when load on the motor is increased, its rotor progressively tends to fall back *in phase* by some angle (called load angle) behind the synchronously-revolving stator magnetic field though it keeps running synchronously. Motor develops maximum torque when its rotor is retarded by an angle of 90° (or in other words, it has shifted backward by a distance equal to half the distance between adjacent poles). Any further increase in load will cause the motor to pull out of step (or synchronism) and stop.

38.12. Power Developed by a Synchronous Motor

In Fig. 38.21,  $OA$  represents supply voltage/phase and  $I_a = I$  is the armature current,  $AB$  is back e.m.f. at a load angle of  $\alpha$ .  $OB$  gives the resultant voltage  $E_R = IZ_S$  (or  $IX_S$  if  $R_a$  is negligible).  $I$  leads  $V$  by  $\phi$  and lags behind  $E_R$  by an angle  $\theta = \tan^{-1}(X_S/R_a)$ . Line  $CD$  is drawn at an angle of  $\theta$  to  $AB$ .  $AC$  and  $ED$  are  $\perp$  to  $CD$  (and hence to  $AE$  also).

Mechanical power per phase developed in the rotor is

$$P_m = E_b I \cos \psi \quad \dots(i)$$

In  $\Delta OBD$ ,  $BD = I Z_S \cos \psi$

Now,  $BD = CD - BC = AE - BC$

$$I Z_S \cos \psi = V \cos (\theta - \alpha) - E_b \cos \theta$$

$$\therefore I \cos \psi = \frac{V}{Z_S} \cos (\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta$$

Substituting this value in (i), we get

$$P_m \text{ per phase} = E_b \left[ \frac{V}{Z_S} \cos (\theta - \alpha) - \frac{E_b}{Z_S} \cos \theta \right] = \frac{E_b V}{Z_S} \cos (\theta - \alpha) - \frac{E_b^2}{Z_S} \cos \theta \quad \dots(ii)$$

This is the expression for the mechanical power developed in terms of the load angle ( $\alpha$ ) and the internal angle ( $\theta$ ) of the motor for a constant voltage  $V$  and  $E_b$  (or excitation because  $E_b$  depends on excitation only).

If  $T_g$  is the gross armature torque developed by the motor, then

$$T_g \times 2 \pi N_s = P_m \text{ or } T_g = P_m / \omega_s = P_m / 2 \pi N_s \quad -N_s \text{ in rps}$$

$$T_g = \frac{P_m}{2 \pi N_s / 60} = \frac{60}{2 \pi} \cdot \frac{P_m}{N_s} = 9.55 \frac{P_m}{N_s} \quad -N_s \text{ in rpm}$$

Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\therefore \frac{d P_m}{d \alpha} = -\frac{E_b V}{Z_S} \sin (\theta - \alpha) = 0 \quad \text{or} \quad \sin (\theta - \alpha) = 0 \quad \therefore \theta = \alpha$$

\* Since  $R_a$  is generally negligible,  $Z_S = X_S$  so that  $\theta \cong 90^\circ$ . Hence

$$P_m = \frac{E_b V}{X_S} \cos (90^\circ - \alpha) = \frac{E_b V}{X_S} \sin \alpha$$

This gives the value of mechanical power developed in terms of  $\alpha$  – the basic variable of a synchronous machine.

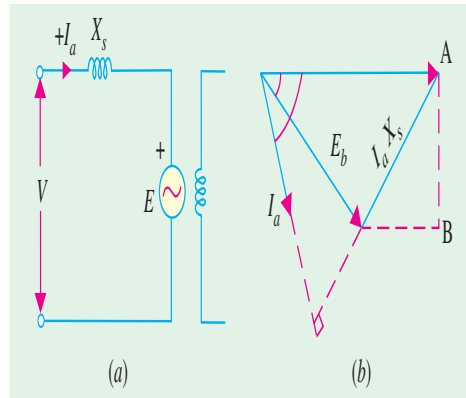


Fig. 38.21

$$\therefore \text{value of maximum power } (P_m)_{max} = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \alpha \text{ or } (P_m)_{max} = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \theta \dots (iii)$$

This shows that the maximum power and hence torque ( $\because$  speed is constant) depends on  $V$  and  $E_b$  i.e., excitation. Maximum value of  $\theta$  (and hence  $\alpha$ ) is  $90^\circ$ . For all values of  $V$  and  $E_b$ , this limiting value of  $\alpha$  is the same but maximum torque will be proportional to the maximum power developed as given in equation (iii). Equation (ii) is plotted in Fig. 38.22.

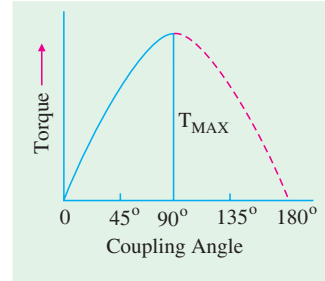


Fig. 38.22

If  $R_a$  is neglected, then  $Z_s \cong X_s$  and  $\theta = 90^\circ \therefore \cos \theta = 0$

$$P_m = \frac{E_b V}{X_s} \sin \alpha \dots (iv) \quad (P_m)_{max} = \frac{E_b V}{X_s} \dots \text{from equation}$$

(iii)\* The same value can be obtained by putting  $\alpha = 90^\circ$  in equation (iv). This corresponds to the ‘pull-out’ torque.

### 38.13. Alternative Expression for Power Developed

In Fig. 38.23, as usual,  $OA$  represents the supply voltage per phase i.e.,  $V$  and  $AB (= OC)$  is the induced or back e.m.f. per phase i.e.,  $E_b$  at an angle  $\alpha$  with  $OA$ . The armature current  $I$  (or  $I_a$ ) lags  $V$  by  $\phi$ .

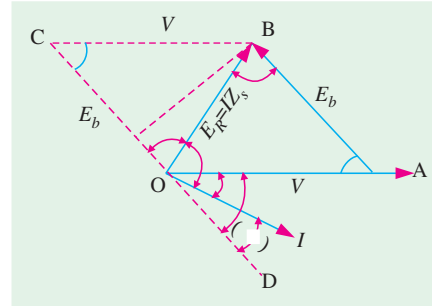


Fig. 38.23

Mechanical power developed is,

$$\begin{aligned} P_m &= E_b \cdot I \times \text{cosine of the angle between } E_b \text{ and } I \\ &= E_b I \cos \angle DOI \\ &= E_b I \cos (\pi - \angle COI) \\ &= -E_b I \cos (\theta + \gamma) \\ &= -E_b \left( \frac{E_R}{Z_s} \right) (\cos \theta \cos \gamma - \sin \theta \sin \gamma) \dots (i) \end{aligned}$$

Now,  $E_R$  and functions of angles  $\theta$  and  $\gamma$  will be eliminated as follows :

$$\text{From } \triangle OAB ; V/\sin \gamma = E_R / \sin \alpha \quad \therefore \sin \gamma = V \sin \alpha / E_R$$

$$\text{From } \triangle OBC ; E_R \cos \gamma + V \cos \alpha = E_b \quad \therefore \cos \gamma = (E_b - V \cos \alpha)/E_R$$

$$\text{Also} \quad \cos \theta = R_a / Z_s \text{ and } \sin \theta = X_s / Z_s$$

Substituting these values in Eq. (i) above, we get

$$\begin{aligned} P_m &= -\frac{E_b \cdot E_R}{Z_s} \left( \frac{R_a}{Z_s} \cdot \frac{E_b - V \cos \alpha}{E_R} - \frac{X_s}{Z_s} \cdot \frac{V \sin \alpha}{E_R} \right) \\ &= \frac{E_b V}{Z_s^2} (R_a \cos \alpha + X_s \sin \alpha) - \frac{E_b^2 R_a}{Z_s^2} \dots (ii) \end{aligned}$$

It is seen that  $P_m$  varies with  $E_b$  (which depends on excitation) and angle  $\alpha$  (which depends on the motor load).

**Note.** If we substitute  $R_a = Z_s \cos \theta$  and  $X_s = Z_s \sin \theta$  in Eq. (ii), we get

$$P_m = \frac{E_b V}{Z_s^2} (Z_s \cos \theta \cos \alpha + Z_s \sin \theta \sin \alpha) - \frac{E_b^2 Z_s \cos \theta}{Z_s^2} = \frac{E_b V}{Z_s} \cos (\theta - \alpha) - \frac{E_b^2}{Z_s} \cos \theta$$

It is the same expression as found in Art. 38.10.

\* It is the same expression as found for an alternator or synchronous generator in Art. 37.37.

38.14. Various Conditions of Maxima

The following two cases may be considered :

(i) **Fixed  $E_b, V, R_a$  and  $X_s$ .** Under these conditions,  $P_m$  will vary with load angle  $\alpha$  and will be maximum when  $dP_m / d\alpha = 0$ . Differentiating Eq. (ii) in Art. 38.11, we have

$$\frac{dP_m}{d\alpha} = \frac{E_b V}{Z_s^2} (X_s \cos \alpha - R_a \sin \alpha) = 0 \quad \text{or} \quad \tan \alpha = X_s / R_a = \tan \theta \quad \text{or} \quad \alpha = \theta$$

Putting  $\alpha = \theta$  in the same Eq. (ii), we get

$$\begin{aligned} (P_m)_{max} &= \frac{E_b V}{Z_s^2} (R_a \cos \theta + X_s \sin \theta) - \frac{E_b^2 R_a}{Z_s^2} = \\ &= \frac{E_b V}{Z_s^2} \left( \frac{R_a^2 + X_s^2}{Z_s} \right) - \frac{E_b^2 R_a}{Z_s^2} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} \end{aligned} \quad \dots(i)$$

This gives the value of power at which the motor falls out of step.

Solving for  $E_b$  from Eq. (i) above, we get

$$E_b = \frac{Z_s}{2R_a} \left[ V \pm \sqrt{V^2 - 4R_a \cdot (P_m)_{max}} \right]$$

The two values of  $E_b$  so obtained represent the excitation limits for any load.

(ii) **Fixed  $V, R_a$  and  $X_s$ .** In this case,  $P_m$  varies with excitation or  $E_b$ . Let us find the value of the excitation or induced e.m.f.  $E_b$  which is necessary for maximum power possible. For this purpose, Eq. (i) above may be differentiated with respect to  $E_b$  and equated to zero.

$$\therefore \frac{d(P_m)_{max}}{d E_b} = \frac{V}{Z_s} - \frac{2R_a E_b}{Z_s^2} = 0; \quad E_b = \frac{V Z_s}{2R_a} \quad \dots(ii)$$

Putting this value of  $E_b$  in Eq. (i) above, maximum power developed becomes

$$(P_m)_{max} = \frac{V^2}{2R_a} - \frac{V^2}{4R_a} = \frac{V^2}{4R_a}$$

38.15. Salient Pole Synchronous Motor

Cylindrical-rotor synchronous motors are much easier to analyse than those having salient-pole rotors. It is due to the fact that cylindrical-rotor motors have a uniform air-gap, whereas in salient-pole motors, air-gap is much greater between the poles than along the poles. Fortunately, cylindrical rotor theory is reasonably accurate in predicting the steady-state performance of salient-pole motors. Hence, salient-pole theory is required only when very high degree of accuracy is needed or when problems concerning transients or power system stability are to be handled.

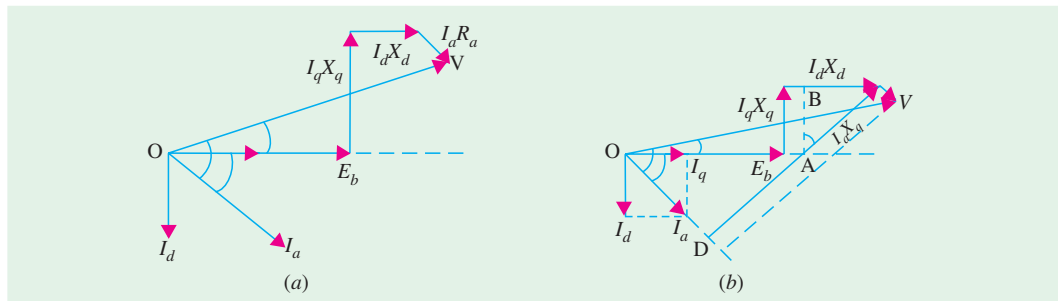


Fig. 38.24

\* This is the value of induced e.m.f. to give maximum power, but it is not the maximum possible value of the generated voltage, at which the motor will operate.



The  $d$ - $q$  currents and reactances for a salient-pole synchronous motor are exactly the same as discussed for salient-pole synchronous generator. The motor has  $d$ -axis reactance  $X_d$  and  $q$ -axis reactance  $X_q$ . Similarly, motor armature current  $I_a$  has two components:  $I_d$  and  $I_q$ . The complete phasor diagram of a salient-pole synchronous motor, for a lagging power factor is shown in Fig. 38.24 (a).

With the help of Fig. 38.24 (b), it can be proved that  $\tan \psi = \frac{V \sin \phi - I_q X_q}{V \cos \phi - I_a R_a}$

If  $R_a$  is negligible, then  $\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$

For an overexcited motor *i.e.*, when motor has leading power factor,

$$\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$$

The power angle  $\alpha$  is given by  $\alpha = \phi - \psi$

The magnitude of the excitation or the back e.m.f.  $E_b$  is given by

$$E_b = V \cos \alpha - I_q R_a - I_d X_d$$

Similarly, as proved earlier for a synchronous generator, it can also be proved from Fig. 38.24 (b) for a synchronous motor with  $R_a = 0$  that

$$\tan \alpha = \frac{I_a X_q \cos \phi}{V - I_a X_q \sin \phi}$$

In case  $R_a$  is not negligible, it can be proved that

$$\tan \alpha = \frac{I_a X_q \cos \phi - I_a R_a \sin \phi}{V - I_a X_q \sin \phi - I_a R_a \cos \alpha}$$

### 38.16. Power Developed by a Salient Pole Synchronous Motor

The expression for the power developed by a salient-pole synchronous generator derived in Chapter 35 also applies to a salient-pole synchronous motor.

$$\begin{aligned} \therefore P_m &= \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \quad \dots \text{per phase} \\ &= 3 \times \left[ \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \right] \dots \text{per three phases} \\ T_g &= 9.55 P_m / N_s \quad \dots N_s \text{ in rps.} \end{aligned}$$

As explained earlier, the power consists of two components, the first component is called excitation power or magnet power and the second is called reluctance power (because when excitation is removed, the motor runs as a reluctance motor).

**Example 38.4.** A 3- $\phi$ , 150-kW, 2300-V, 50-Hz, 1000-rpm salient-pole synchronous motor has  $X_d = 32 \Omega / \text{phase}$  and  $X_q = 20 \Omega / \text{phase}$ . Neglecting losses, calculate the torque developed by the motor if field excitation is so adjusted as to make the back e.m.f. twice the applied voltage and  $\alpha = 16^\circ$ .

**Solution.**

$$V = 2300 / \sqrt{3} = 1328 \text{ V}; E_b = 2 \times 1328 = 2656 \text{ V}$$

$$\text{Excitation power / phase} = \frac{E_b V}{X_d} \sin \alpha = \frac{2656 \times 1328}{32} \sin 16^\circ = 30,382 \text{ W}$$

$$\text{Reluctance power / phase} = \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha = \frac{1328^2 (32 - 20)}{2 \times 32 \times 20} \sin 32^\circ = 8760 \text{ W}$$

$$\text{Total power developed, } P_m = 3 (30382 + 8760) = 117,425 \text{ W}$$

$$T_g = 9.55 \times 117,425 / 1000 = \mathbf{1120 \text{ N-m}}$$

**Example 38.5.** A 3300-V, 1.5-MW, 3- $\phi$ , Y-connected synchronous motor has  $X_d = 4\Omega$ /phase and  $X_q = 3\Omega$ /phase. Neglecting all losses, calculate the excitation e.m.f. when motor supplies rated load at unity p.f. Calculate the maximum mechanical power which the motor would develop for this field excitation. (Similar Example, Swami Ramanand Teertha Marathwada Univ. Nanded 2001)

**Solution.**

$$V = 3300 / \sqrt{3} = 1905 \text{ V}; \cos \phi = 1; \sin \phi = 0; \phi = 0^\circ$$

$$I_a = 1.5 \times 10^6 / \sqrt{3} \times 3300 \times 1 = 262 \text{ A}$$

$$\tan \psi = \frac{V \sin \phi - I_a X_q}{V \cos \phi} = \frac{1905 \times 0 - 262 \times 3}{1905} = -0.4125; \psi = -22.4^\circ$$

$$\alpha = \phi - \psi = 0 - (-22.4^\circ) = 22.4^\circ$$

$$I_d = 262 \times \sin(-22.4^\circ) = -100 \text{ A}; I_q = 262 \cos(-22.4^\circ) = 242 \text{ A}$$

$$E_b = V \cos \alpha - I_d X_d = 1905 \cos(-22.4^\circ) - (-100 \times 4) = 2160 \text{ V}$$

$$= 1029 \sin \alpha + 151 \sin 2\alpha$$

$$P_m = \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \quad \dots \text{ per phase}$$

$$= \frac{2160 \times 1905}{4 \times 1000} + \frac{1905^2 (4 - 3)}{2 \times 4 \times 3 \times 1000} \sin 2\alpha \quad \dots \text{ kW/phase}$$

$$= 1029 \sin \alpha + 151 \sin 2\alpha \quad \dots \text{ kW/phase}$$

If developed power has to achieve maximum value, then

$$\frac{dP_m}{d\alpha} = 1029 \cos \alpha + 2 \times 151 \cos 2\alpha = 0$$

$$\therefore 1029 \cos \alpha + 302 (2 \cos^2 \alpha - 1) = 0 \quad \text{or} \quad 604 \cos^2 \alpha + 1029 \cos \alpha - 302 = 0$$

$$\therefore \cos \alpha = \frac{-1029 \pm \sqrt{1029^2 + 4 \times 604 \times 302}}{2 \times 604} = 0.285; \alpha = 73.4^\circ$$

$$\therefore \text{maximum } P_m = 1029 \sin 73.4^\circ + 151 \sin 2 \times 73.4^\circ = 1070 \text{ kW/phase}$$

Hence, maximum power developed for three phases =  $3 \times 1070 = 3210 \text{ kW}$

**Example 38.6.** The input to an 11000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find (i) the power supplied to the motor (ii) mechanical power developed and (iii) induced emf for a power factor of 0.8 leading. (Elect. Engg. AMIETE (New Scheme) June 1990)

**Solution.** (i) Motor power input =  $\sqrt{3} \times 11000 \times 60 \times 0.8 = 915 \text{ kW}$

(ii) stator Cu loss/phase =  $60^2 \times 1 = 3600 \text{ W}$ ; Cu loss for three phases =  $3 \times 3600 = 10.8 \text{ kW}$

$$P_m = P_2 - \text{rotor Cu loss} = 915 - 10.8 = 904.2 \text{ kW}$$

$$V_p = 11000 / \sqrt{3} = 6350 \text{ V}; \phi = \cos^{-1} 0.8 = 36.9^\circ;$$

$$\theta = \tan^{-1} (30/1) = 88.1^\circ$$

$$Z_s \cong 30 \Omega; \text{stator impedance drop / phase} = I_a Z_s$$

$$= 60 \times 30 = 1800 \text{ V}$$

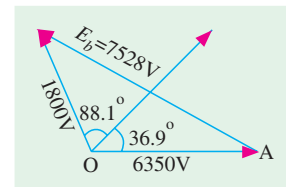


Fig. 38.25

As seen from Fig. 38.25,

$$E_b^2 = 6350^2 + 1800^2 - 2 \times 6350 \times 1800 \times \cos (88.1^\circ + 36.9^\circ)$$

$$= 6350^2 + 1800^2 - 2 \times 6350 \times 1800 \times -0.572$$

$$\therefore E_b = 7528 \text{ V}; \text{line value of } E_b = 7528 \times \sqrt{3} = 13042$$

**Special Example 38.7. Case of Salient - Pole Machines**

A synchronous machine is operated as below :

As a Generator : 3 -Phase,  $V_{ph} = 400$ ,  $I_{ph} = 32$ , unity p.f.

As a Motor : 3 -Phase,  $V_{ph} = 400$ ,  $I_{ph} = 32$ , unity p.f.

Machine parameters :  $X_d = 10 \Omega$ ,  $X_q = 6.5 \Omega$

Calculate excitation emf and  $\delta$  in the two modes and deal with the term power in these two cases.

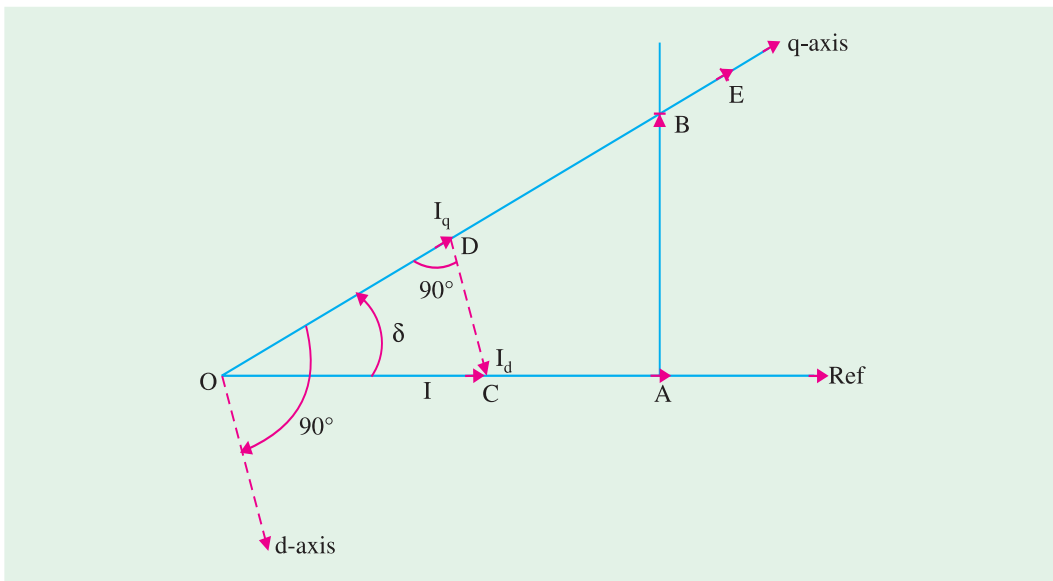


Fig. 38.26 (a) Generator-action

**Solution.**

**Generating Mode :**

**Voltages :**

$$OA = 400 \text{ V}, AB = IX_q \\ = 32 \times 6.5 = 208 \text{ V}$$

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V},$$

$$\delta = \tan^{-1} \frac{AB}{OA} = \tan^{-1} \frac{208}{400} = 27.5^\circ$$

$$BE = I_d(X_d - X_q) \\ = 14.8 \times 3.5 = 51.8 \text{ V},$$

$$E = OE = OB + BE = 502.8 \text{ V}$$

**Currents :**  $I = OC = 32$ ,  $I_q = I \cos \delta = OD = 28.4$  amp.,  $I_d = DC = I \sin \delta = 14.8$  amp.  $E$  leads  $V$  in case of generator, as shown in Fig. 38.26 (a)

$$\text{Power (by one formula)} = 3 \times 400 \times 32 \times 10^{-3} = 38.4 \text{ kW}$$

$$\text{or Power (by another formula)} = 3 \times \left[ \frac{400 \times 502.8}{10} \sin 27.5^\circ + \frac{400^2}{2} \times \left( \frac{3.5}{65} \right) \times \sin 55^\circ \right] \\ = 38.44 \text{ kW}$$

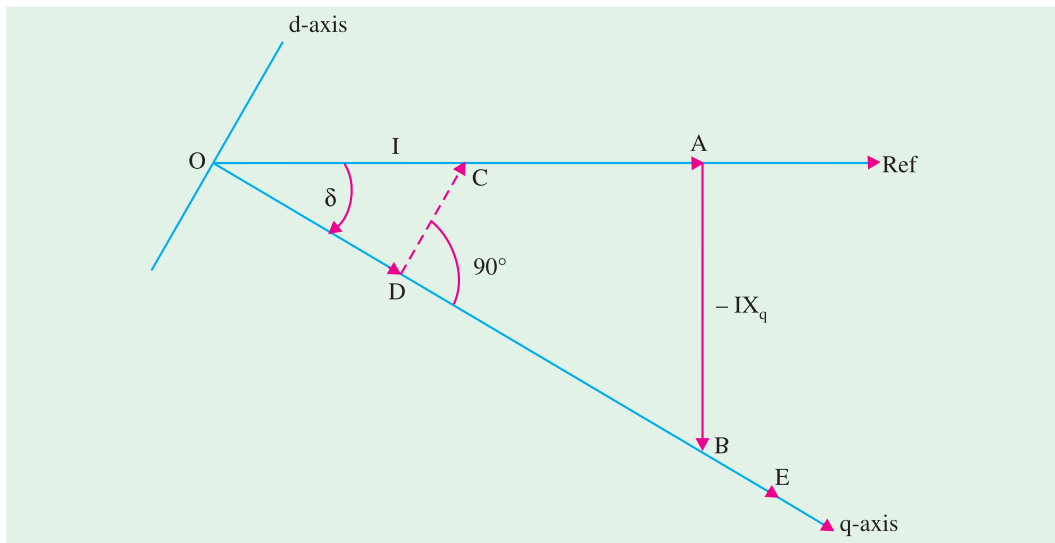


Fig. 38.26 (b) Phasor diagram : Motoring mode

**Motoring mode of a salient pole synchronous machine**

**Voltages :**  $OA = 400 \text{ V}, AB = -IX_q = 208 \text{ V}$

$OB = \sqrt{400^2 + 208^2} = 451 \text{ V}$

$\delta = \tan^{-1} \frac{AB}{OA} = \tan^{-1} \frac{208}{400} = 27.5^\circ$  as before but now  $E$  lags behind  $V$ .

$BE = I_d(X_d - X_q) = 51.8 \text{ V}$  in the direction shown.  $OE = 502.8 \text{ V}$  as before

**Currents :**  $OC = 32 \text{ amp. } OD = 28.4 \text{ amp. } DC = 14.8 \text{ amp.}$  Naturally,  $I_q = 28.4 \text{ amp.}$  and  $I_d = 14.8 \text{ amp}$

Power (by one formula) = 38.4 kW

Power (by another formula) = 38.44 kW

**Note.** Numerical values of  $E$  and  $\delta$  are same in cases of generator-mode and motor-mode, due to unity p.f.  $\delta$  has different signs in the two cases.

**Example 38.8.** A 500-V, 1-phase synchronous motor gives a net output mechanical power of 7.46 kW and operates at 0.9 p.f. lagging. Its effective resistance is  $0.8 \Omega$ . If the iron and friction losses are 500 W and excitation losses are 800 W, estimate the armature current. Calculate the commercial efficiency. (Electrical Machines-I, Gujarat Univ. 1988)

**Solution.** Motor input =  $VI_a \cos \phi$ ; Armature Cu loss =  $I_a^2 R_a$

Power developed in armature is  $P_m = VI_a \cos \phi - I_a^2 R_a$

$$\therefore I_a^2 R_a - VI_a \cos \phi + P_m = 0 \quad \text{or} \quad I_a = \frac{V \cos \phi \pm \sqrt{V^2 \cos^2 \phi - 4 R_a P_m}}{2 R_a}$$

Now,

$P_{out} = 7.46 \text{ kW} = 7,460 \text{ W}$

$P_m = P_{out} + \text{iron and friction losses} + \text{excitation losses}$   
 $= 7460 + 500 + 800 = 8760 \text{ W} \quad \dots \text{ Art. 38.5}$

$I_a = \frac{500 \times 0.9 \pm \sqrt{(500 \times 0.9)^2 - 4 \times 0.8 \times 3760}}{2 \times 0.8}$

$= \frac{450 \pm \sqrt{202,500 - 28,030}}{1.6} = \frac{450 \pm 417.7}{1.6} = \frac{32.3}{1.6} = 20.2 \text{ A}$

Motor input =  $500 \times 20.2 \times 0.9 = 9090 \text{ W}$

$\eta_c = \text{net output} / \text{input} = 7460 / 9090 = 0.8206$  or **82.06%**.

**Example 38.9.** A 2,300-V, 3-phase, star-connected synchronous motor has a resistance of 0.2 ohm per phase and a synchronous reactance of 2.2 ohm per phase. The motor is operating at 0.5 power factor leading with a line current of 200 A. Determine the value of the generated e.m.f. per phase. (Elect. Engg.-I, Nagpur Univ. 1993)

**Solution.** Here,  $\phi = \cos^{-1}(0.5) = 60^\circ$  (lead)  
 $\theta = \tan^{-1}(2.2/0.2) = 84.8^\circ$   
 $\therefore (\theta + \phi) = 84.8^\circ + 60^\circ = 144.8^\circ$   
 $\cos 144.8^\circ = -\cos 35.2^\circ$   
 $V = 2300 / \sqrt{3} = 1328 \text{ volt}$   
 $Z_s = \sqrt{0.2^2 + 2.2^2} = 2.209 \Omega$   
 $I Z_s = 200 \times 2.209 = 442 \text{ V}$

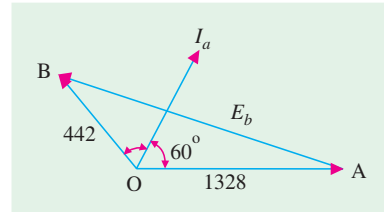


Fig. 38.27

The vector diagram is shown in Fig. 38.27.

$$E_b = \sqrt{V^2 + E_R^2 - 2V \cdot E_R \cos(\theta + \phi)}$$

$$= \sqrt{1328^2 + 442^2 + 2 \times 1328 \times 442 \times \cos 35.2^\circ} = \mathbf{1708 \text{ Volt / Phase}}$$

**Example 38.10.** A 3-phase, 6,600-volts, 50-Hz, star-connected synchronous motor takes 50 A current. The resistance and synchronous reactance per phase are 1 ohm and 20 ohm respectively. Find the power supplied to the motor and induced emf for a power factor of (i) 0.8 lagging and (ii) 0.8 leading. (Eect. Engg. II pune Univ. 1988)

**Solution.** (i) **p.f. = 0.8 lag** (Fig. 38.28 (a)).

Power input =  $\sqrt{3} \times 6600 \times 50 \times 0.8 = 457,248 \text{ W}$

Supply voltage / phase =  $6600 / \sqrt{3} = 3810 \text{ V}$

$\phi = \cos^{-1}(0.8) = 36^\circ 52'$ ;  $\theta = \tan^{-1}(X_s / R_a) = (20/1) = 87.8'$

$Z_s = \sqrt{20^2 + 1^2} = 20 \Omega$  (approx.)

Impedance drop =  $I_a Z_s = 50 \times 20 = 1000 \text{ V/phase}$

$\therefore E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times \cos(87^\circ 8' - 36^\circ 52')$   $\therefore E_b = 3263 \text{ V / phase}$

Line induced e.m.f. =  $3263 \times \sqrt{3} = \mathbf{5651 \text{ V}}$

(ii) Power input would remain the same.

As shown in Fig. 38.28 (b), the current vector is drawn at a **leading** angle of

$\phi = 36^\circ 52'$

Now,  $(\theta + \phi) = 87^\circ 8' + 36^\circ 52' = 124^\circ$ ,  
 $\cos 124^\circ = -\cos 56^\circ$

$\therefore E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times -\cos 56^\circ$   $\therefore E_b = 4447 \text{ V / phase}$

Line induced e.m.f. =  $\sqrt{3} \times 4447$   
 = **7,700 V**

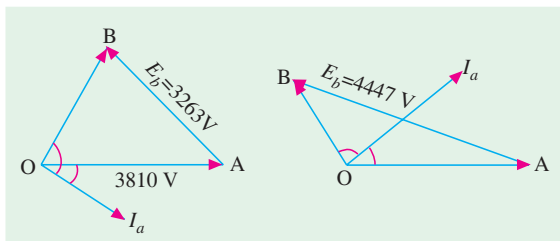


Fig. 38.28 (a)

Fig. 38.28 (b)

**Note.** It may be noted that if  $E_b > V$ , then motor has a leading power factor and if  $E_b < V$ .

**Example 38.11.** A synchronous motor having 40% reactance and a negligible resistance is to be operated at rated load at (i) u.p.f. (ii) 0.8 p.f. lag (iii) 0.8 p.f. lead. What are the values of induced e.m.f.? Indicate assumptions made, if any. (Electrical Machines-II, Indore Univ. 1990)

**Solution.** Let  $V = 100 \text{ V}$ , then reactance drop  $= I_a X_S = 40 \text{ V}$

(i) At unity p.f.

Here,  $\theta = 90^\circ$ ,  $E_b = \sqrt{100^2 + 40^2} = 108 \text{ V}$  ...Fig. 38.29 (a)

(ii) At p.f. 0.8 (lag.) Here  $\angle BOA = \theta - \phi = 90^\circ - 36^\circ 54' = 53^\circ 6'$

$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 53^\circ 6'$ ;  $E_b = 82.5 \text{ V}$ , as in Fig. 38.29 (b)

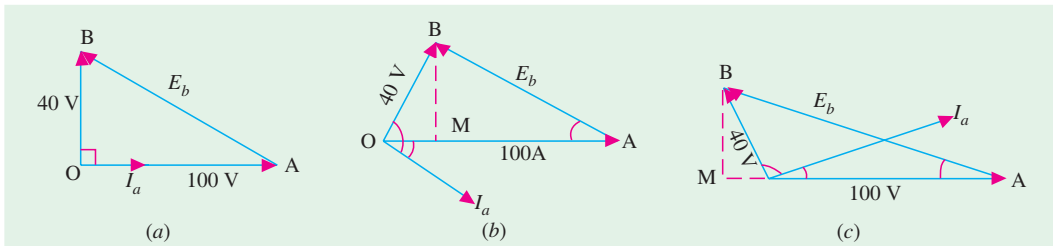


Fig. 38.29

Alternatively,  $E_b = AB = \sqrt{AM^2 + MB^2} = \sqrt{76^2 + 32^2} = 82.5 \text{ V}$

(iii) At p.f. 0.8 (lead.) Here,  $(\theta + \phi) = 90^\circ + 36.9^\circ = 126.9^\circ$

$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 126.9^\circ = 128^2$

Again from Fig. 38.29 (c),  $E_b^2 = (OM + OA)^2 + MB^2 = 124^2 + 32^2$ ;  $E_b = 128 \text{ V}$ .

**Example 38.12.** A 1,000-kVA, 11,000-V, 3- $\phi$ , star-connected synchronous motor has an armature resistance and reactance per phase of  $3.5 \Omega$  and  $40 \Omega$  respectively. Determine the induced e.m.f. and angular retardation of the rotor when fully loaded at (a) unity p.f. (b) 0.8 p.f. lagging (c) 0.8 p.f. leading. (Elect. Engineering-II, Bangalore Univ. 1992)

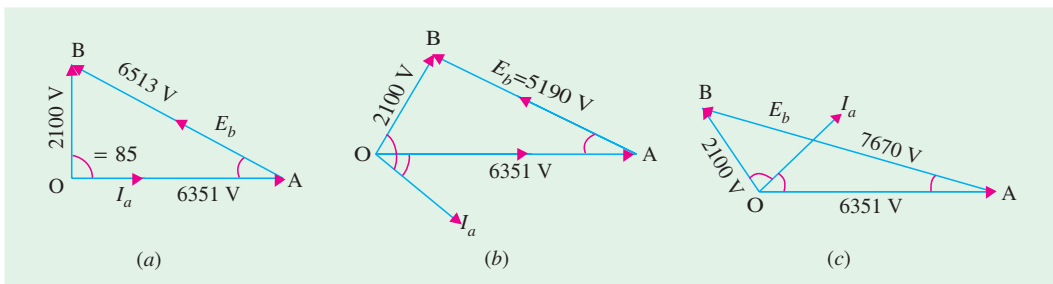


Fig. 38.30

**Solution.** Full-load armature current  $= 1,000 \times 1,000 / \sqrt{3} \times 11,000 = 52.5 \text{ A}$

Voltage / phase  $= 11,000 / \sqrt{3} = 6,351 \text{ V}$ ;  $\cos \phi = 0.8 \therefore \phi = 36^\circ 53'$

Armature resistance drop / phase  $= I_a R_a = 3.5 \times 52.5 = 184 \text{ V}$

reactance drop / phase  $= I_a X_S = 40 \times 52.5 = 2,100 \text{ V}$

$\therefore$  impedance drop / phase  $= I_a Z_S = \sqrt{(184^2 + 2100^2)} = 2,100 \text{ V (approx.)}$

$\tan \theta = X_S / R_a \therefore \theta = \tan^{-1} (40 / 3.5) = 85^\circ$

(a) At unity p.f. Vector diagram is shown in Fig. 38.30 (a)

$$E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \cos 85^\circ; E_b = 6,513 \text{ V per phase}$$

$$\text{Induced line voltage} = 6,513 \times \sqrt{3} = \mathbf{11,280 \text{ V}}$$

$$\text{From } \triangle OAB, \frac{2100}{\sin \alpha} = \frac{6153}{\sin 85^\circ} = \frac{6153}{0.9961}$$

$$\sin \alpha = 2,100 \times 0.9961 / 6,513 = 0.3212 \quad \therefore \alpha = \mathbf{18^\circ 44'}$$

(b) At p.f. 0.8 lagging – Fig. 38.30 (b)

$$\angle BOA = \theta - \phi = 85^\circ - 36^\circ 53' = 48^\circ 7'$$

$$E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \times \cos 48^\circ 7'$$

$$E_b = 5,190 \text{ V per phase}$$

$$\text{Induced line voltage} = 5,190 \times \sqrt{3} = \mathbf{8,989 \text{ V}}$$

Again from the  $\triangle OAB$  of Fig. 36.30 (b)

$$\frac{2100}{\sin \alpha} = \frac{5190}{\sin 48^\circ 7'} = \frac{5,190}{0.7443}$$

$$\therefore \sin \alpha = 2100 \times 0.7443 / 5190 = 0.3012 \quad \therefore \alpha = \mathbf{17^\circ 32'}$$

(c) At p.f. 0.8 leading [Fig. 38.30 (c)]

$$\angle BOA = \theta + \phi = 85^\circ + 36^\circ 53' = 121^\circ 53'$$

$$\therefore E_b^2 = 6,351^2 + 2,100^2 - 2 \times 6,351 \times 2,100 \times \cos 121^\circ 53'$$

$$\therefore E_b = 7,670 \text{ volt per phase.}$$

$$\text{Induced line e.m.f} = 7,670 \times \sqrt{3} = \mathbf{13,280 \text{ V}}$$

$$\text{Also, } \frac{2,100}{\sin \alpha} = \frac{7,670}{\sin 121^\circ 53'} = \frac{7,670}{0.8493}$$

$$\therefore \sin \alpha = 2,100 \times 0.8493 / 7,670 = 0.2325 \quad \therefore \alpha = \mathbf{13^\circ 27'}$$

**Special Example 38.13.** Both the modes of operation : Phase - angle = 20° Lag

Part (a) : A three phase star-connected synchronous generator supplies a current of 10 A having a phase angle of 20° lagging at 400 volts/phase. Find the load angle and components of armature current (namely  $I_d$  and  $I_q$ ) if  $X_d = 10$  ohms,  $X_q = 6.5$  ohms. Neglect  $r_a$ . Calculate voltage regulation.

**Solution.** The phasor diagram is drawn in Fig 38.31 (a)

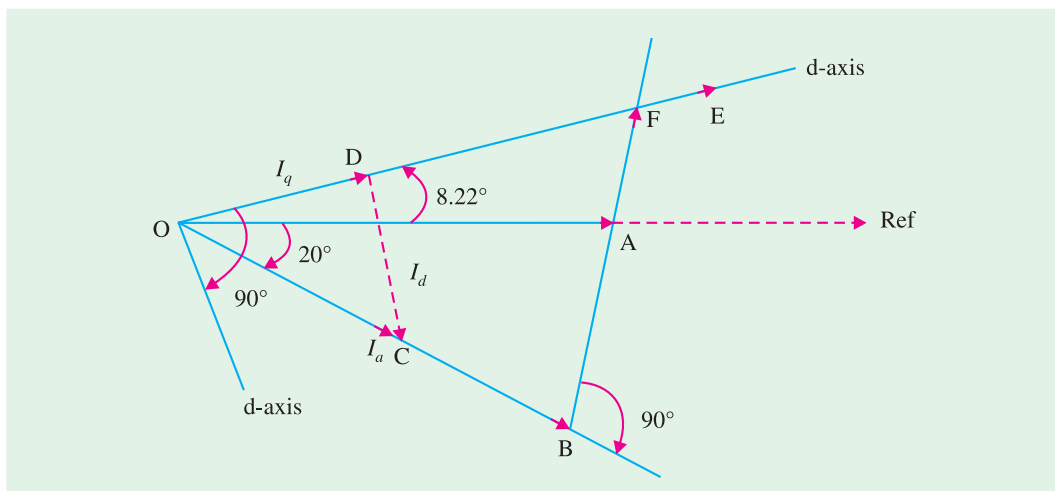


Fig. 38.31 (a) : Generator-mode

$$\begin{aligned}
 OA &= 400 \text{ V}, OB = 400 \cos 20^\circ = 376 \text{ V}, AB = 400 \sin 20^\circ = 136.8 \text{ V} \\
 AF &= IX_q = 10 \times 6.5 = 65 \text{ V}, BF = BA + AF = 201.8 \text{ V} \\
 OF &= \sqrt{376^2 + 201.8^2} = 426.7 \text{ V}, \delta = 8.22^\circ \\
 DC &= I_d = I_a \sin 28.22^\circ = 4.73 \text{ amp}, DC \text{ perpendicular to } OD, \\
 OD &= I_q = I_a \cos 28.22^\circ = 8.81 \text{ amp} \\
 FE &= I_d(X_d - X_q) = 4.73 \times 3.5 = 16.56 \text{ V. This is along the direction of '+q' -axis} \\
 E &= OE = OF + FE = 426.7 + 16.56 = 443.3 \text{ V}
 \end{aligned}$$

$$\% \text{ Regulation} = \frac{443 - 400}{400} \times 100\% = \mathbf{10.75\%}$$

If the same machine is now worked as a synchronous motor with terminal voltage, supply-current and its power-factor kept unaltered, find the excitation emf and the load angle.

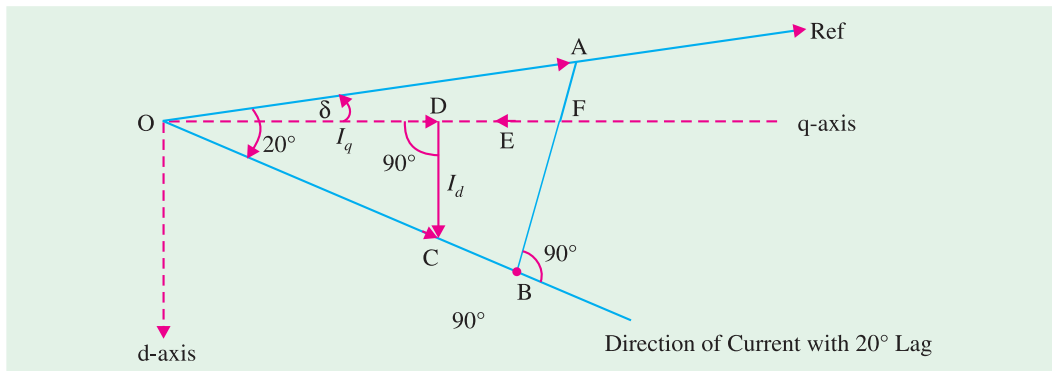


Fig. 38.31 (b) Motoring-mode

$$\begin{aligned}
 AF &= -I_a X_q = -65 \text{ V}, AB = 136.8 \text{ V}, FB = 71.8 \text{ V} \\
 OB &= 400 \cos 20^\circ = 376 \text{ V} \\
 OF &= \sqrt{376^2 + 71.8^2} = 382.8 \text{ V} \\
 20^\circ - \delta &= \tan^{-1} BF/OB = \tan^{-1} 71.8/376 = 10.8^\circ, \delta = 9.2^\circ \\
 FE &= -I_d(X_d - X_q) = -1.874 \times (3.5) = -6.56 \text{ volts, as shown in Fig.38.31 (b)} \\
 E &= OE = OF + FE = 382.8 - 6.56 = 376.24 \text{ volts}
 \end{aligned}$$

**Currents :**

$$\begin{aligned}
 I_a &= OC = 10 \text{ amp} \\
 I_q &= OD = 10 \cos 10.8^\circ = 9.823 \text{ amp}, I_d = DC = \sin 10.8^\circ = 1.874 \text{ amp}
 \end{aligned}$$

**Note.**  $I_d$  is in downward direction.

Hence,  $-I_d(X_d - X_q)$  will be from  $F$  towards  $O$  i.e., along '-q' direction.  
Thus, Excitation emf = 376.24 Volts, Load angle =  $9.2^\circ$

**(Note.** With respect to the generator mode,  $E$  has decreased, while  $\delta$  has increased.)

Power (by one formula) = 11276 watts, as before

Power (by another formula)

$$\begin{aligned}
 &= 3 [(V E/X_d) \sin \delta + (V^2/2) \{(1/X_q) \sin 2\delta\}] \\
 &= 3 [(400 \times 376.24/10) \sin 9.2^\circ + (400 \times 400/2) (3.5/65) \sin 18.4^\circ] \\
 &= 3 \times [2406 + 1360] = 11298 \text{ watts.}
 \end{aligned}$$

[This matches quite closely to the previous value calculated by other formula.]



**Example 38.14.** A 1- $\phi$  alternator has armature impedance of  $(0.5 + j0.866)$ . When running as a synchronous motor on 200-V supply, it provides a net output of 6 kW. The iron and friction losses amount to 500 W. If current drawn by the motor is 50 A, find the two possible phase angles of current and two possible induced e.m.fs. (Elec. Machines-I, Nagpur Univ. 1990)

**Solution.** Arm. Cu loss/phase =  $I_a^2 R_a = 50^2 \times 0.5 = 1250$  W  
 Motor intake =  $6000 + 500 + 1250 = 7750$  W  
 p.f. =  $\cos \phi = \text{Watts} / \text{VA} = 7750 / 200 \times 50 = 0.775 \therefore \phi = 39^\circ$  lag or lead.  
 $\theta = \tan^{-1} (X_S / R_a) = \tan^{-1} (0.866 / 0.5) = 60^\circ$  ;

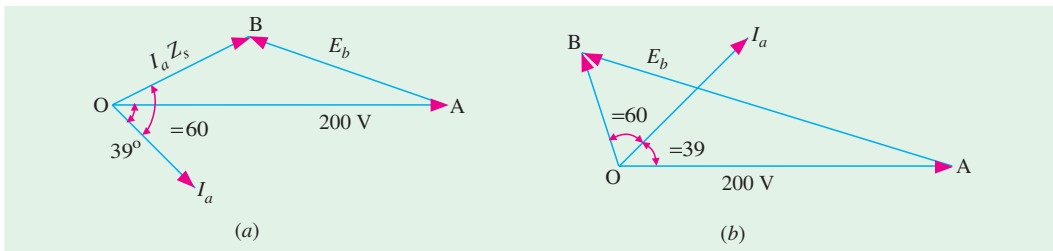


Fig. 38.32

$\angle BOA = 60^\circ - 39^\circ = 21^\circ$  – Fig. 38.32 (a)

$Z_S = \sqrt{0.5^2 + 0.866^2} = 1 \Omega ; I_a Z_S = 50 \times 1 = 50$  V

$AB = E_b = \sqrt{200^2 + 50^2 - 2 \times 200 \times 50 \cos 21^\circ} ; E_b = 154$  V.

In Fig. 38.32 (b),  $\angle BOA = 60^\circ + 39^\circ = 99^\circ$

$\therefore AB = E_b = \sqrt{(200^2 + 50^2) - 2 \times 200 \times 50 \cos 99^\circ} ; E_b = 214$  V.

**Example 38.15.** A 2200-V, 3- $\phi$ , Y-connected, 50-Hz, 8-pole synchronous motor has  $Z_S = (0.4 + j 6)$  ohm/phase. When the motor runs at no-load, the field excitation is adjusted so that E is made equal to V. When the motor is loaded, the rotor is retarded by  $3^\circ$  mechanical.

Draw the phasor diagram and calculate the armature current, power factor and power of the motor. What is the maximum power the motor can supply without falling out of step?

(Power Apparatus-II, Delhi Univ. 1988)

**Solution.** Per phase  $E_b = V = 2200 / \sqrt{3} = 1270$  V

$\alpha = 3^\circ$  (mech) =  $3^\circ \times (8/2) = 12^\circ$  (elect).

As seen from Fig 38.33 (a).

$E_R = (1270^2 + 1270^2 - 2 \times 1270 \times 1270 \times \cos 12^\circ)^{1/2}$   
 $= 266$  V;  $Z_S = \sqrt{0.4^2 + 6^2} = 6.013 \Omega$

$I_a = E_R / Z_S = 266 / 6.013 = 44.2$  A. From  $\Delta OAB$ ,

we get,  $\frac{1270}{\sin(\theta - \phi)} = \frac{266}{\sin 12^\circ}$

$\therefore \sin(\theta - \phi) = 1270 \times 0.2079 / 266 = 0.9926 \therefore (\theta - \phi) = 83^\circ$

Now,  $\theta = \tan^{-1} (X_S / R_a) = \tan^{-1} (6 / 0.4) = 86.18^\circ$

$\phi = 86.18^\circ - 83^\circ = 3.18^\circ \therefore \text{p.f.} = \cos 3.18^\circ = 0.998(\text{lag})$

Total motor power input =  $3 V I_a \cos \phi = 3 \times 1270 \times 44.2 \times 0.998 = 168$  kW

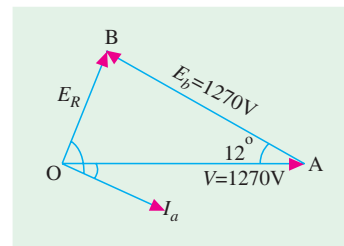


Fig. 38.33 (a)

$$\begin{aligned} \text{Total Cu loss} &= 3 I_a^2 R_a = 3 \times 44.2^2 \times 0.4 = 2.34 \text{ kW} \\ \text{Power developed by motor} &= 168 - 2.34 = \mathbf{165.66 \text{ kW}} \end{aligned}$$

$$P_{m(\max)} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} = \frac{1270 \times 1270}{6.013} - \frac{1270^2 \times 0.4}{6.013^2} = \mathbf{250 \text{ kW}}$$

**Example 38.16.** A 1- $\phi$ , synchronous motor has a back e.m.f. of 250 V, leading by 150 electrical degrees over the applied voltage of 200 volts. The synchronous reactance of the armature is 2.5 times its resistance. Find the power factor at which the motor is operating and state whether the current drawn by the motor is leading or lagging.

**Solution.** As induced e.m.f. of 250 V is greater than the applied voltage of 200 V, it is clear that the motor is over-excited, hence it must be working with a leading power factor.

In the vector diagram of Fig. 38.33 (b), OA represents applied voltage, AB is back e.m.f. at an angle of 30° because  $\angle AOC = 150^\circ$  and  $\angle COD = \angle BAO = 30^\circ$ . OB represents resultant of voltage V and  $E_b$  i.e.  $E_R$

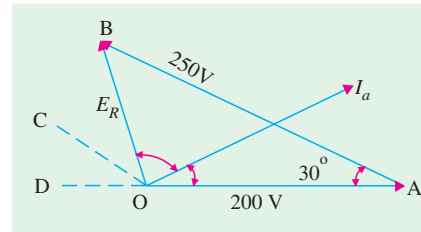


Fig. 38.33 (b)

In  $\triangle OBA$ ,

$$\begin{aligned} E_R &= \sqrt{(V^2 + E_b^2 - 2VE_b \cos 30^\circ)} \\ &= \sqrt{(200^2 + 250^2 - 2 \times 200 \times 250 \times 0.866)} = 126 \text{ V} \end{aligned}$$

Now, 
$$\frac{E_R}{\sin 30^\circ} = \frac{E_b}{\sin (\theta + \phi)} \quad \text{or} \quad \frac{126}{0.5} = \frac{250}{\sin (\theta + \phi)}$$

$\therefore \sin (\theta + \phi) = 125/126$  (approx.)  $\therefore (\theta + \phi) = 90^\circ$

Now  $\tan \theta = 2.5 \quad \therefore \theta = 68^\circ 12'$   $\therefore \phi = 90^\circ - 68^\circ 12' = 21^\circ 48'$

$\therefore$  p.f. of motor =  $\cos 21^\circ 48' = \mathbf{0.9285}$  (leading)

**Example 38.17.** The synchronous reactance per phase of a 3-phase star-connected 6,600 V synchronous motor is 10  $\Omega$ . For a certain load, the input is 900 kW and the induced line e.m.f. is 8,900 V. (line value). Evaluate the line current. Neglect resistance.

(Basic Elect. Machines, Nagpur Univ. (1993))

**Solution.** Applied voltage / phase =  $6,600 / \sqrt{3} = 3,810 \text{ V}$

Back e.m.f. / phase =  $8,900 / \sqrt{3} = 5,140 \text{ V}$

Input =  $\sqrt{3} V_L \cdot I \cos \phi = 900,000$

$\therefore I \cos \phi = 9 \times 10^5 / \sqrt{3} \times 6,600 = 78.74 \text{ A}$

In  $\triangle ABC$  of vector diagram in Fig. 38.34, we have  $AB^2 = AC^2 + BC^2$

Now  $OB = I \cdot X_s = 10 I$

$$\begin{aligned} BC &= OB \cos \phi = 10 I \cos \phi \\ &= 10 \times 78.74 = 787.4 \text{ V} \end{aligned}$$

$\therefore 5,140^2 = 787.4^2 + AC^2 \quad \therefore AC = 5,079 \text{ V}$

$\therefore OC = 5,079 - 3,810 = 1,269 \text{ V}$

$\tan \phi = 1269/787.4 = 1.612; \phi = 58.2^\circ, \cos \phi = 0.527$

Now  $I \cos \phi = 78.74; I = 78.74/0.527 = \mathbf{149.4 \text{ A}}$

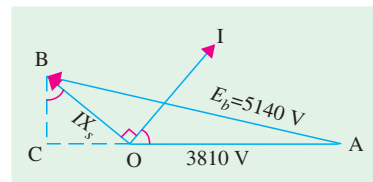


Fig. 38.34

**Example 38.18.** A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohms per phase and armature resistance negligible when the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.

(Elect. Machines, AMIE Sec. B 1991)

**Solution.** When Power Input is 1000 kW (Fig. 38.35 (a))

$$\begin{aligned} \sqrt{3} \times 6600 \times I_{a1} \times 0.8 &= 1000,000; I_{a1} = 109.3 \text{ A} \\ Z_S &= X_S = 20 \ \Omega; I_{a1} Z_S = 109.3 \times 20 = 2186 \text{ V}; \phi_1 = \cos^{-1} 0.8 = 36.9^\circ; \theta = 90^\circ \\ E_b^2 &= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times \cos(90^\circ + 36.9^\circ) \\ &= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times -\cos 53.1^\circ; \therefore E_b = 5410 \text{ V} \end{aligned}$$

Since excitation remains constant,  $E_b$  in the second case would remain the same i.e., 5410 V.

**When Power Input is 1500 kW :**

$$\begin{aligned} \sqrt{3} \times 6600 \times I_{a2} \cos \phi_2 &= 1500,000; I_{a2} \cos \phi_2 = 131.2 \text{ A} \\ \text{As seen from Fig. 38.35 (b),} \\ OB &= I_{a2} Z_S = 20 I_{a2} \\ BC &= OB \cos \phi_2 = 20 I_{a2} \cos \phi_2 \\ \cos \phi_2 &= 20 \times 131.2 = 2624 \text{ V} \\ \text{In } \Delta ABC, \text{ we have, } AB^2 &= AC^2 + BC^2 \end{aligned}$$

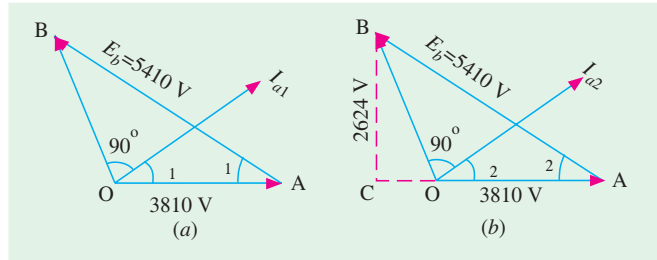


Fig. 38.35

$$\begin{aligned} \therefore AC &= 4730 \text{ V}; OC = 4730 - 3810 = 920 \text{ V} \\ \tan \phi_2 &= 920 / 2624; \phi_2 = 19.4^\circ; \text{p.f.} = \cos \phi_2 = \cos 19.4^\circ = 0.9432 \text{ (lead)} \\ \tan \alpha_2 &= BC / AC = 2624 / 4730; \alpha_2 = 29^\circ \end{aligned}$$

**Example 38.19.** A 3-phase, star-connected 400-V synchronous motor takes a power input of 5472 watts at rated voltage. Its synchronous reactance is 10  $\Omega$  per phase and resistance is negligible. If its excitation voltage is adjusted equal to the rated voltage of 400 V, calculate the load angle, power factor and the armature current. (Elect. Machines AMIE Sec. B, 1990)

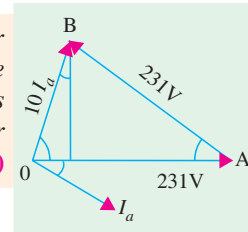


Fig. 38.36

$$\begin{aligned} \text{Solution. } \sqrt{3} \times 400 \times I_a \cos \phi &= 5472; I_a \cos \phi = 7.9 \text{ A} \\ Z_S &= 10 \ \Omega; E_R = I_a Z_S = 10 I_a \\ \text{As seen from Fig. 38.36, } BC &= OB \cos \phi = 10, I_a \cos \phi = 7.9 \text{ V} \\ AC &= \sqrt{231^2 - 79^2} = 217 \text{ V}; OC = 231 - 217 = 14 \text{ V} \\ \tan \phi &= 14 / 79; \phi = 10^\circ; \cos \phi = 0.985 \text{ (lag)} \\ I_a \cos \phi &= 7.9; I_a = 7.9 / 0.985 = 8 \text{ A}; \tan \alpha = BC / AC = 79 / 217; \alpha = 20^\circ \end{aligned}$$

**Example 38.20.** A 2,000-V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0.2  $\Omega$  and 2.2  $\Omega$  respectively. The input is 800 kW at normal voltage and the induced e.m.f. is 2,500 V. Calculate the line current and power factor. (Elect. Engg. A.M.I.E.T.E., June 1992)

**Solution.** Since the induced e.m.f. is greater than the applied voltage, the motor must be running with a leading p.f. If the motor current is  $I$ , then its in-phase or power component is  $I \cos \phi$  and reactive component is  $I \sin \phi$ .

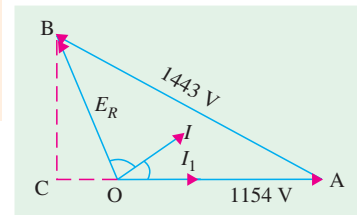


Fig. 38.37

Let  $I \cos \phi = I_1$  and  $I \sin \phi = I_2$  so that  $I = (I_1 + jI_2)$

$$I \cos \phi = I_1 = 800,00 / \sqrt{3} = 231 \text{ A}$$

$$\text{Applied voltage / phase} = 2,000 / \sqrt{3} = 1,154 \text{ V}$$

$$\text{Induced e.m.f. / phase} = 2500 / \sqrt{3} = 1,443 \text{ V}$$

In Fig. 38.37  $OA = 1154 \text{ V}$  and

$$AB = 1443 \text{ V, } OI \text{ leads } OA \text{ by } \phi$$

$$E_R = I Z_S \text{ and } \theta = \tan^{-1} (2.2 / 0.2) = 84.8^\circ$$

$BC$  is  $\perp AO$  produced.

Now,  $E_R = I Z_S = (I_1 + jI_2) (0.2 + j2.2)$

$$= (231 + jI_2) (0.2 + j2.2) = (46.2 - 2.2 I_2) + j (508.2 + 0.2 I_2)$$

Obviously,  $OC = (46.2 - 2.2 I_2)$ ;  $BC = j (508.2 + 0.2 I_2)$

From the right-angled  $\Delta ABC$ , we have

$$AB^2 = BC^2 + AC^2 = BC^2 + (AO + OC)^2$$

$$\text{or } 1443^2 = (508.2 + 0.2 I_2)^2 + (1154 + 46.2 - 2.2 I_2)^2$$

Solving the above quadratic equation, we get  $I_2 = 71 \text{ A}$

$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{231^2 + 71^2} = 242 \text{ A}$$

$$\text{p.f.} = I_1 / I = 231 / 242 = 0.95 \text{ (lead)}$$

**Example 38.21.** A 3 phase, 440-V, 50 Hz, star-connected synchronous motor takes 7.46 kW from the three phase mains. The resistance per phase of the armature winding is 0.5 ohm. The motor operates at a p. f. of 0.75 lag. Iron and mechanical losses amount to 500 watts. The excitation loss is 650 watts. Assume the source for excitation to be a separate one.

Calculate. (i) armature current, (ii) power supplied to the motor, (iii) efficiency of the motor (Amravati University 1999)

**Solution.** A 3-phase synchronous motor receives power from two sources :

(a) 3-phase a. c. source feeding power to the armature.

(b) D.C. source for the excitation, feeding electrical power only to the field winding.

Thus, power received from the d. c. source is utilized only to meet the copper-losses of the field winding.

3 Phase a.c. source feeds electrical power to the armature for following components of power:

(i) Net mechanical power output from the shaft

(ii) Copper-losses in armature winding

(iii) Friction, and armature-core-losses.

In case of the given problem

$$\sqrt{3} \times I_a \times 440 \times 0.75 = 7460$$

$$I_a = 13.052 \text{ amp}$$

$$\text{Total copper-loss in armature winding} = 3 \times 13.052^2 \times 0.50 = 255 \text{ watts}$$

$$\text{Power supplied to the motor} = 7460 + 650 = 8110 \text{ watts}$$

$$\text{efficiency of the motor} = \frac{\text{Output}}{\text{Input}}$$

$$\text{Output from shaft} = (\text{Armature Input}) - (\text{Copper losses in armature winding}) - (\text{friction and iron losses})$$

$$= 7460 - 255 - 500 = 6705 \text{ watts}$$

$$\text{Efficiency of the motor} = \frac{6705}{8110} \times 100\% = 82.7\%$$

**Example 38.22.** Consider a 3300 V delta connected synchronous motor having a synchronous reactance per phase of 18 ohm. It operates at a leading pf of 0.707 when drawing 800 kW from mains. Calculate its excitation emf and the rotor angle (= delta), explaining the latter term.  
(Elect. Machines Nagpur Univ. 1993)

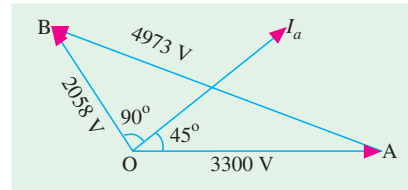


Fig. 38.38

**Solution.**  $\sqrt{3} \times 3300 \times I_a \times 0.707 = 800,000$

$\therefore$  Line current = 198 A, phase current,  $I_a = 198 / \sqrt{3} = 114.3$  A;

$$Z_s = 18 \Omega ; I_a Z_s = 114.3 \times 18 = 2058 \text{ V}$$

$$\phi = \cos^{-1} 0.707; \phi = 45^\circ; \theta = 90^\circ;$$

$$\cos(\theta + \phi) = \cos 135^\circ = -\cos 45^\circ = -0.707$$

From Fig. 38.38, we find

$$E_b^2 = 3300^2 + 2058^2 - 2 \times 3300 \times 2058 \times -0.707$$

$$\therefore E_b = 4973 \text{ V}$$

From  $\Delta OAB$ , we get  $2058/\sin \alpha = 4973/\sin 135^\circ$ . Hence,  $\alpha = 17^\circ$

**Example 38.23.** A 75-kW, 400-V, 4-pole, 3-phase star connected synchronous motor has a resistance and synchronous reactance per phase of 0.04 ohm and 0.4 ohm respectively. Compute for full-load 0.8 p.f. lead the open circuit e.m.f. per phase and mechanical power developed. Assume an efficiency of 92.5%.  
(Elect. Machines AMIE Sec. B 1991)

**Solution.** Motor input =  $75,000 / 0.925 = 81,080 \text{ W}$

$$I_a = 81,080 / (\sqrt{3} \times 400 \times 0.8) = 146.3 \text{ A}; Z_s = \sqrt{0.04^2 + 0.4^2} = 0.402 \Omega$$

$$I_a Z_s = 146.3 \times 0.402 = 58.8 \text{ V}; \tan \phi = 0.4 / 0.04 = 10 ;$$

$$\theta = 84.3^\circ; \phi = \cos^{-1} 0.8 ; \phi = 36.9^\circ; (\theta + \phi) = 121.2^\circ;$$

$$V_{ph} = 400 / \sqrt{3} = 231 \text{ V}$$

As seen from Fig. 36.39,

$$E_b^2 = 231^2 + 58.8^2 - 2 \times 231 \times 58.8 \times \cos 121.2^\circ; E_b / \text{phase} = 266 \text{ V}$$

Stator Cu loss for 3 phases =  $3 \times 146.3^2 \times 0.04 = 2570 \text{ W};$

$$N_s = 120 \times 50/40 = 1500 \text{ r.p.m.}$$

$$P_m = 81080 - 2570 = 78510 \text{ W}; T_g = 9.55 \times 78510/1500 = 500 \text{ N-m.}$$

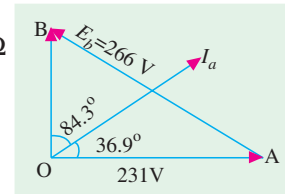


Fig. 38.39

**Example 38.24.** A 400-V, 3-phase, 50-Hz, Y-connected synchronous motor has a resistance and synchronous impedance of 0.5 ohm and 4 ohm per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is increased to 60 A, the field current remaining unchanged, calculate the gross torque developed and the new power factor.  
(Elect. Machines, AMIE Sec. B 1992)

**Solution.** The conditions corresponding to the first case are shown in Fig. 38.40.

$$\text{Voltage/phase} = 400/\sqrt{3} = 231 \text{ V}; I_a Z_s = OB = 15 \times 4 = 60 \text{ V}$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{4^2 - 0.5^2} = 3.968 \Omega$$

$$\theta = \tan^{-1} (3.968/0.5) = \tan^{-1} (7.936) = 81.8^\circ$$

$$E_b^2 = 231^2 + 60^2 - 2 \times 231 \times 60 \times \cos 81.48^\circ; E_b = 231 \text{ V}$$

It is obvious that motor is running with normal excitation because  $E_b = V$

When the motor load is increased, the phase angle between the applied voltage and the induced (or back) e.m.f. is increased. Art (38.7). The vector diagram is as shown in Fig. 38.41.

Let  $\phi$  be the new phase angle.

$$I_a Z_S = 60 \times 4 = 240 \text{ V}$$

$$\angle BOA = (81^\circ 48' - \phi).$$

Since the field current remains constant, the value of  $E_b$  remains the same.

$$\therefore 231^2 = 231^2 + 240^2 - 2 \times 231 \times 240 \cos (81^\circ 48' - \phi)$$

$$\therefore \cos (81^\circ 48' - \phi) = 0.4325 \text{ or } 81^\circ 48' - \phi = 64^\circ 24'$$

$$\therefore \phi = 81^\circ 48' - 64^\circ 24' = 17^\circ 24'. \text{ New p.f.} = \cos 17^\circ 24' = \mathbf{0.954 \text{ (lag)}}$$

$$\text{Motor input} = \sqrt{3} \times 400 \times 60 \times 0.954 = 39,660 \text{ W}$$

$$\text{Total armature Cu loss} = 3 \times 60^2 \times 0.5 = 5,400 \text{ W}$$

$$\text{Electrical power converted into mechanical power} = 39,660 - 5,400 - 34,260 \text{ W}$$

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m. } T_g = 9.55 \times 34,260/1000 = \mathbf{327 \text{ N-m}}$$

**Example 38.25.** A 400-V, 10 h.p. (7.46 kW), 3-phase synchronous motor has negligible armature resistance and a synchronous reactance of 10  $\Omega$  / phase. Determine the minimum current and the corresponding induced e.m.f. for full-load conditions. Assume an efficiency of 85%.

(A.C. Machines-I, Jadavpur Univ. 1987)

**Solution.** The current is minimum when the power factor is unity i.e., when  $\cos \phi = 1$ . The vector diagram is as shown in Fig. 38.42.

$$\text{Motor input} = 7460 / 0.85 = 8,775 \text{ W}$$

$$\text{Motor line current} = 8,775 / \sqrt{3} \times 400 \times 1 = \mathbf{12.67 \text{ A}}$$

$$\text{Impedance drop} = I_a X_S = 10 \times 12.67 = 126.7 \text{ V}$$

$$\text{Voltage / phase} = 400 / \sqrt{3} = 231 \text{ V}$$

$$E_b = \sqrt{231^2 + 126.7^2} = \mathbf{263.4 \text{ V}}$$

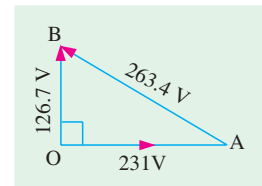


Fig. 38.42

**Example 38.26.** A 400-V, 50-Hz, 3-phase, 37.5 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is  $(0.2 + j 1.6)$  ohm per phase. If the excitation of the motor is adjusted to give a leading power factor of 0.9, calculate the following for full load :

(i) the excitation e.m.f.

(ii) the total mechanical power developed

(Elect.Machines, A.M.I.E. Sec. B, 1989)

**Solution.** Motor input =  $37.5/0.88 = 42.61 \text{ kW}$ ;  $I_a = 42,610 / \sqrt{3} \times 400 \times 0.9 = 68.3 \text{ A}$

$$V = 400 / \sqrt{3} = 231 \text{ V}; Z_S = 0.2 + j 1.6 = 1.612 \angle 82.87^\circ$$

$$E_R = I_a Z_S = 68.3 \times 1.612 = 110 \text{ V};$$

$$\phi = \cos^{-1}(0.9) = 25.84^\circ$$

$$\text{Now, } (\phi + \theta) = 25.84^\circ + 82.87^\circ = 108.71^\circ$$

$$\cos (\phi + \theta) = \cos 108.71^\circ = -0.32$$

$$\mathbf{(a) \therefore E_b^2 = V^2 + E_R^2 - 2 V E_R \cos 108.71^\circ \text{ or } E_b = 286 \text{ V}}$$

$$\text{Line value of excitation voltage} = \sqrt{3} \times 285 = \mathbf{495 \text{ V}}$$

**(b)** From  $\Delta OAB$ , (Fig. 38.43)  $E_R / \sin \alpha = E_b / \sin (\phi + \theta)$ ,  $\alpha = 21.4^\circ$

$$P_m = 3 \frac{E_b V}{Z_S} \sin \alpha = 3 \frac{286 \times 231}{1.612} \sin 21.4^\circ = \mathbf{14,954 \text{ W}}$$

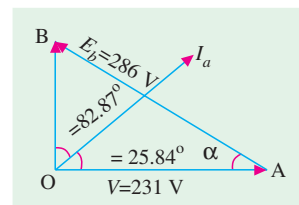


Fig. 38.43

**Example 38.27.** A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohm per phase and armature resistance negligible. When the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.

(Elect. Machines, A.M.I.E., Sec. B, 1991)

**Solution.**  $V = 6600 / \sqrt{3} = 3810 \text{ V}$ ,  $I_a = 1000 \times 10^3 / \sqrt{3} \times 6600 \times 0.8 = 109.3 \text{ A}$

The phasor diagram is shown in Fig. 38.44. Since  $R_a$  is negligible,  $\theta = 90^\circ$

$$E_R = I_a X_S = 109.3 \times 20 = 2186 \text{ V}$$

$$\cos \phi = 0.8, \phi = 36.87^\circ$$

$$E_b^2 = V^2 + E_R^2 - 2E_b V \cos (90^\circ + 36.87^\circ);$$

$$E_b = 5410 \text{ V}$$

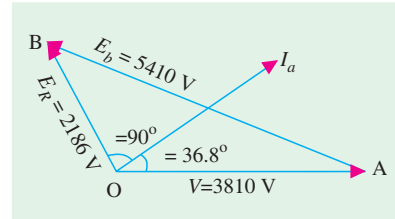


Fig. 38.44

Now, excitation has been kept constant but power has been increased to 1500 kW

$$\therefore 3 \frac{E_b V}{Z_S} \sin \alpha = P; 3 \times \frac{5410 \times 3810}{20} \sin \alpha = 1500 \times 10^3; \alpha = 29^\circ$$

Also, 
$$\frac{E_b}{(\sin 90^\circ + \phi)} = \frac{V}{\sin [180^\circ - (\alpha + 90 + \phi)]} = \frac{V}{\cos (\alpha + \phi)}$$

or 
$$\frac{E_b}{\cos \phi} = \frac{V}{\cos (\alpha + \phi)} \text{ or } \frac{V}{E_b} = \frac{\cos (29^\circ + \phi)}{\cos \phi} = 0.3521$$

$$\therefore \phi = 19.39^\circ, \cos \phi = \cos 19.39^\circ = \mathbf{0.94 \text{ (lead)}}$$

**Example 38.28.** A 400-V, 50-Hz, 6-pole, 3-phase, Y-connected synchronous motor has a synchronous reactance of 4 ohm/phase and a resistance of 0.5 ohm/phase. On full-load, the excitation is adjusted so that machine takes an armature current of 60 ampere at 0.866 p.f. leading.

Keeping the excitation unchanged, find the maximum power output. Excitation, friction, windage and iron losses total 2 kW. (Electrical Machinery-III, Bangalore Univ. 1990)

**Solution.**  $V = 400 / \sqrt{3} = 231 \text{ V/phase}$ ;  $Z_S = 0.5 + j4 = 4.03 \angle 82.9^\circ$ ;  $\theta = 82.9^\circ$

$$I_a Z_S = 60 \times 4.03 = 242 \text{ V}; \cos \phi = 0.866$$

$$\phi = 30^\circ \text{ (lead)}$$

As seen from Fig. 36.45,

$$E_b^2 = 231^2 + 242^2 - 2 \times 231 \times 242 \cos 112.9^\circ$$

$$E_b = \mathbf{394 \text{ V}}$$

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{394 \times 231}{4.03} - \frac{394^2 \times 0.5}{4.03^2}$$

$$= 17,804 \text{ W/phase.} \quad \text{—Art. 38.12}$$

Maximum power developed in armature for 3 phases

$$= 3 \times 17,804 = 52,412 \text{ W}$$

$$\text{Net output} = 52,412 - 2,000 = 50,412 \text{ W} = \mathbf{50.4 \text{ kW}}$$

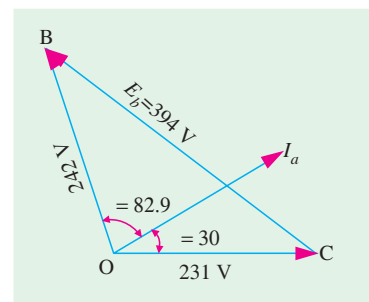


Fig. 38.45

**Example 38.29.** A 6-pole synchronous motor has an armature impedance of 10 Ω and a resistance of 0.5 Ω. When running on 2,000 volts, 25-Hz supply mains, its field excitation is such that the e.m.f. induced in the machine is 1600 V. Calculate the maximum total torque in N-m developed before the machine drops out of synchronism.

**Solution.** Assuming a three-phase motor,

$$V = 2000 \text{ V}, E_b = 1600 \text{ V}; R_a = 0.5 \text{ } \Omega; Z_s = 10 \text{ } \Omega; \cos \theta = 0.5/10 = 1/20$$

Using equation (iii) of Art. 37-10, the total max. power for 3 phases is

$$(P_m)_{max} = \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \theta = \frac{2000 \times 1600}{10} - \frac{1600^2 \times 1}{10 \times 20} = 307,200 \text{ watt}$$

Now,  $N_s = 120 f / P = 120 \times 25/6 = 500 \text{ r.p.m.}$

Let  $T_{g,max}$  be the maximum gross torque, then

$$T_{g,max} = 9.55 \times \frac{307200}{500} = \mathbf{5,868 \text{ N-m}}$$

**Example. 38.30.** A 2,000-V, 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m. The excitation is constant and corresponds to an open-circuit terminal voltage of 2,000 V. The resistance is negligible as compared with synchronous reactance of 3  $\Omega$  per phase. Determine the power input, power factor and torque developed for an armature current of 200 A.

(Elet. Engg.-I, Nagpur Univ. 1993)

**Solution.** Voltage/phase =  $2000/\sqrt{3} = 1150 \text{ V}$

Induced e.m.f. = 1150 V – given

Impedance drop =  $200 \times 3 = 600 \text{ V}$

As shown in Fig. 38.46, the armature current is assumed to lag behind V by an angle  $\phi$ . Since  $R_a$  is negligible,  $\theta = 90^\circ$ .

$$\angle BOA = (90^\circ - \phi)$$

Considering  $\Delta BOA$ , we have

$$1150^2 = 1150^2 + 600^2 - 2 \times 600 \times 1150 \cos (90 - \phi^\circ)$$

$$\sin \phi = 0.2605; \phi = 16.2^\circ; \text{p.f.} = \cos 16.2^\circ = \mathbf{0.965 \text{ (lag)}}$$

Power input =  $\sqrt{3} \times 2,000 \times 200 \times 0.965 = \mathbf{668.5 \text{ kW}}$

$$N_s = 1500 \text{ r.p.m.} \quad \therefore T_g = 9.55 \times 66,850/1500 = \mathbf{4,255 \text{ N-m.}}$$

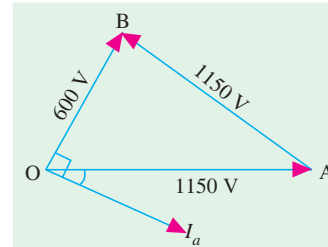


Fig. 38.46

**Example 38.31.** A 3- $\phi$ , 3300-V, Y-connected synchronous motor has an effective resistance and synchronous reactance of 2.0  $\Omega$  and 18.0  $\Omega$  per phase respectively. If the open-circuit generated e.m.f. is 3800 V between lines, calculate (i) the maximum total mechanical power that the motor can develop and (ii) the current and p.f. at the maximum mechanical power.

(Electrical Machines-III, Gujarat Univ. 1988)

**Solution.**  $\theta = \tan^{-1} (18/2) = 83.7^\circ; V_{ph} = 3300 / \sqrt{3} = 1905 \text{ V}; E_b = 3800 / \sqrt{3} = 2195 \text{ V}$

Remembering that  $\alpha = \theta$  for maximum power development (Ar. 38-10)

$$E_R = (1905^2 + 2195^2 - 2 \times 1905 \times 2195 \times \cos 83.7^\circ)^{1/2} = 2744 \text{ volt per phase}$$

$$\therefore I_a Z_s = 2,744; \text{ Now, } Z_s = \sqrt{2^2 + 18^2} = 18.11 \text{ } \Omega$$

$$\therefore I_a = 2744/18.11 = 152 \text{ A/phase; line current} = \mathbf{152 \text{ A}}$$

$$(P_m)_{max} \text{ per phase} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} = \frac{2195 \times 1905}{18.11} - \frac{2195^2 \times 2}{18.11^2}$$

$$= 230,900 - 29,380 = 201520 \text{ W per phase}$$

Maximum power for three phases that the motor can develop in its armature

$$= 201,520 \times 3 = \mathbf{604,560 \text{ W}}$$

$$\text{Total Cu losses} = 3 \times 152^2 \times 2 = 138,700 \text{ W}$$

$$\text{Motor input} = 604,560 + 138,700 = 743,260 \text{ W}$$

$$\therefore \sqrt{3} \times 3300 \times 152 \times \cos \phi = 743,260 \quad \therefore \cos \phi = \mathbf{0.855 \text{ (lead).}}$$



**Example 38.32.** The excitation of a 415-V, 3-phase, mesh-connected synchronous motor is such that the induced e.m.f. is 520 V. The impedance per phase is  $(0.5 + j4.0)$  ohm. If the friction and iron losses are constant at 1000 W, calculate the power output, line current, power factor and efficiency for maximum power output. (Elect. Machines-I, Madras Univ. 1987)

**Solution.** As seen from Art. 38-12, for fixed  $E_b$ ,  $V$ ,  $R_a$  and  $X_s$ , maximum power is developed when  $\alpha = \theta$ .

$$\text{Now, } \theta = \tan^{-1} (4/0.5) = \tan^{-1} (8) = 82.90^\circ = \alpha$$

$$E_R = \sqrt{415^2 + 520^2 - 2 \times 415 \times 520 \times \cos 82.9^\circ} = 625 \text{ V per phase}$$

$$\text{Now, } IZ_s = 625; \quad Z_s = \sqrt{4^2 + 0.5^2} = 4.03 \Omega \quad \therefore I = 625/4.03 = 155 \text{ A}$$

$$\text{Line current} = \quad \times 155 = 268.5 \text{ A}$$

$$(P_m)_{\max} = \frac{E_b V}{Z_s} - \frac{E_b^2 R_a}{Z_s^2} = \frac{520 \times 415}{4.03} - \frac{520^2 \times 0.5}{16.25} = 45,230 \text{ W}$$

$$\text{Max. power for 3 phases} = 3 \times 45,230 = 135,690 \text{ W}$$

$$\begin{aligned} \text{Power output} &= \text{power developed} - \text{iron and friction losses} \\ &= 135,690 - 1000 = 134,690 \text{ W} = \mathbf{134.69 \text{ kW}} \end{aligned}$$

$$\text{Total Cu loss} = 3 \times 155^2 \times 0.5 = 36,080 \text{ W}$$

$$\text{Total motor input} = 135,690 + 36,080 = 171,770 \text{ W}$$

$$\therefore \sqrt{3} \times 415 \times 268.5 \times \cos \phi = 171,770; \quad \cos \phi = \mathbf{0.89 \text{ (lead)}}$$

$$\text{Efficiency} = 134,690/171,770 = 0.7845 \text{ or } \mathbf{78.45\%}$$

### Tutorial Problems 38.1

- A 3-phase, 400-V, synchronous motor takes 52.5 A at a power factor of 0.8 leading. Calculate the power supplied and the induced e.m.f. The motor impedance per phase is  $(0.25 + j3.2)$  ohm. [29.1 kW; 670V]
- The input to a 11-kV, 3 $\phi$ , Y-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are  $1 \Omega$  and  $30 \Omega$  respectively. Find (a) power supplied to the motor and (b) the induced e.m.f. for a p.f. of 0.8 leading. [(a) 915 kW (b) 13kV] (Grad. I.E.T.E. Dec. 1978)
- A 2,200-V, 3-phase, star-connected synchronous motor has a resistance of  $0.6 \Omega$  and a synchronous reactance of  $6 \Omega$ . Find the generated e.m.f. and the angular retardation of the motor when the input is 200 kW at (a) power factor unity and (b) power factor 0.8 leading. [(a) 2.21 kV; 14.3° (b) 2.62 kV; 12.8°]
- A 3-phase, 220-V, 50-Hz, 1500 r.p.m., mesh-connected synchronous motor has a synchronous impedance of 4 ohm per phase. It receives an input line current of 30 A at a leading power factor of 0.8. Find the line value of the induced e.m.f. and the load angle expressed in mechanical degrees. If the mechanical load is thrown off without change of excitation, determine the magnitude of the current under the new conditions. Neglect losses. [268 V; 6°, 20.6 A]
- A 400-V, 3-phase, Y-connected synchronous motor takes 3.73 kW at normal voltage and has an impedance of  $(1 + j8)$  ohm per phase. Calculate the current and p.f. if the induced e.m.f. is 460 V. [6.28 A; 0.86 lead] (Electrical Engineering, Madras Univ. April 1979)
- The input to 6600-V, 3-phase, star-connected synchronous motor is 900 kW. The synchronous reactance per phase is  $20 \Omega$  and the effective resistance is negligible. If the generated voltage is 8,900 V

(line), calculate the motor current and its power factor.

[Hint. See solved Ex. 38.17 ] (*Electrotechnics, M.S. Univ. April 1979*)

7. A 3-phase synchronous motor connected to 6,600-V mains has a star-connected armature with an impedance of  $(2.5 + j15)$  ohm per phase. The excitation of machine gives 7000 V. The iron, friction and excitation losses are 12 kW. Find the maximum output of the motor. [153.68 kW]
8. A 3300-V, 3-phase, 50-Hz, star-connected synchronous motor has a synchronous impedance of  $(2 + j15)$  ohm. Operating with an excitation corresponding to an e.m.f. of 2,500 V between lines, it just falls out of step at full-load. To what open-circuit e.m.f. will it have to be excited to stand a 50% excess torque. [4 kV]
9. A 6.6 kV, star-connected, 3-phase, synchronous motor works at constant voltage and constant excitation. Its synchronous impedance is  $(2.0 + j 20)$  per phase. When the input is 1000 kW, its power factor is 0.8 leading. Find the power factor when the input is increased to 1500 kW (solve graphically or otherwise).  
[0.925 lead] (*AMIE Sec. B Advanced Elect. Machines (E-9) Summer 1991*)
10. A 2200-V, 373 kW, 3-phase, star-connected synchronous motor has a resistance of  $0.3 \Omega$  and a synchronous reactance of  $3.0 \Omega$  per phase respectively. Determine the induced e.m.f. per phase if the motor works on full-load with an efficiency of 94 per cent and a p.f. of 0.8 leading.  
[1510 V] (*Electrical Machinery, Mysore Univ. 1992*)
11. The synchronous reactance per phase of a 3-phase star-connected 6600 V synchronous motor is  $20 \Omega$ . For a certain load, the input is 915 kW at normal voltage and the induced line e.m.f. is 8,942 V. Evaluate the line current and the p.f. Neglect resistance. [97 A; 0.8258 (lead)]
12. A synchronous motor has an equivalent armature reactance of  $3.3 \Omega$ . The exciting current is adjusted to such a value that the generated e.m.f. is 950 V. Find the power factor at which the motor would operate when taking 80 kW from a 800-V supply mains. [0.965 leading] (*City & Guilds, London*)
13. The input to an 11000 V, 3-phase star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohms. Find the power supplied to the motor and the induced electromotive force for a power factor of 0.8 leading.  
[914.5 kW, 13 kV] (*Elect. Machines, A.M.I.E. Sec. B, 1990*)
14. A 400-V, 6-pole, 3-phase, 50-Hz, star-connected synchronous motor has a resistance and synchronous reactance of 0.5 ohm per phase and 4-ohm per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 60 A, the field current remaining unchanged, find the gross torque developed, and the new power factor. [354 Nm; 0.93] (*Elect. Engg. AMIETE Dec. 1990*)
15. The input to a 11,000-V, 3-phase, star-connected synchronous motor is 60 amperes. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find the power supplied to the motor and the induced e.m.f. for power factor of 0.8 (a) leading and (b) lagging.  
[915 kW (a) 13 kV (b) 9.36 kV] (*Elect. Machines-II, South Gujarat Univ. 1981*)
16. Describe with the aid of a phasor diagram the behaviour of a synchronous motor starting from no-load to the pull-out point.  
What is the output corresponding to a maximum input to a 3- $\phi$  delta-connected 250-V, 14.92 kW synchronous motor when the generated e.m.f. is 320 V ? The effective resistance and synchronous reactance per phase are  $0.3 \Omega$  and  $4.5 \Omega$  respectively. The friction, windage, iron and excitation losses total 800 watts and are assumed to remain constant. Give values for (i) output (ii) line current (iii) p.f.  
[(i) 47.52 kW (ii) 161 A (iii) 0.804] (*Elect. Machines, Indore Univ. Feb. 1982*)
17. A synchronous motor takes 25 kW from 400 V supply mains. The synchronous reactance of the motor is 4 ohms. Calculate the power factor at which the motor would operate when the field excitation is so adjusted that the generated EMF is 500 volts.  
[ 0.666 Leading] (*Rajiv Gandhi Technical University, Bhopal, 2000*)

### 38.17. Effect of Excitation on Armature Current and Power Factor

The value of excitation for which back e.m.f.  $E_b$  is equal (in magnitude) to applied voltage  $V$  is known as 100% excitation. We will now discuss what happens when motor is either over-excited or under-excited although we have already touched this point in Art. 38-8.

Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected).

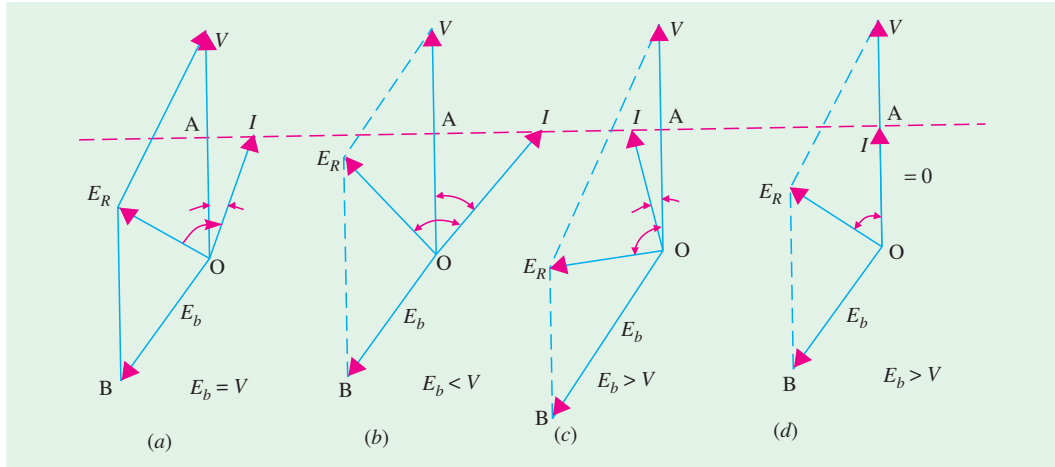


Fig. 38.47

Fig. 38.47 (a) shows the case for 100% excitation *i.e.*, when  $E_b = V$ . The armature current  $I$  lags behind  $V$  by a small angle  $\phi$ . Its angle  $\theta$  with  $E_R$  is fixed by stator constants *i.e.*  $\tan \theta = X_S / R_a$ .

In Fig. 38.47 (b)\* excitation is less than 100% *i.e.*,  $E_b < V$ . Here,  $E_R$  is advanced clockwise and so is armature current (because it lags behind  $E_R$  by fixed angle  $\theta$ ). We note that the magnitude of  $I$  is increased but its power factor is decreased ( $\phi$  has increased). Because input as well as  $V$  are constant, hence the power component of  $I$  *i.e.*,  $I \cos \phi$  remains the same as before, but wattless component  $I \sin \phi$  is increased. Hence, as excitation is decreased,  $I$  will increase but p.f. will decrease so that power component of  $I$  *i.e.*,  $I \cos \phi = OA$  will remain constant. In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.

Incidentally, it may be noted that when field current is reduced, the motor pull-out torque is also reduced in proportion.

Fig. 38.47 (c) represents the condition for overexcited motor *i.e.* when  $E_b > V$ . Here, the resultant voltage vector  $E_R$  is pulled anticlockwise and so is  $I$ . It is seen that now motor is drawing a leading current. It may also happen for some value of excitation, that  $I$  may be in phase with  $V$  *i.e.*, p.f. is unity [Fig. 38.47 (d)]. At that time, the current drawn by the motor would be **minimum**.

Two important points stand out clearly from the above discussion :

- (i) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of  $I$  with excitation are shown in Fig. 38.48 (a) which are known as 'V' curves because of their shape.
- (ii) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly. When over-excited, motor runs with leading p.f. and with lagging p.f. when under-excited. In between, the p.f. is unity. The variations of *p.f.* with excitation

\* These are the same diagrams as given in Fig. 38.7 and 8 except that vector for  $V$  has been shown vertical.

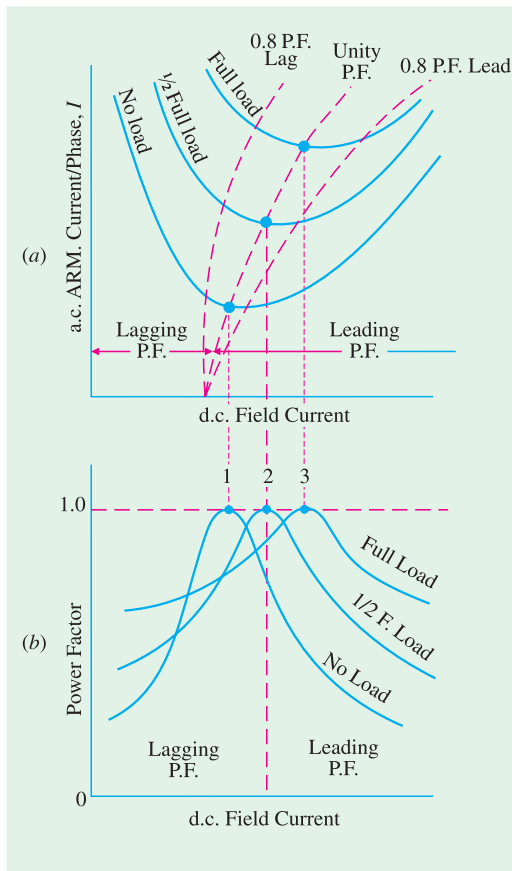


Fig. 38.48



Inductor motor

are shown in Fig. 38.48 (b). The curve for p.f. looks like inverted 'V' curve. It would be noted that **minimum armature current corresponds to unity power factor**.

It is seen (and it was pointed out in Art. 38.1) that an over-excited motor can be run with leading power factor. This property of the motor renders it extremely useful for phase advancing (and so power factor correcting) purposes in the case of industrial loads driven by induction motors (Fig. 38.49) and lighting and heating loads supplied through transformers. Both transformers and induction motors draw lagging currents from the line. Especially on light loads, the power drawn by them has a large reactive component and the power factor has a very low value. This reactive component, though essential for operating the electric machinery, entails appreciable loss in many ways. By using synchronous motors in conjunction with induction motors and transformers, the lagging reactive power required by the

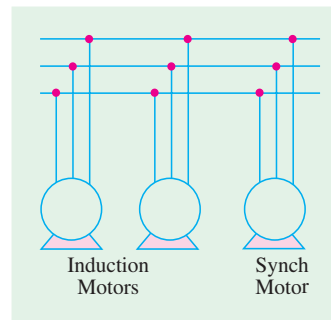


Fig. 38.49

latter is supplied locally by the leading reactive component taken by the former, thereby relieving the line and generators of much of the reactive component. Hence, they now supply only the active component of the load current. When used in this way, a synchronous motor is called a **synchronous capacitor**, because it draws, like a capacitor, leading current from the line. Most synchronous capacitors are rated between 20 MVAR and 200 MVAR and many are hydrogen-cooled.

**Example 38.33.** Describe briefly the effect of varying excitation upon the armature current and p.f. of a synchronous motor when input power to the motor is maintained constant.

A 400-V, 50-Hz, 3- $\phi$ , 37.3 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is  $(0.2 + j 1.6) \Omega$  per phase. If the excitation of the motor is adjusted to give a leading p.f. of 0.9, calculate for full-load (a) the induced e.m.f. (b) the total mechanical power developed.

**Solution.** Voltage / phase =  $400 / \sqrt{3} = 231 \text{ V}$ ;

$$Z_S = \sqrt{(1.6^2 + 0.2^2)} = 1.61 \Omega;$$

$$\text{Full-load current} = \frac{37,300}{\sqrt{3} \times 400 \times 0.88 \times 0.9} = 68 \text{ A}$$

$$\therefore IZ_S = 1.61 \times 68 = 109.5 \text{ V}$$

With reference to Fig. 38.50

$$\tan \theta = 1.6 / 0.2 = 8, \theta = 82^\circ 54'$$

$$\cos \phi = 0.9, \phi = 25^\circ 50'$$

$$\therefore (\theta + \phi) = 82^\circ 54' + 25^\circ 50' = 108^\circ 44'$$

$$\text{Now } \cos 108^\circ 44' = -0.3212$$

$$(a) \text{ In } \triangle OAB, E_b^2 = 231^2 + 109.5^2 - 2 \times 231 \times 109.5 \times (-0.3212) = 285.6^2; E_b = 285.6 \text{ V}$$

$$\text{Line value of } E_b = \sqrt{3} \times 285.6 = 495 \text{ V}$$

$$(b) \text{ Total motor input} = 37,300 / 0.88 = 42,380 \text{ W}$$

$$\text{Total Cu losses} = 3 \times I^2 R_a = 3 \times 68^2 \times 0.2 = 2,774 \text{ W}$$

$$\therefore \text{Electric power converted into mechanical power} = 42,380 - 2,774 = 39.3 \text{ kW.}$$

**Example 38.34.** A 3- $\phi$ , star-connected synchronous motor takes 48 kW at 693 V, the power factor being 0.8 lagging. The induced e.m.f. is increased by 30%, the power taken remaining the same. Find the current and the p.f. The machine has a synchronous reactance of 2  $\Omega$  per phase and negligible resistance.

**Solution.** Full-load current

$$= 48,000 / \sqrt{3} \times 693 \times 0.8 = 50 \text{ A}$$

$$\text{Voltage/phase} = 693 / \sqrt{3} = 400 \text{ V}$$

$$Z_S = X_S = 2 \Omega \quad \therefore IZ_S = 50 \times 2 = 100 \text{ V}$$

$$\tan \theta = 2/0 = \infty \quad \therefore \theta = 90^\circ; \cos \phi = 0.8, \sin \phi = 0.6$$

The vector diagram is shown in Fig. 38.51. In  $\triangle OAB$ ,

$$E_b^2 = 400^2 + 100^2 - 2 \times 400 \times 100 \times \cos (90^\circ - \phi)$$

$$= 400^2 + 100^2 - 2 \times 400 \times 100 \times 0.6 = 349^2 \quad \therefore E_b = 349 \text{ V.}$$

The vector diagram for increased e.m.f. is shown in Fig. 38.52. Now,  $E_b = 1.3 \times 349 = 454 \text{ V}$ . It can be safely assumed that in the second case, current is leading  $V$  by some angle  $\phi'$ .

Let the new current and the leading angle of current by  $I'$  and  $\phi'$  respectively. As power input remains the same and  $V$  is also constant,  $I \cos \phi$  should be the same for the same input.

$$\therefore I \cos \phi = 50 \times 0.8 = 40 = I' \cos \phi'$$

$$\text{In } \triangle ABC, AB^2 = AC^2 + BC^2$$

$$\text{Now } BC = I' X_S \cos \phi' \quad (\because OB = I' X_S)$$

$$= 40 \times 2 = 80 \text{ V}$$

$$\therefore 454^2 = 400^2 + 80^2 \text{ or } AC = 447 \text{ V}$$

$$\therefore OC = 447 - 400 = 47 \text{ V}$$

$$\therefore \tan \phi' = 47/80, \phi' = 30^\circ 26'$$

$$\therefore \text{New p.f.} = \cos 30^\circ 26' = 0.8623 \text{ (leading)}$$

$$\text{Also, } I' \cos \phi' = 40 \quad \therefore I' = 40/0.8623 = 46.4 \text{ A.}$$

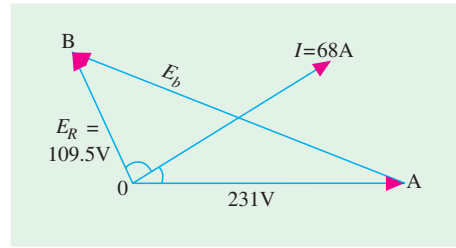


Fig. 38.50

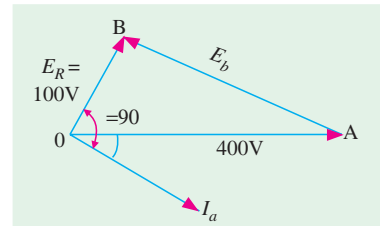


Fig. 38.51

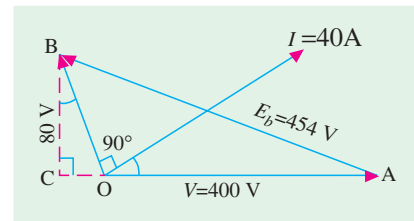


Fig. 38.52

**Example 38.35.** A synchronous motor absorbing 60 kW is connected in parallel with a factory load of 240 kW having a lagging p.f. of 0.8. If the combined load has a p.f. of 0.9, what is the value of the leading kVAR supplied by the motor and at what p.f. is it working ?

(Electrical Engineering-II, Bangalore Univ. 1990)

**Solution.** Load connections and phase relationships are shown in Fig. 38.53.

$$\text{Total load} = 240 + 60 = 300 \text{ kW; combined p.f.} = 0.9 \text{ (lag)}$$

$$\phi = 25.8^\circ, \tan \phi = 0.4834, \text{ combined kVAR} = 300 \times 0.4834 = 145 \text{ (lag)}$$

**Factory Load**

$$\cos \phi_L = 0.8, \phi_L = 36.9^\circ, \tan \phi_L = 0.75, \text{ load kVAR} = 240 \times 0.75 = 180 \text{ (lag)}$$

or  $\text{load kVA} = 240/0.8 = 300, \text{ kVAR} = 300 \times \sin \phi_L = 300 \times 0.6 = 180$

$\therefore$  leading kVAR supplied by synchronous motor =  $180 - 145 = 35$ .

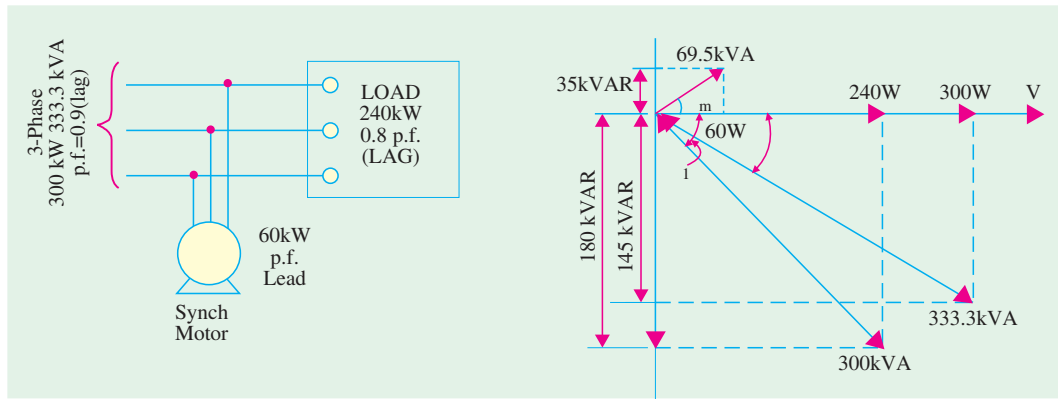


Fig. 38.53

**For Synchronous Motor**

$$\text{kW} = 60, \text{ leading kVAR} = 35, \tan \phi_m = 35/60; \phi_m = 30.3^\circ; \cos 30.3^\circ = 0.863$$

$\therefore$  motor p.f. = **0.863 (lead)**. Incidentally, motor kVA =  $\sqrt{60^2 + 35^2} = 69.5$ .

**38.18. Constant-power Lines**

In Fig. 38.54,  $OA$  represents applied voltage / phase of the motor and  $AB$  is the back e.m.f. / phase,  $E_b$ .  $OB$  is their resultant voltage  $E_R$ . The armature current is  $OI$  lagging behind  $E_R$  by an angle  $\theta = \tan^{-1} X_S / R_a$ . Value of  $I = E_R / Z_S$ . Since  $Z_S$  is constant,  $E_R$  or vector  $OB$  represents (to some suitable scale) the main current  $I$ .  $OY$  is drawn at an angle  $\phi$  with  $OB$  (or at an angle  $\theta$  with  $CA$ ).  $BL$  is drawn perpendicular to  $OY$  which is at right angles to  $OY$ . Vector  $OB$ , when referred to  $OY$ , also represents, on a different scale, the current both in magnitude and phase.

Hence,  $OB \cos \phi = I \cos \phi = BL$

The power input / phase of the motor  
 $= VI \cos \phi = V \times BL$ .

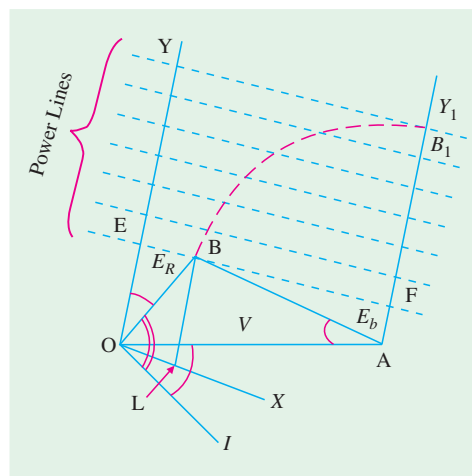


Fig. 38.54

As  $V$  is constant, power input is dependent on  $BL$ . If motor is working with a constant intake, then locus of  $B$  is a straight line  $\parallel$  to  $OX$  and  $\perp$  to  $OY$  i.e. line  $EF$  for which  $BL$  is constant. Hence,  $EF$ , represents a constant-power input line for a given voltage but varying excitation. Similarly, a series of such parallel lines can be drawn each representing a definite power intake of the motor. As regards these constant-power lines, it is to be noted that

1. for equal increase in intake, the power lines are parallel and equally-spaced
2. zero power line runs along  $OX$
3. the perpendicular distance from  $B$  to  $OX$  (or zero power line) represents the motor intake
4. If excitation is fixed i.e.  $AB$  is constant in length, then as the load on motor is increased, increases. In other words, locus of  $B$  is a circle with radius =  $AB$  and centre at  $A$ . With increasing load,  $B$  goes on to lines of higher power till point  $B_1$  is reached. Any further increase in load on the motor will bring point  $B$  down to a lower line. It means that as load increases beyond the value corresponding to point  $B_1$ , the motor intake decreases which is impossible. The area to the right of  $AY_1$  represents unstable conditions. For a given voltage and excitation, the maximum power the motor can develop, is determined by the location of point  $B_1$  beyond which the motor pulls out of synchronism.

### 38.19. Construction of V-curves

The V-curves of a synchronous motor show how armature current varies with its field current when motor **input is kept constant**. These are obtained by plotting a.c. armature current against d.c. field current while motor input is kept constant and are so called because of their shape (Fig. 38.55). There is a family of such curves, each corresponding to a definite power intake.

In order to draw these curves experimentally, the motor is run from constant voltage and constant-frequency bus-bars. Power input to motor is kept constant at a definite value. Next, field current is increased in small steps and corresponding armature currents are noted. When plotted, we get a V-curve for a particular constant motor input. Similar curves can be drawn by keeping motor input constant at different values. A family of such curves is shown in Fig. 38.55.

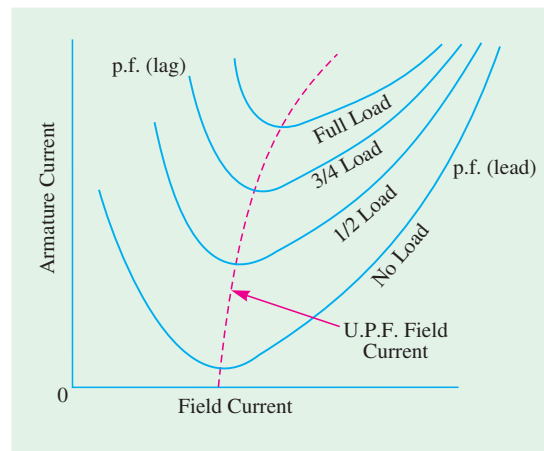


Fig. 38.55

Detailed procedure for graphic construction of V-curves is given below :

1. First, constant-power lines are drawn as discussed in Art. 38.14.
2. Then, with  $A$  as the centre, concentric circles of different radii  $AB, AB_1, AB_2$ , etc. are drawn where  $AB, AB_1, AB_2$ , etc., are the back e.m.fs corresponding to different excitations. The intersections of these circles with lines of constant power give positions of the working points for specific loads and excitations (hence back e.m.fs). The vectors  $OB, OB_1, OB_2$  etc., represent different values of  $E_R$  (and hence currents) for different excitations. Back e.m.f. vectors  $AB, AB_1$  etc., have not been drawn purposely in order to avoid confusion (Fig. 38.56).

3. The different values of back e.m.fs like  $A B$ ,  $A B_1$ ,  $A B_2$ , etc., are projected on the magnetisation and corresponding values of the field (or exciting) amperes are read from it.
4. The field amperes are plotted against the corresponding armature currents, giving us 'V' curves.

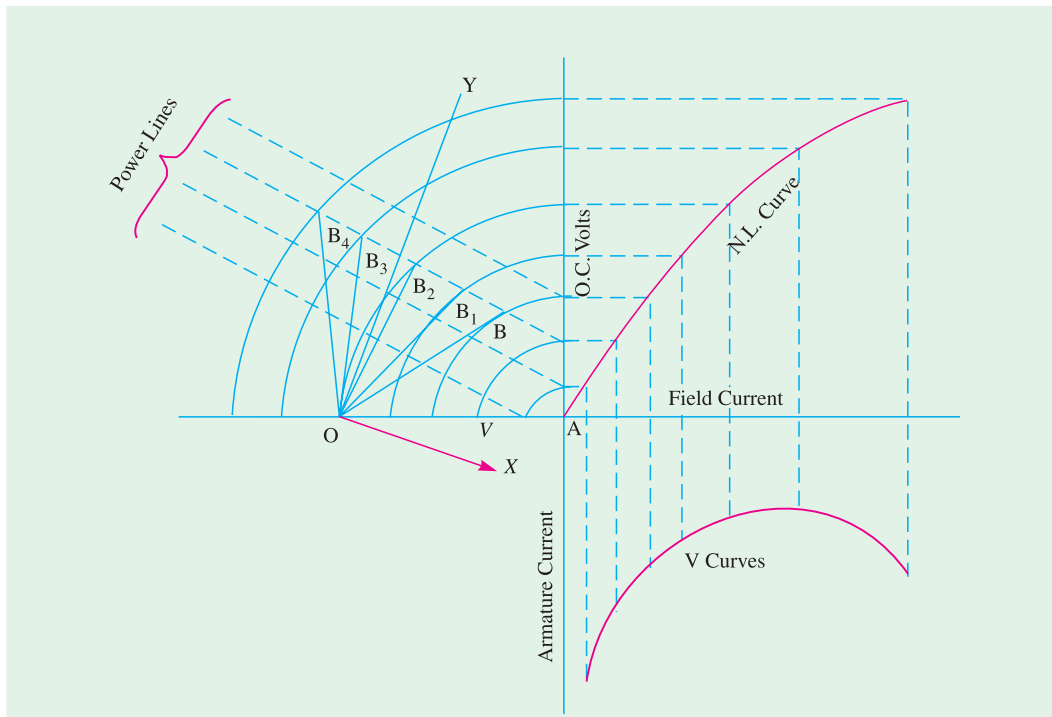


Fig. 38.56

### 38.20. Hunting or Surging or Phase Swinging

When a synchronous motor is used for driving a varying load, then a condition known as hunting is produced. Hunting may also be caused if supply frequency is pulsating (as in the case of generators driven by reciprocating internal combustion engines).

We know that when a synchronous motor is loaded (such as punch presses, shears, compressors and pumps etc.), its rotor falls back in phase by the coupling angle  $\alpha$ . As load is progressively increased, this angle also increases so as to produce more torque for coping with the increased load. If now, there is sudden decrease in the motor load, the motor is immediately pulled up or advanced to a new value of  $\alpha$  corresponding to the new load. But in this process, the rotor overshoots and hence is again pulled back. In this way, the rotor starts oscillating (like a pendulum) about its new position of

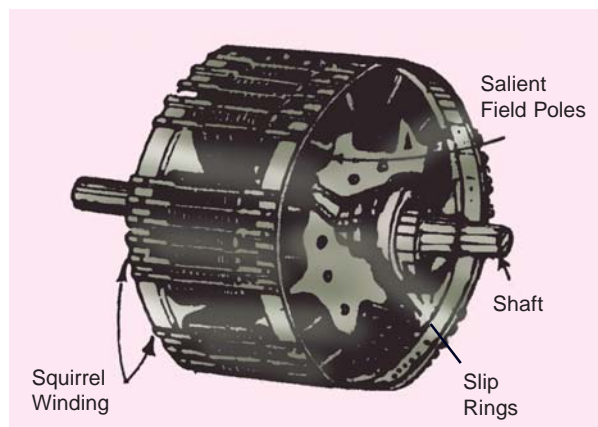
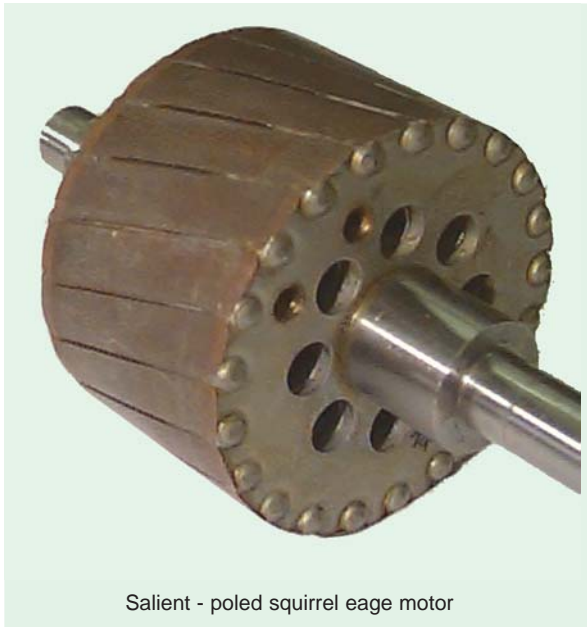


Fig. 38.57





Salient - poled squirrel cage motor

equilibrium corresponding to the new load. If the time period of these oscillations happens to be equal to the natural time period of the machine (refer Art. 37.36) then mechanical resonance is set up. The amplitude of these oscillations is built up to a large value and may eventually become so great as to throw the machine out of synchronism. To stop the build-up of these oscillations, dampers or damping grids (also known as squirrel-cage winding) are employed. These dampers consist of short-circuited Cu bars embedded in the faces of the field poles of the motor (Fig. 38.57). The oscillatory motion of the rotor sets up eddy currents in the dampers which flow in such a way as to suppress these oscillations.

But it should be clearly understood that dampers do not completely prevent hunt-

ing because their operation depends upon the presence of some oscillatory motion. However, they serve the additional purpose of making the synchronous motor self-starting.

### 38.21. Methods of Starting

As said above, almost all synchronous motors are equipped with dampers or squirrel cage windings consisting of Cu bars embedded in the pole-shoes and short-circuited at both ends. Such a motor starts readily, acting as an induction motor during the starting period. The procedure is as follows :

The line voltage is applied to the armature (stator) terminals and the field circuit is left *unexcited*. Motor starts as an induction motor and while it reaches nearly 95% of its synchronous speed, the d.c. field is excited. At that moment the stator and rotor poles get engaged or interlocked with each other and hence pull the motor into synchronism.

However, two points should be noted :

1. At the beginning, when voltage is applied, the rotor is stationary. The rotating field of the stator winding induces a very large e.m.f. in the rotor during the starting period, though the value of this e.m.f. goes on decreasing as the rotor gathers speed.

Normally, the field windings are meant for 110-V (or 250 V for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.

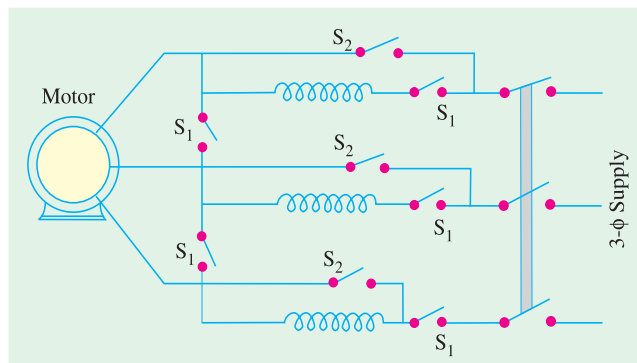


Fig. 38.58

2. When full line voltage is switched on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor. In some cases, this may not be objectionable but where it is, the applied voltage at starting, is reduced by using auto-transformers (Fig. 38.58). However, the voltage should not be reduced to a very low value because the starting torque of an induction motor varies approximately as the square of the applied voltage. Usually, a value of 50% to 80% of the full-line voltage is satisfactory. Auto-transformer connections are shown in Fig. 38.58. For reducing the supply voltage, the switches  $S_1$  are closed and  $S_2$  are kept open. When the motor has been speeded-up,  $S_2$  are closed and  $S_1$  opened to cut out the transformers.

### 38.22. Procedure for Starting a Synchronous Motor

While starting a modern synchronous motor provided with damper windings, following procedure is adopted.

1. First, main field winding is short-circuited.
2. Reduced voltage with the help of auto-transformers is applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak d.c. excitation is applied by removing the short-circuit on the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the auto-transformers.
5. The motor may be operated at any desired power factor by changing the d.c. excitation.

### 38.23. Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.
4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

### 38.24. Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation

#### (a) Power factor correction

Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors (Fig. 38.49) and other devices having lagging p.f. such as welders and fluorescent lights etc.

#### (b) Constant-speed applications

Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc.

Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

#### (c) Voltage regulation

The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor with a field regulator (for varying its excitation), this voltage rise can be controlled.

When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.

### QUESTIONS AND ANSWERS ON SYNCHRONOUS MOTORS

**Q. 1. Does change in excitation affect the synchronous motor speed ?**

**Ans.** No.

**Q. 2. The power factor ?**

**Ans.** Yes.

**Q. 3. How ?**

**Ans.** When over-excited, synchronous motor has leading power factor. However, when underexcited, it has lagging power factor.

**Q. 4. For what service are synchronous motors especially suited ?**

**Ans.** For high voltage service.

**Q. 5. Which has more efficiency; synchronous or induction motor ?**

**Ans.** Synchronous motor.

**Q. 6. Mention some specific applications of synchronous motor ?**

**Ans.** 1. constant speed load service      2. reciprocating compressor drives  
3. power factor correction              4. voltage regulation of transmission lines.

**Q. 7. What is a synchronous capacitor ?**

**Ans.** An overexcited synchronous motor is called synchronous capacitor, because, like a capacitor, it takes a leading current.

**Q. 8. What are the causes of faulty starting of a synchronous motor ?**

**Ans.** It could be due to the following causes :

1. voltage may be too low – at least half voltage is required for starting
2. there may be open-circuit in one phase – due to which motor may heat up
3. static friction may be large – either due to high belt tension or too tight bearings
4. stator windings may be incorrectly connected
5. field excitation may be too strong.

**Q. 9. What could be the reasons if a synchronous motor fails to start ?**

**Ans.** It is usually due to the following reasons :

1. voltage may be too low
2. some faulty connection in auxiliary apparatus
3. too much starting load
4. open-circuit in one phase or short-circuit
5. field excitation may be excessive.

**Q. 10. A synchronous motor starts as usual but fails to develop its full torque. What could it be due to ?**

**Ans.** 1. exciter voltage may be too low 2. field spool may be reversed 3. there may be either open-circuit or short-circuit in the field.

**Q. 11. Will the motor start with the field excited ?**

**Ans.** No.

**Q. 12. Under which conditions a synchronous motor will fail to pull into step ?**

**Ans.** 1. no field excitation 2. excessive load 3. excessive load inertia

**OBJECTIVE TESTS – 38**

1. In a synchronous motor, damper winding is provided in order to
  - (a) stabilize rotor motion
  - (b) suppress rotor oscillations
  - (c) develop necessary starting torque
  - (d) both (b) and (c)
2. In a synchronous motor, the magnitude of stator back e.m.f.  $E_b$  depends on
  - (a) speed of the motor
  - (b) load on the motor
  - (c) both the speed and rotor flux
  - (d) d.c. excitation only
3. An electric motor in which both the rotor and stator fields rotates with the same speed is called a/an .....motor.
  - (a) d.c.
  - (b) chrage
  - (c) synchronous
  - (d) universal
4. While running, a synchronous motor is compelled to run at synchronous speed because of
  - (a) damper winding in its pole faces
  - (b) magnetic locking between stator and rotor poles
  - (c) induced e.m.f. in rotor field winding by stator flux
  - (d) compulsion due to Lenz's law
5. The direction of rotation of a synchronous motor can be reversed by reversing
  - (a) current to the field winding
  - (b) supply phase sequence
  - (c) polarity of rotor poles
  - (d) none of the above

6. When running under no-load condition and with normal excitation, armature current  $I_a$  drawn by a synchronous motor
- leads the back e.m.f.  $E_b$  by a small angle
  - is large
  - lags the applied voltage  $V$  by a small angle
  - lags the resultant voltage  $E_R$  by  $90^\circ$ .
7. The angle between the synchronously-rotating stator flux and rotor poles of a synchronous motor is called..... angle.
- synchronizing
  - torque
  - power factor
  - slip
8. If load angle of a 4-pole synchronous motor is  $8^\circ$  (elect), its value in mechanical degrees is .....
- 4
  - 2
  - 0.5
  - 0.25
9. The maximum value of torque angle  $\alpha$  in a synchronous motor is .....degrees electrical.
- 45
  - 90
  - between 45 and 90
  - below 60
10. A synchronous motor running with normal excitation adjusts to load increases *essentially* by increase in its
- power factor
  - torque angle
  - back e.m.f.
  - armature current.
11. When load on a synchronous motor running with normal excitation is increased, armature current drawn by it increases because
- back e.m.f.  $E_b$  becomes less than applied voltage  $V$
  - power factor is decreased
  - net resultant voltage  $E_R$  in armature is increased
  - motor speed is reduced
12. When load on a normally-excited synchronous motor is increased, its power factor tends to
- approach unity
  - become increasingly lagging
  - become increasingly leading
  - remain unchanged.
13. The effect of increasing load on a synchronous motor running with normal excitation is to
- increase both its  $I_a$  and p.f.
  - decrease  $I_a$  but increase p.f.
  - increase  $I_a$  but decrease p.f.
  - decrease both  $I_a$  and p.f.
14. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is increased, its torque angle must necessarily
- decrease
  - increase
  - remain constant
  - become twice the no-load value.
15. If the field of a synchronous motor is under-excited, the power factor will be
- lagging
  - leading
  - unity
  - more than unity
16. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is decreased from its normal value, it leads to
- increase in  $I_a$  but decrease in  $E_b$
  - increase in  $E_b$  but decrease in  $I_a$
  - increase in both  $I_a$  and p.f. which is lagging
  - increase in both  $I_a$  and  $\phi$
17. A synchronous motor connected to infinite bus-bars has at *constant* full-load, 100% excitation and unity p.f. On changing the excitation only, the armature current will have
- leading p.f. with under-excitation
  - leading p.f. with over-excitation
  - lagging p.f. with over-excitation
  - no change of p.f.
- (Power App.-II, Delhi Univ. Jan 1987)**
18. The V-curves of a synchronous motor show relationship between
- excitation current and back e.m.f.
  - field current and p.f.
  - d.c. field current and a.c. armature current
  - armature current and supply voltage.

19. When load on a synchronous motor is increased, its armature currents is increased provided it is  
 (a) normally-excited  
 (b) over-excited  
 (c) under-excited  
 (d) all of the above
20. If main field current of a salient-pole synchronous motor fed from an infinite bus and running at no-load is reduced to zero, it would  
 (a) come to a stop  
 (b) continue running at synchronous speed  
 (c) run at sub-synchronous speed  
 (d) run at super-synchronous speed
21. In a synchronous machine when the rotor speed becomes more than the synchronous speed during hunting, the damping bars develop  
 (a) synchronous motor torque  
 (b) d.c. motor torque  
 (c) induction motor torque  
 (d) induction generator torque  
 (Power App.-II, Delhi Univ. Jan. 1987)
22. In a synchronous motor, the rotor Cu losses are met by  
 (a) motor input  
 (b) armature input  
 (c) supply lines  
 (d) d.c. source
23. A synchronous machine is called a doubly-excited machine because  
 (a) it can be overexcited  
 (b) it has two sets of rotor poles  
 (c) both its rotor and stator are excited  
 (d) it needs twice the normal exciting current.
24. Synchronous capacitor is  
 (a) an ordinary static capacitor bank  
 (b) an over-excited synchronous motor driving mechanical load  
 (c) an over-excited synchronous motor running without mechanical load  
 (d) none of the above 623  
 (Elect. Machines, A.M.I.E. Sec. B, 1993)

**ANSWERS**

1. d    2. d    3. c    4. b    5. b    6. c    7. b    8. a    9. b    10. d    11. c  
 12. b    13. c    14. a    15. a    16. d    17. b    18. c    19. d    20. b    21. d    22. d  
 23. c    24. c